

Universal laws and architectures:

Theory and lessons from

grids, brains, bugs, nets,
planes, docs, fire, bodies, fashion,
earthquakes, turbulence, music, buildings, cities,
art, running, throwing, **Synesthesia**, spacecraft, statistical mechanics

“plug and play”

John Doyle 道陽

Jean-Lou Chameau Professor

Control and Dynamical Systems, EE, & BioE

Ca#1tech

Universal laws and architectures:

Theory and lessons from

grids, brains, bugs, nets,

fire, bodies, fashion,

intelligence, music, buildings, cities,

anesthesia, space, statistical mechanics

**Few words are
more misused
and confused**

“plug and play”

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Control and Dynamical Systems, EE, & BioE

Ca#1tech

Efficiency and robustness in sustainable infrastructure

- Accept previous talks as givens
 - New efficiencies and instability/fragility
 - Needs distributed/layered/complex/active control
- What could go wrong (longterm)?
- How to fix/avoid (technical) problems.
- Persistent errors/confusion in science & engineering

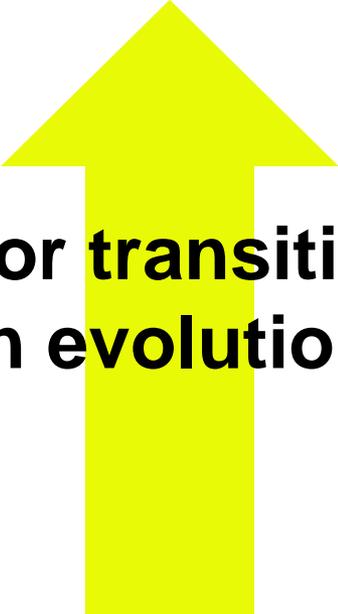
Efficiency and robustness

- **Efficient** use of resources
 - Less inertia and damping \Rightarrow instability
- **Robustness** on all scales
 - **Fluctuations** in supply and demand
 - **Component** uncertainty and failure*
 - **Adaptability** to large changes*
 - **Evolvability** on long time scales*

* Aspects of “plug and play” modularity

Efficiency/instability/layers/feedback

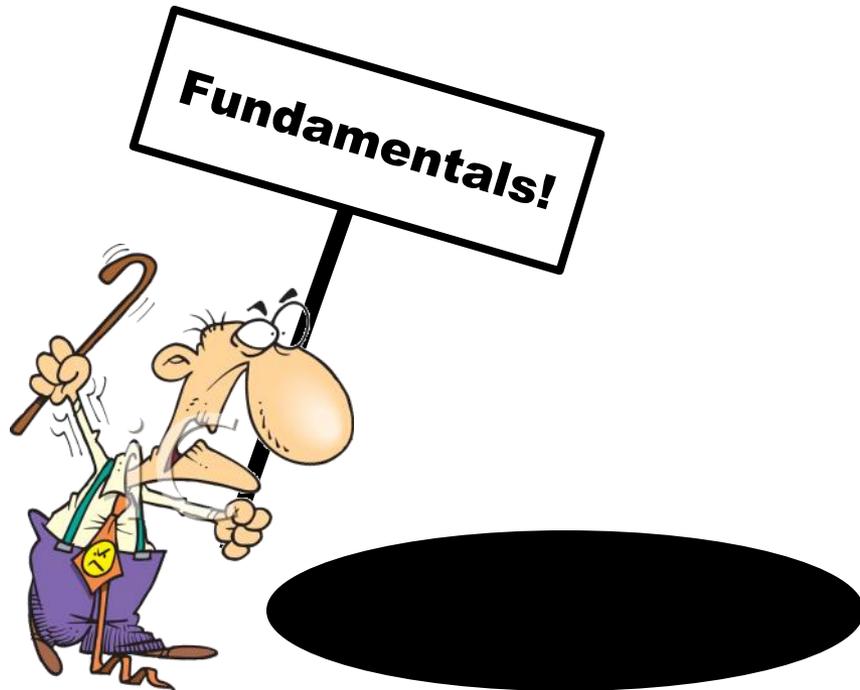
- All create new efficiencies but also unstable/fragile
- Needs new distributed/layered/complex/active control
- Persistent errors/confusion in science & engineering
- **Sustainable infrastructure? (e.g. smartgrids)**
- Money/finance/lobbyists/etc
- Industrialization
- Society/agriculture/weapons/etc
- Bipedalism
- Maternal care
- Warm blood
- Flight
- Mitochondria
- Oxygen
- Translation (ribosomes)
- Glycolysis (2011 *Science*)



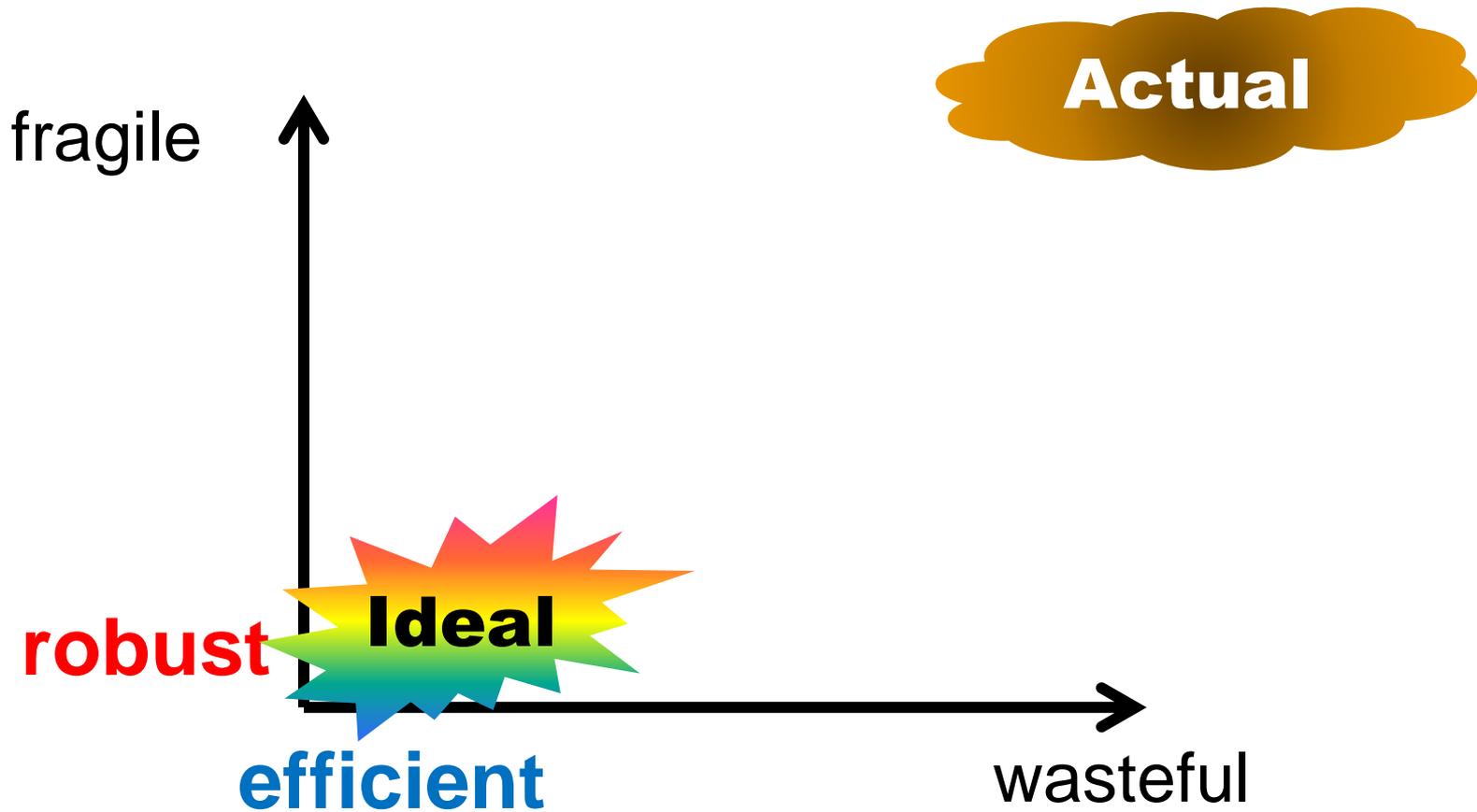
**Major transitions
in evolution**

I {Case Study}

- **Nets/Grids (cyberphys)**
- **Brains**
- **Bugs (microbes, ants)**
- Medical physiology (tomorrow)
- Lots of aerospace
- Wildfire ecology
- Earthquakes
- Physics:
 - turbulence,
 - stat mech (QM?)
- “Toy”:
 - Lego
 - clothing, fashion
- Buildings, cities
- **Synesthesia**



The main tradeoff



Efficiency/instability/layers/feedback

- New efficiencies but also instability/fragility
- New distributed/layered/complex/active control

- **Sustainable infrastructure? (e.g. smartgrids)**

- Money/finance/lobbyists/etc
- Industrialization
- Society/agriculture/weapons/etc

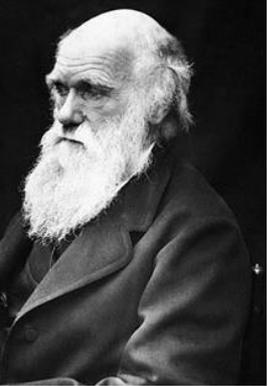
- **Bipedalism**

- Maternal care
- Warm blood
- Flight
- Mitochondria
- Oxygen
- Translation (ribosomes)
- Glycolysis (2011 *Science*)



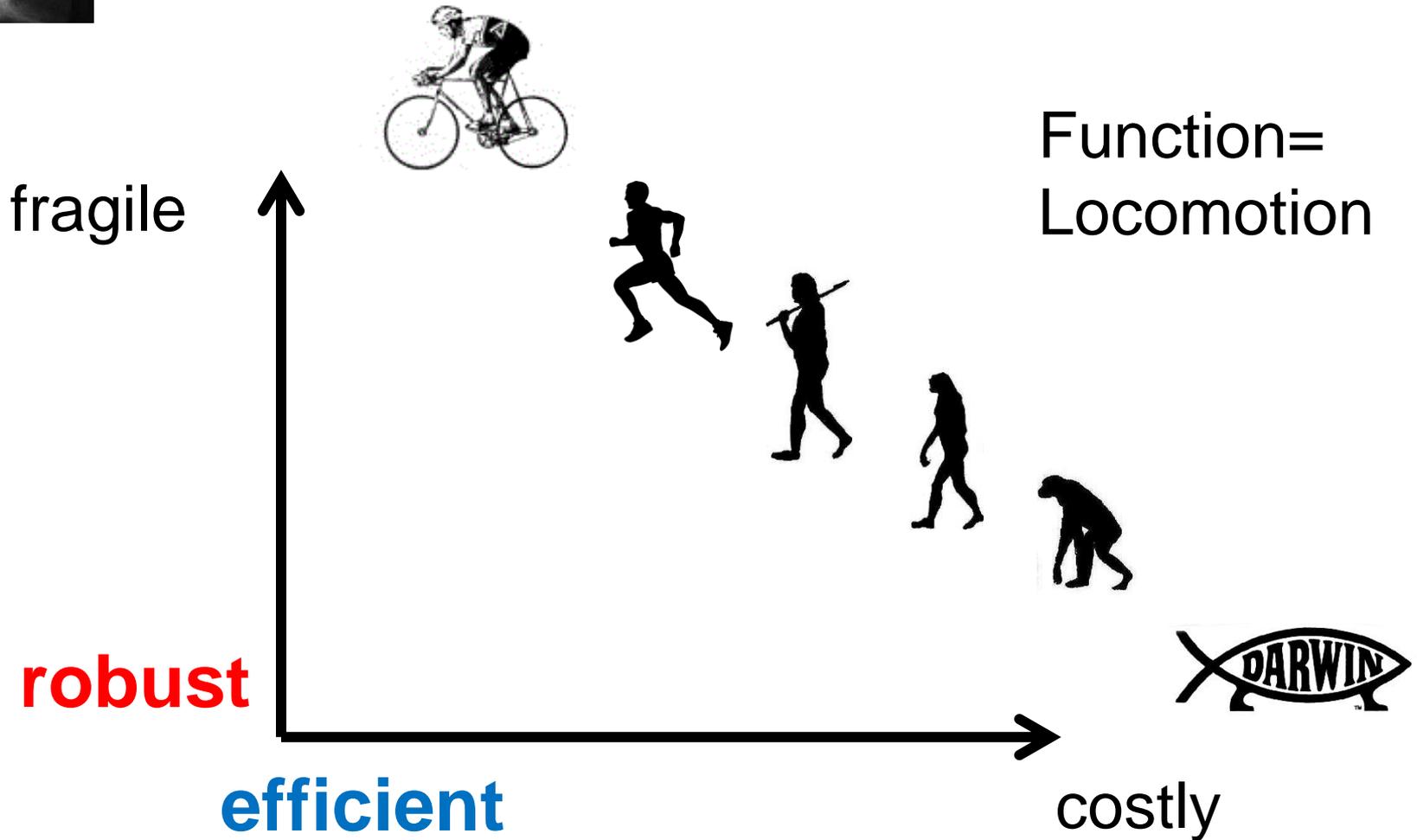
Live demo?



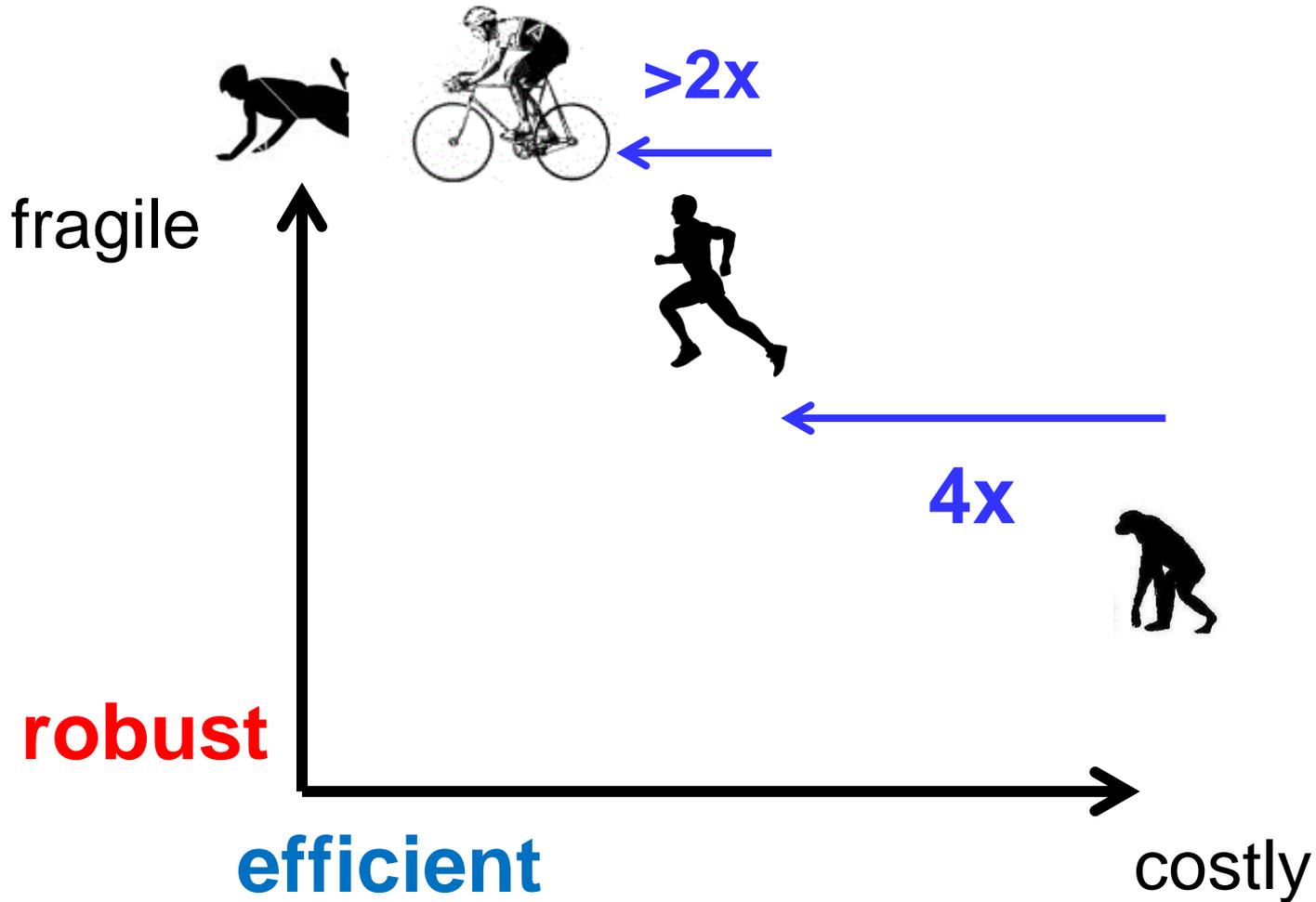


Tradeoffs

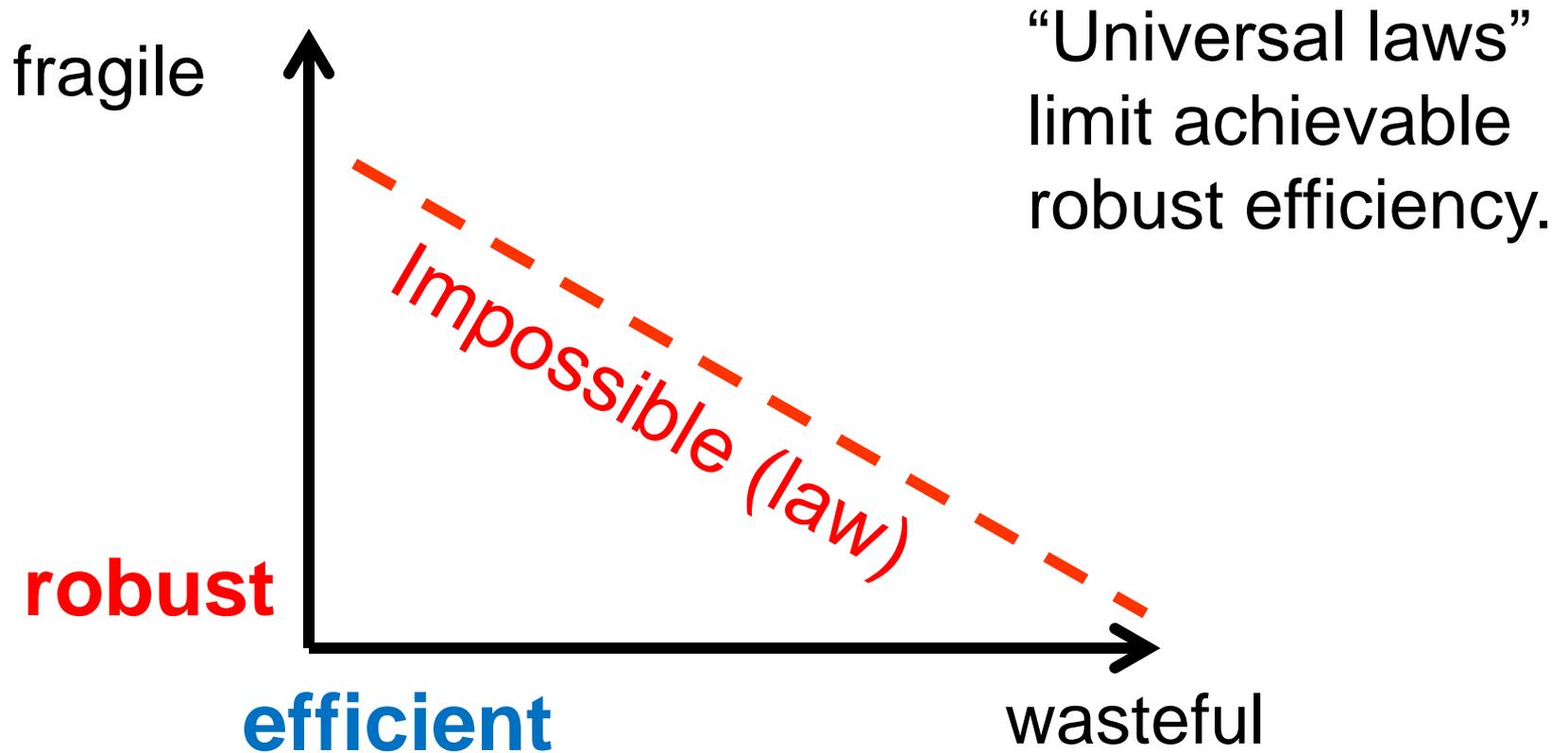
(swim/crawl to run/bike)



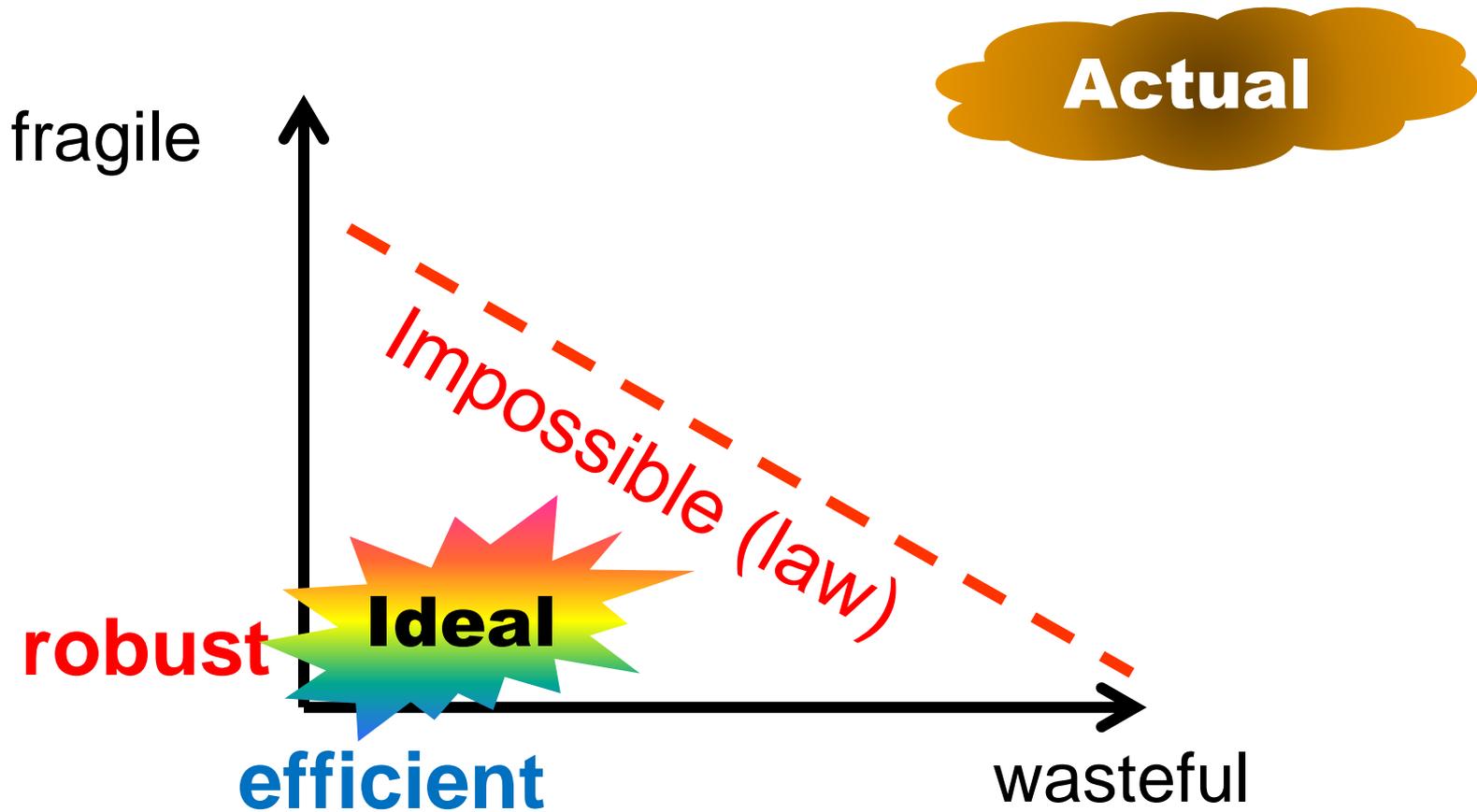
Tradeoffs



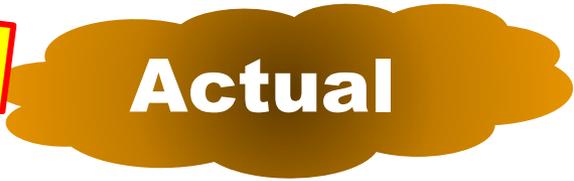
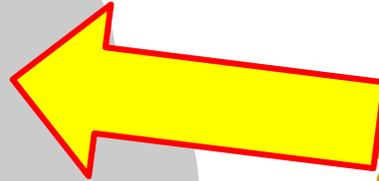
Universal laws



Universal laws

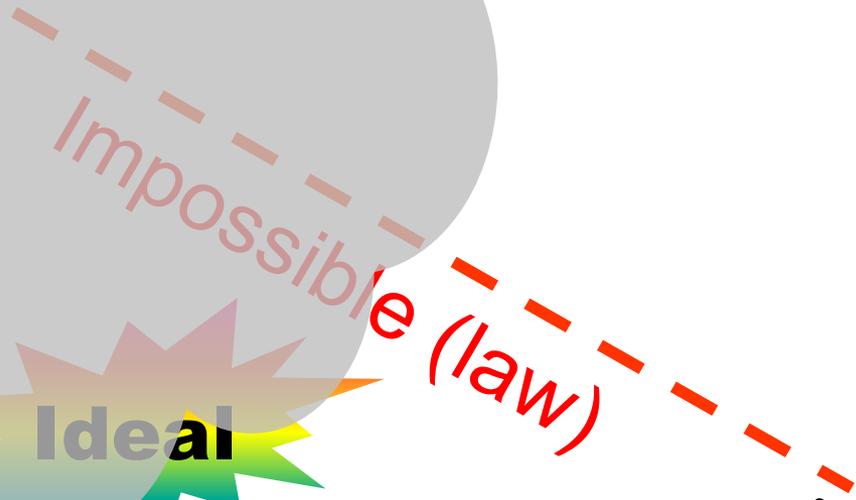


The risk



Actual

fragile



Impossible (law)

robust

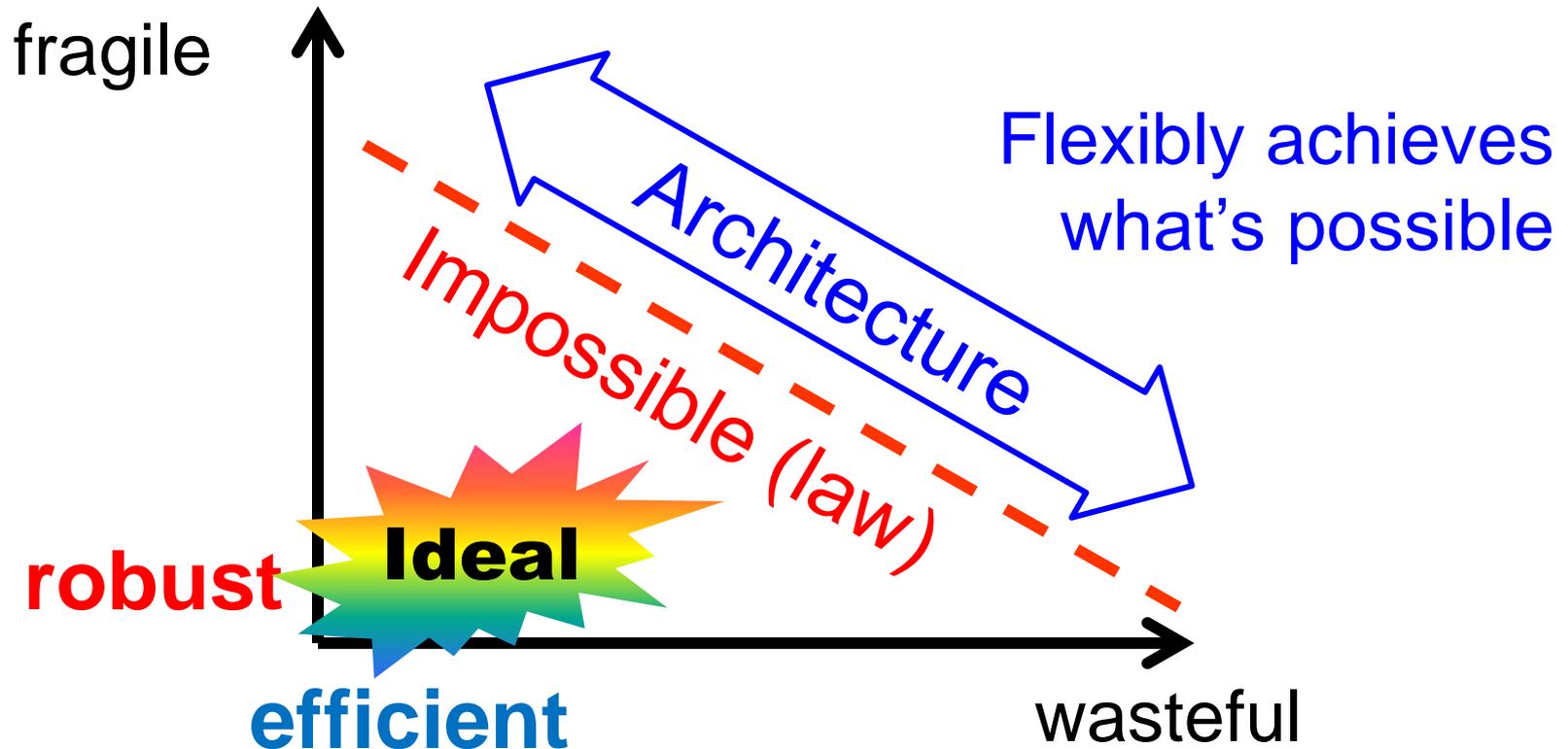
Ideal

efficient

wasteful



Universal laws and architectures

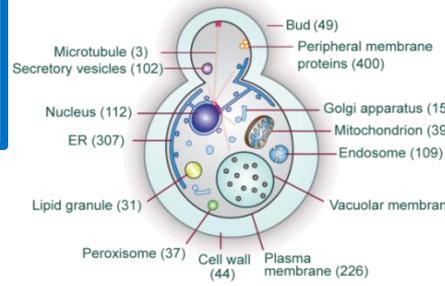
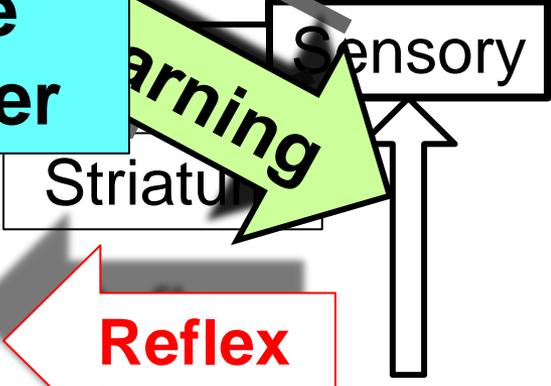
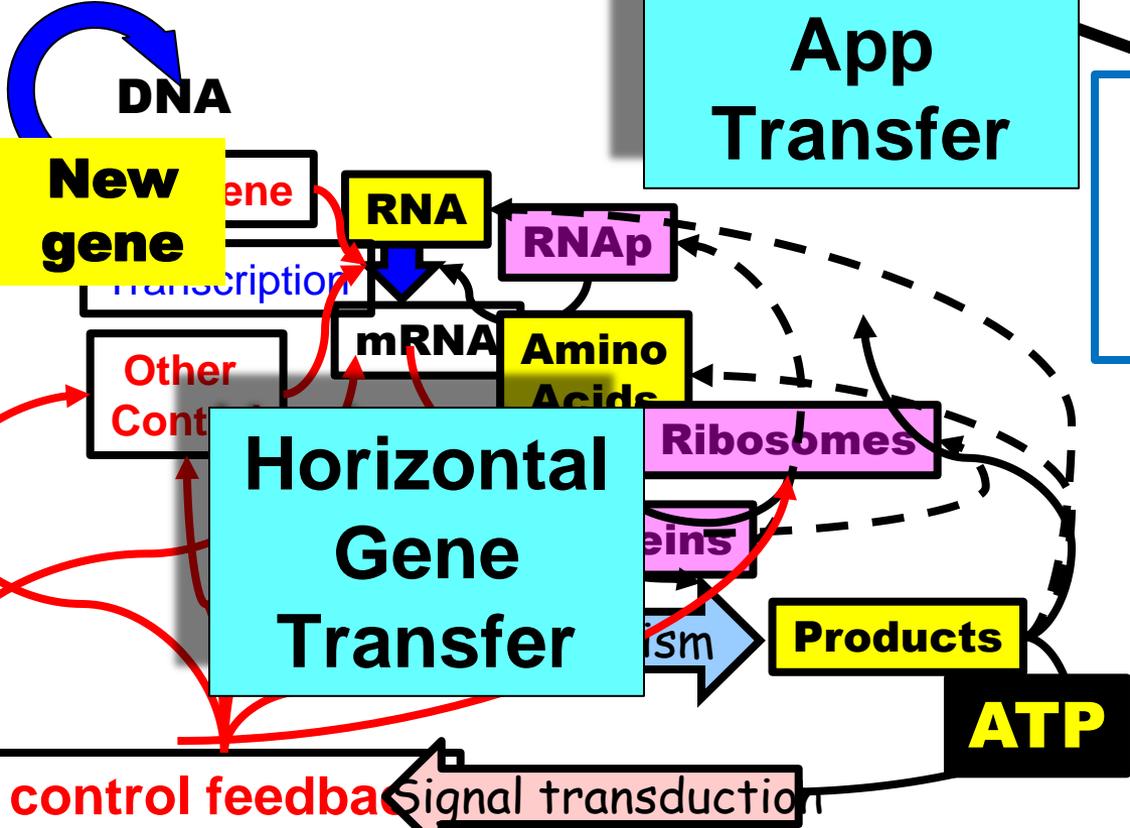


Evolvable architectures

Horizontal Meme Transfer

Horizontal App Transfer

Digital Analog



Requires shared "OS"

Learning

bike



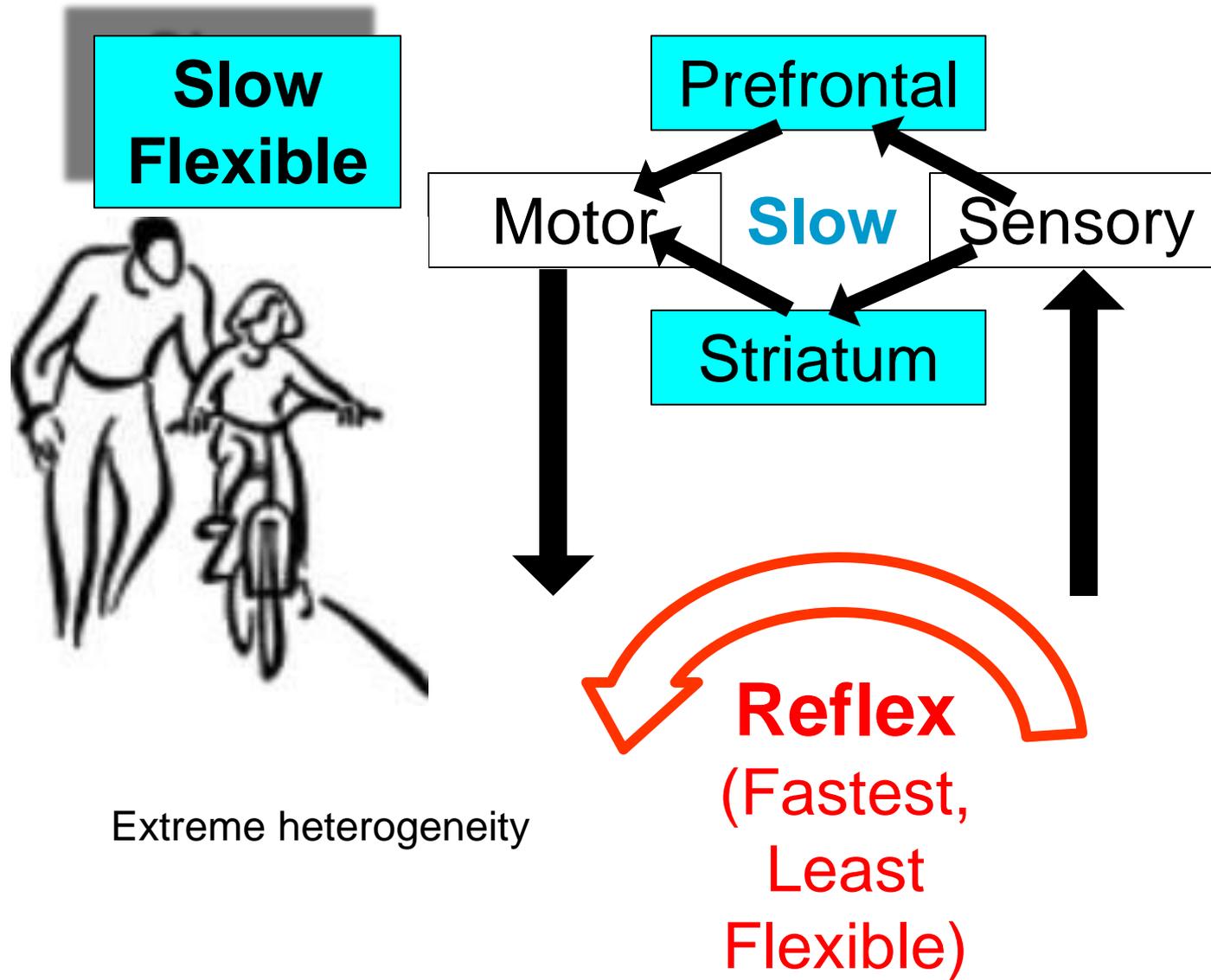
walk



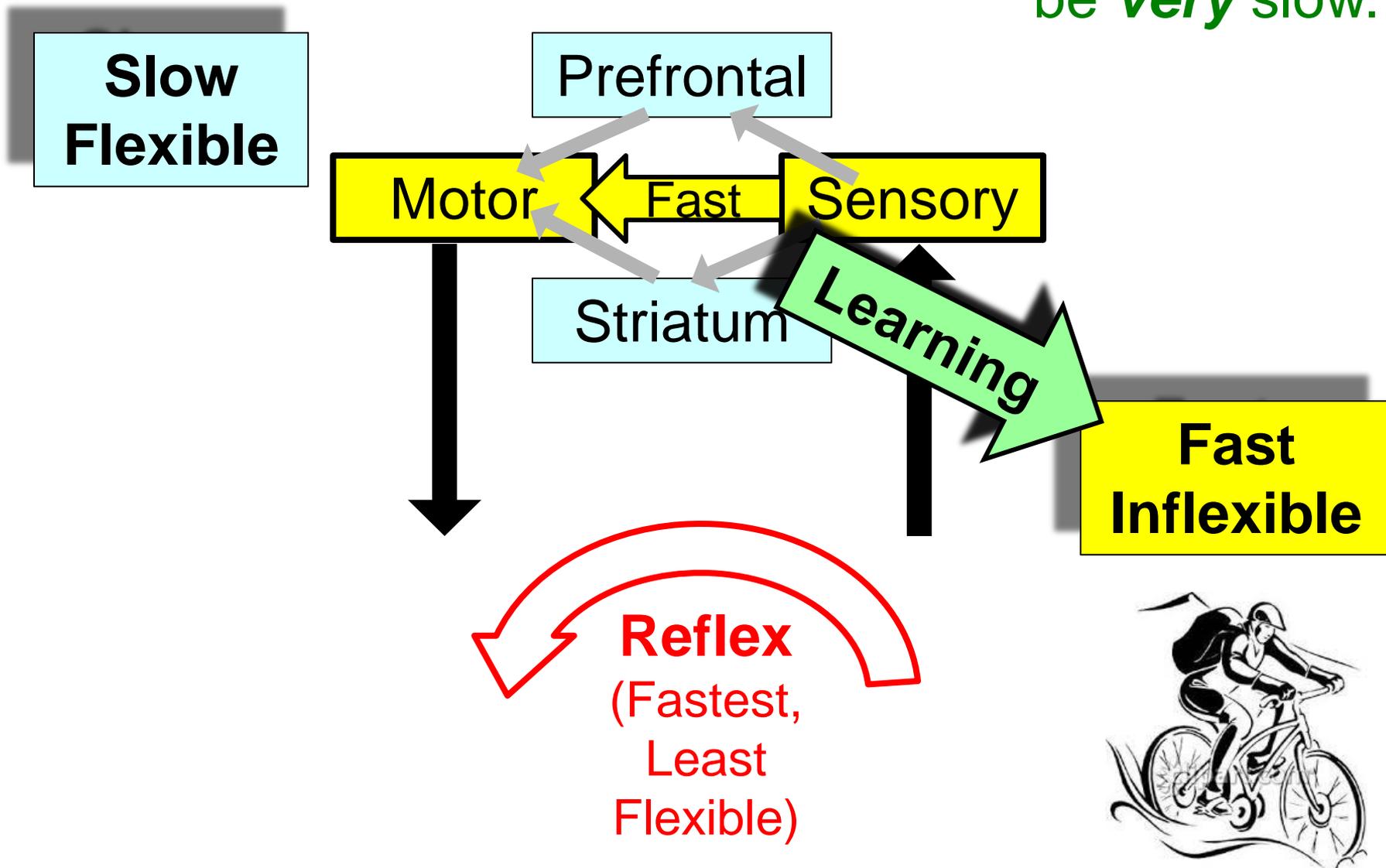
crawl



Learning



Learning can be **very** slow.





HMT

**Horizontal
Meme
Transfer**

Teach

Prefrontal

Learn

Motor

Fast Sense

Evolve

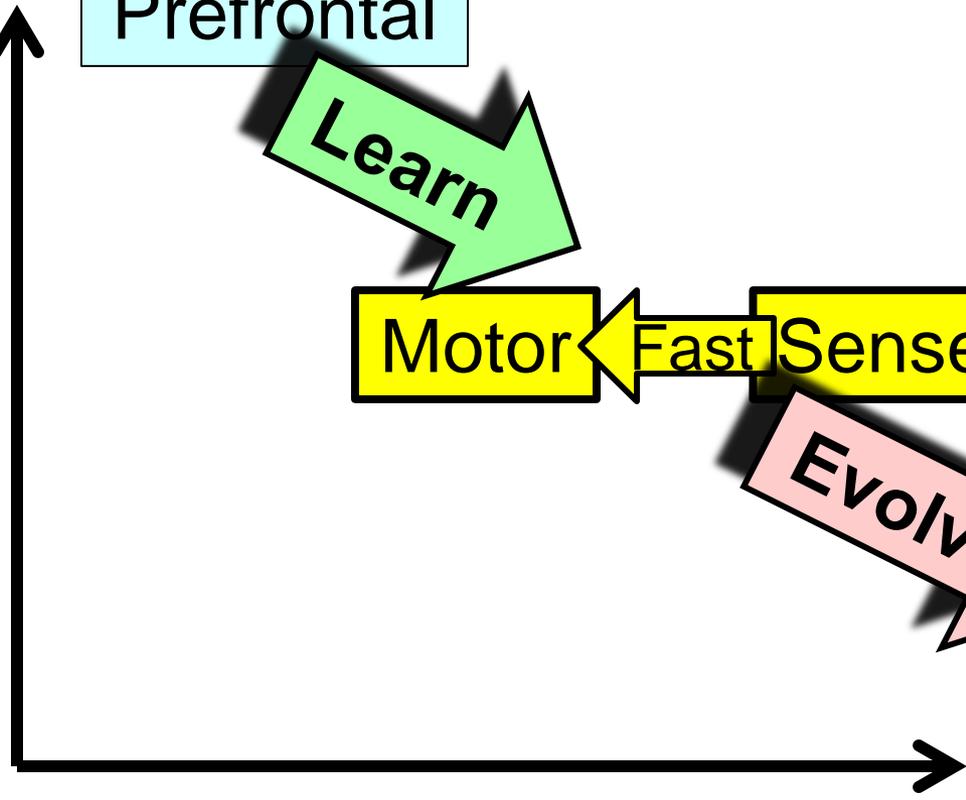
Reflex

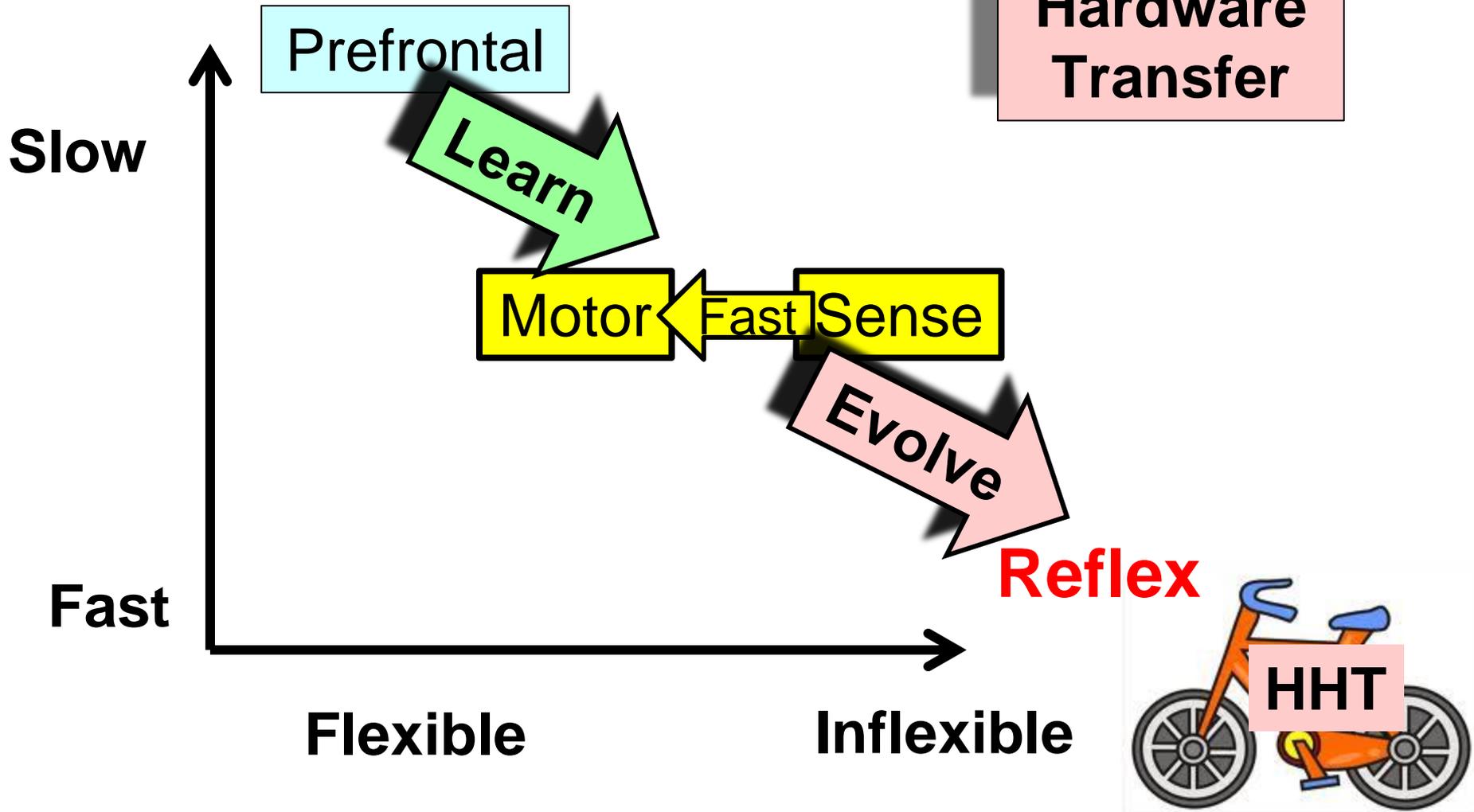
Slow

Fast

Flexible

Inflexible







HMT

Apps



© www.ClipProject.info

Prefrontal

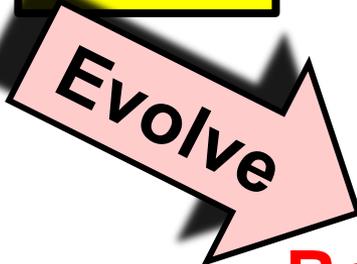
OS



Motor

Fast

Sense



HW

Reflex



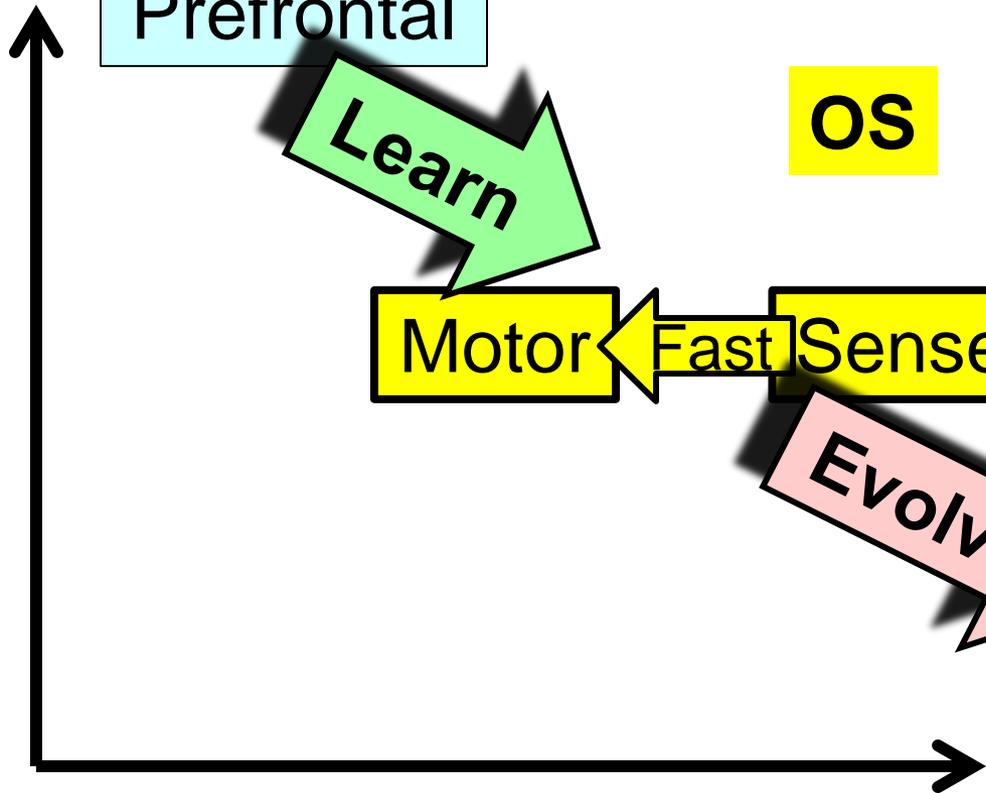
HHT

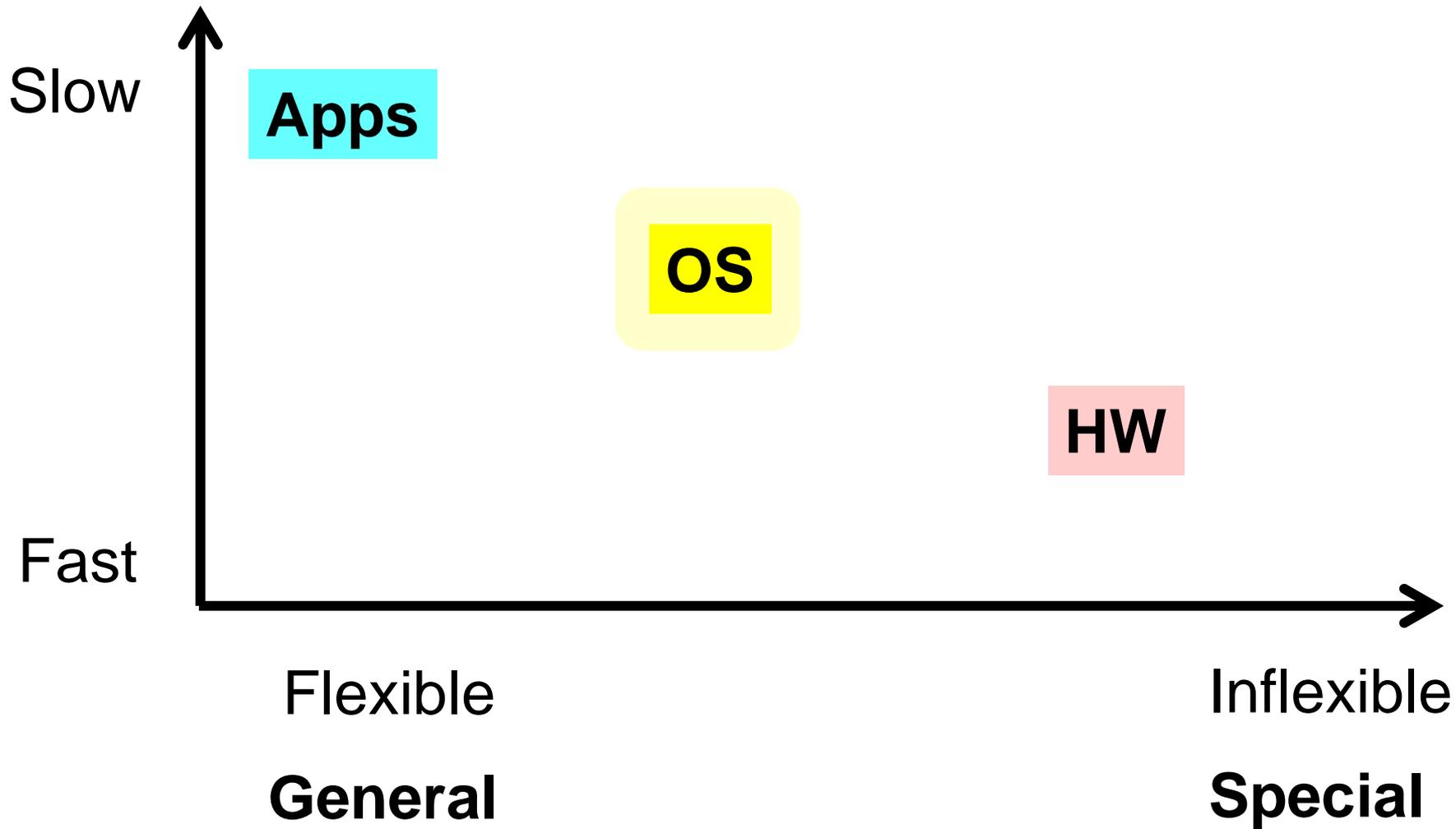
Slow

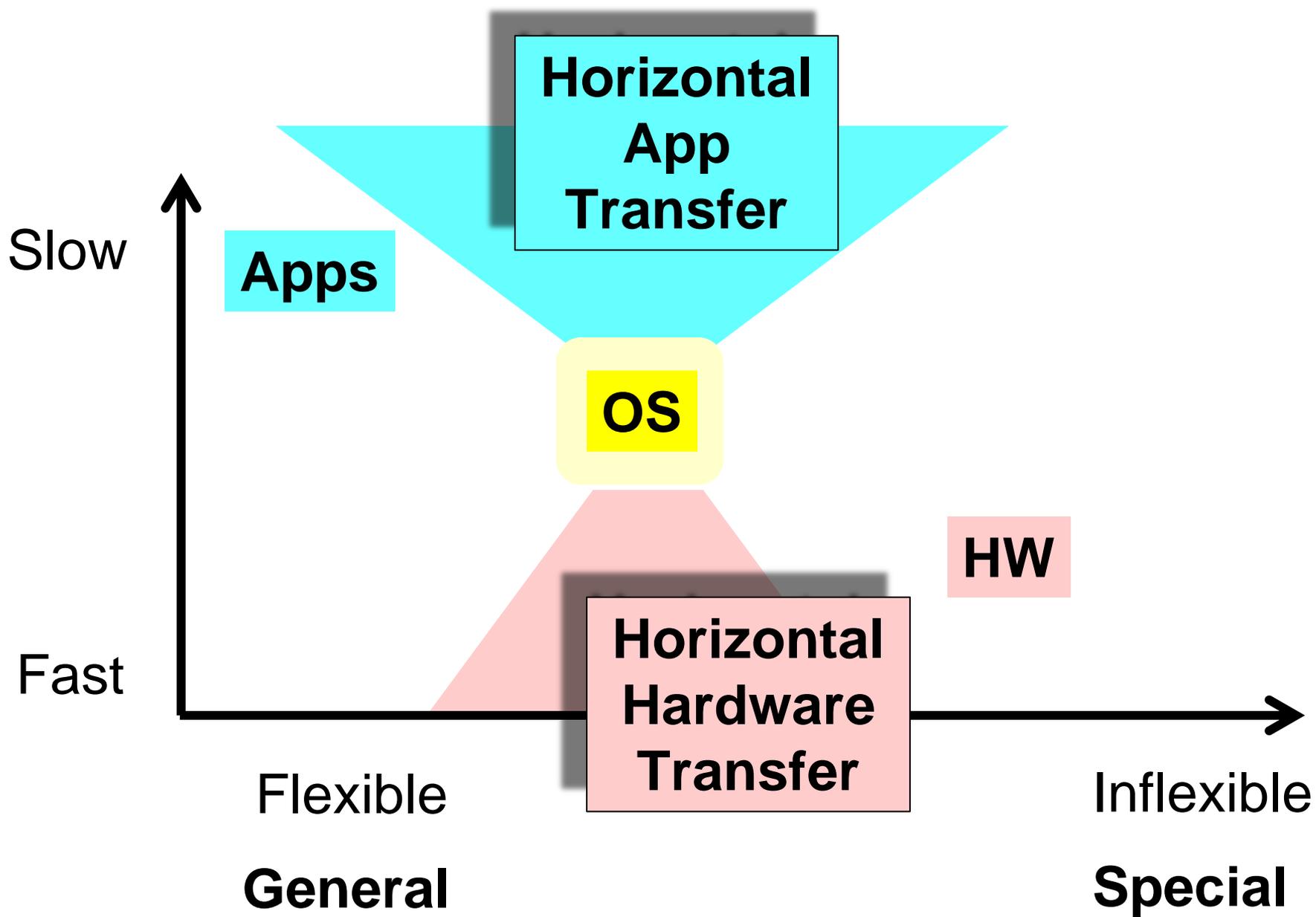
Fast

Flexible

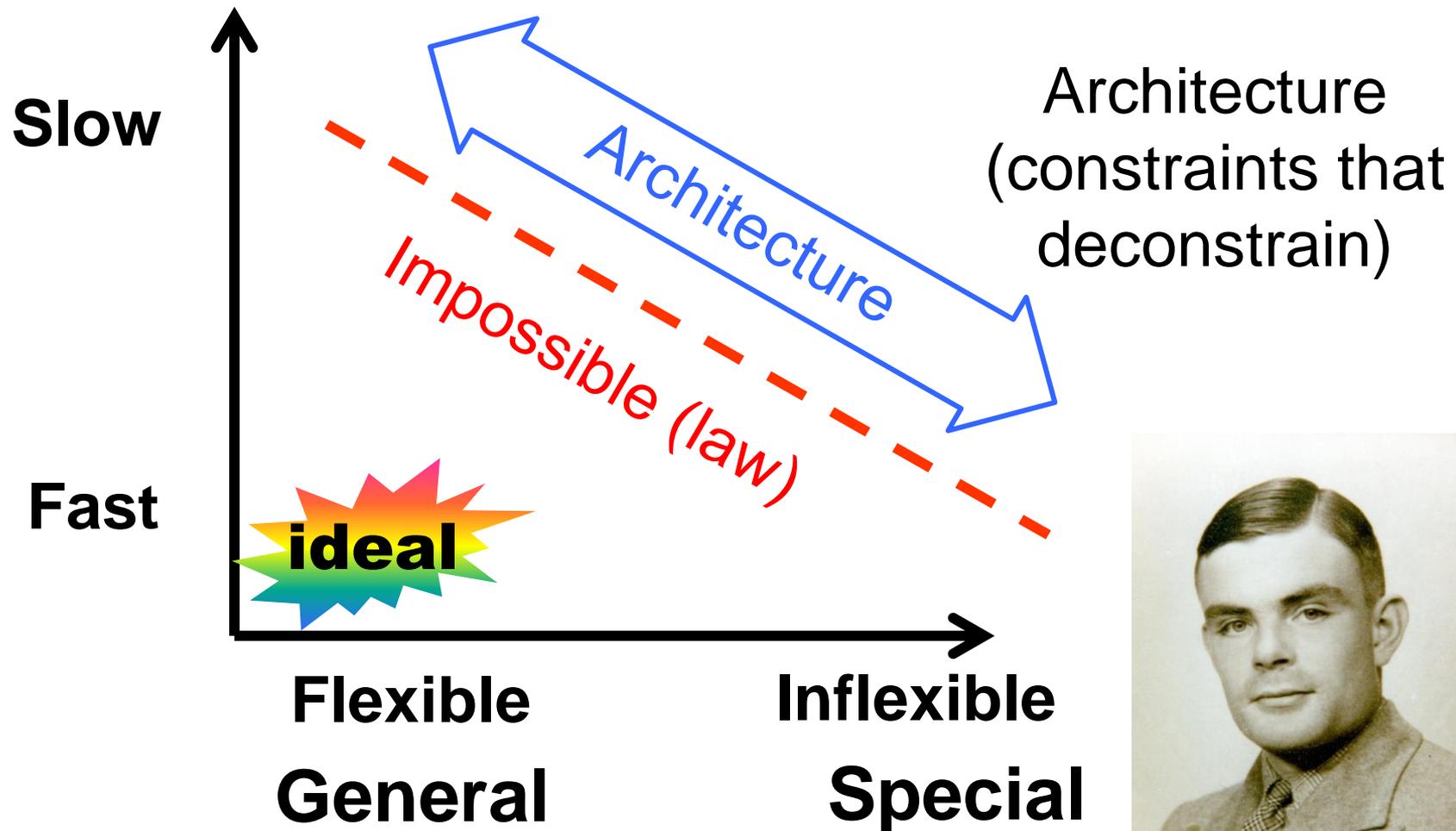
Inflexible



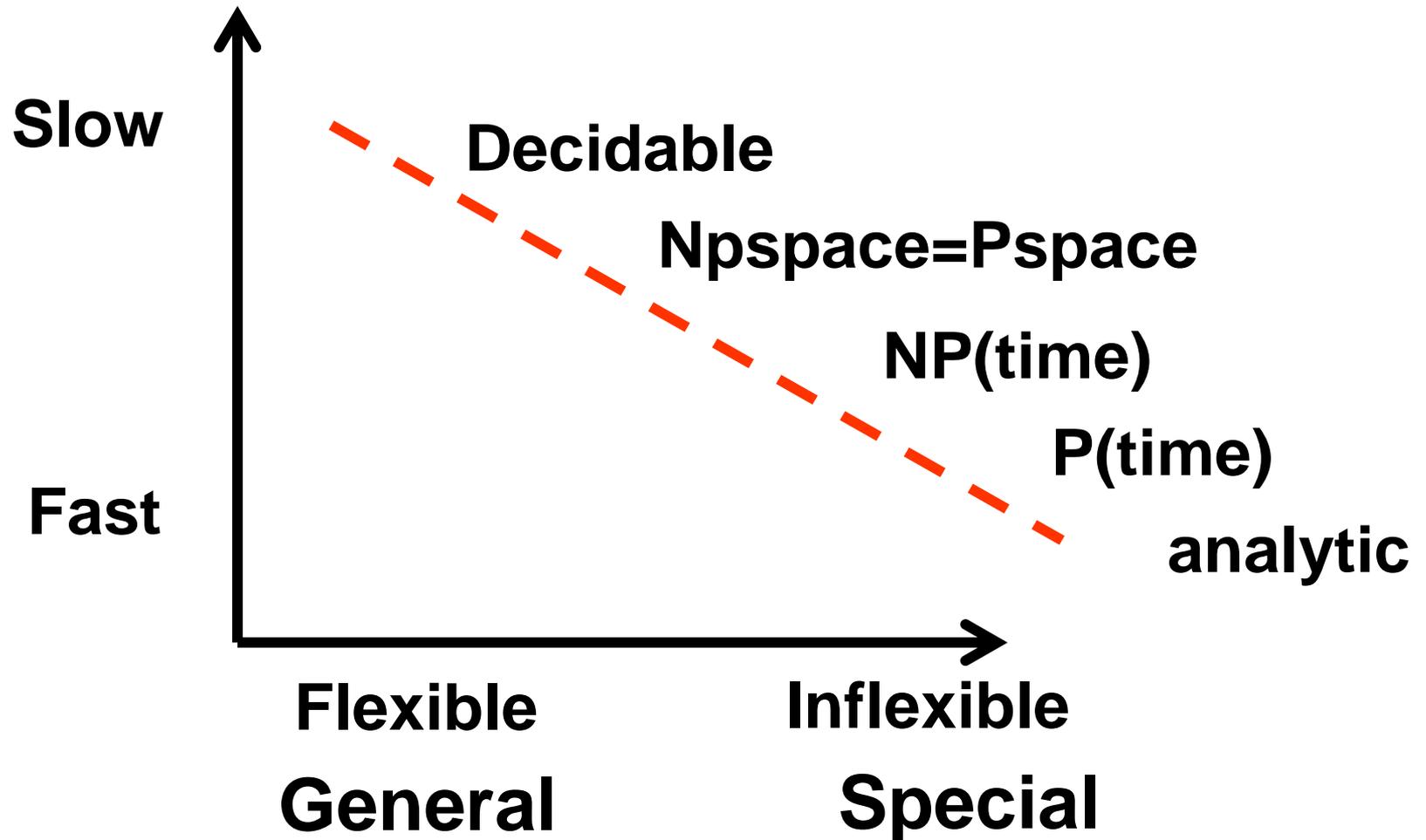




Universal laws and architectures (Turing)



Computation (on and off-line)



**Exploiting
layered
architecture**

Meme

**Horizontal
Bad Meme
Transfer**

Virus

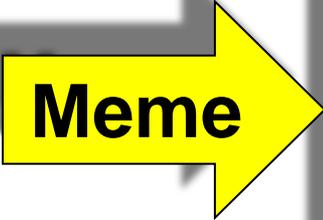
**Horizontal
Bad App
Transfer**

Fragility?

**Horizontal
Bad Gene
Transfer**

Virus

**Parasites
&
Hijacking**



Meme



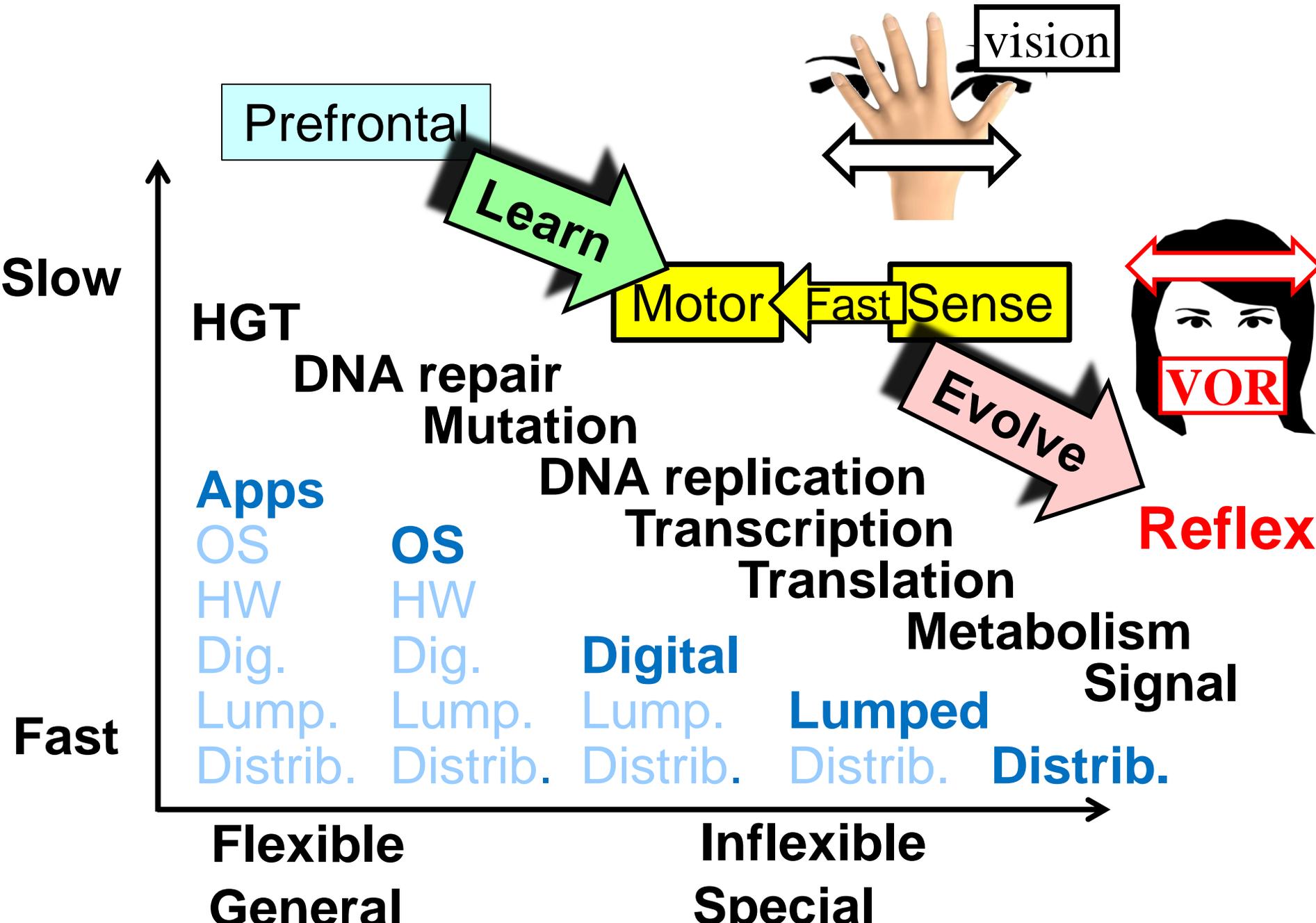
Horizontal
Bad Meme
Transfer

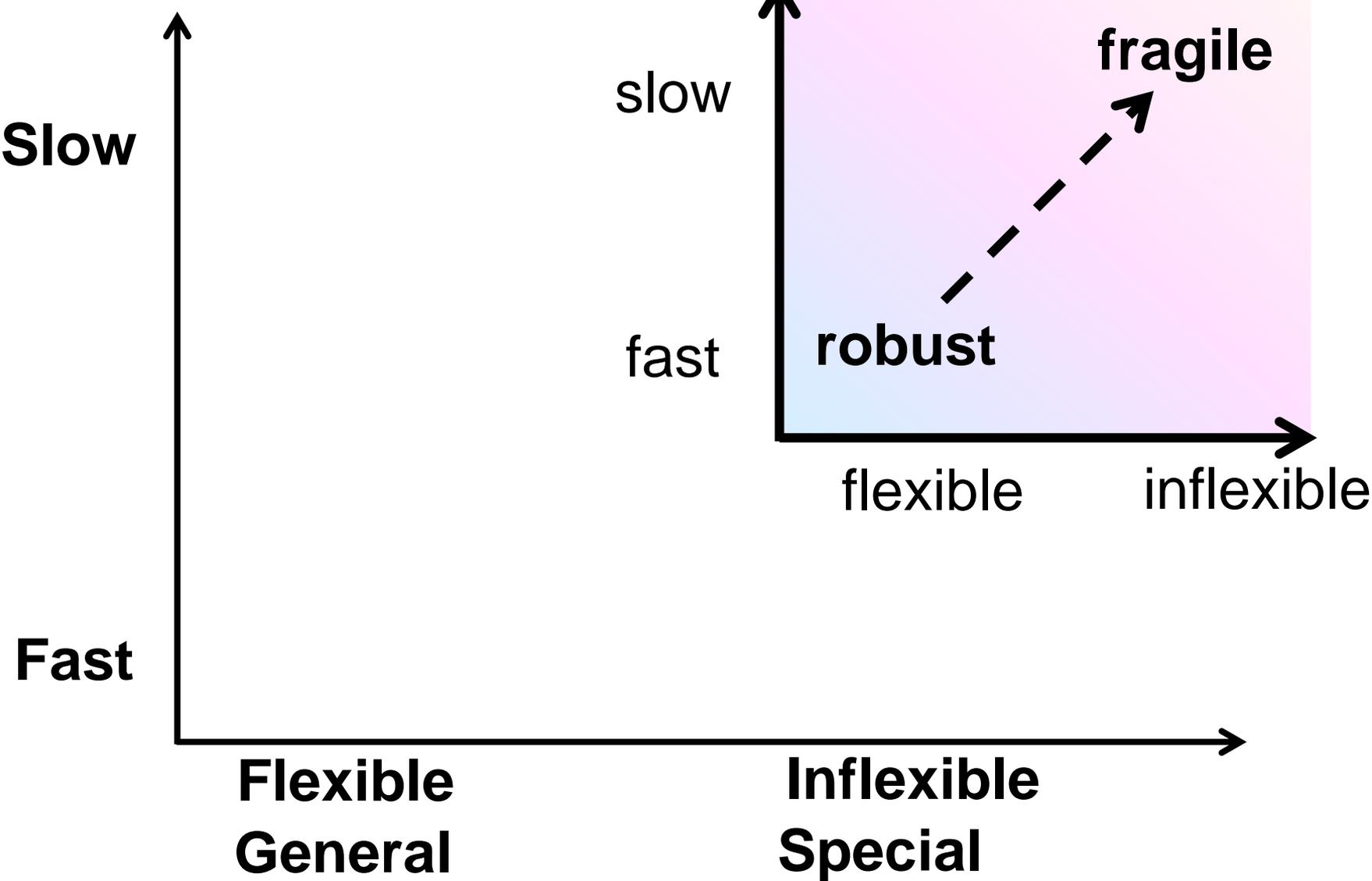
Many human
beliefs are:

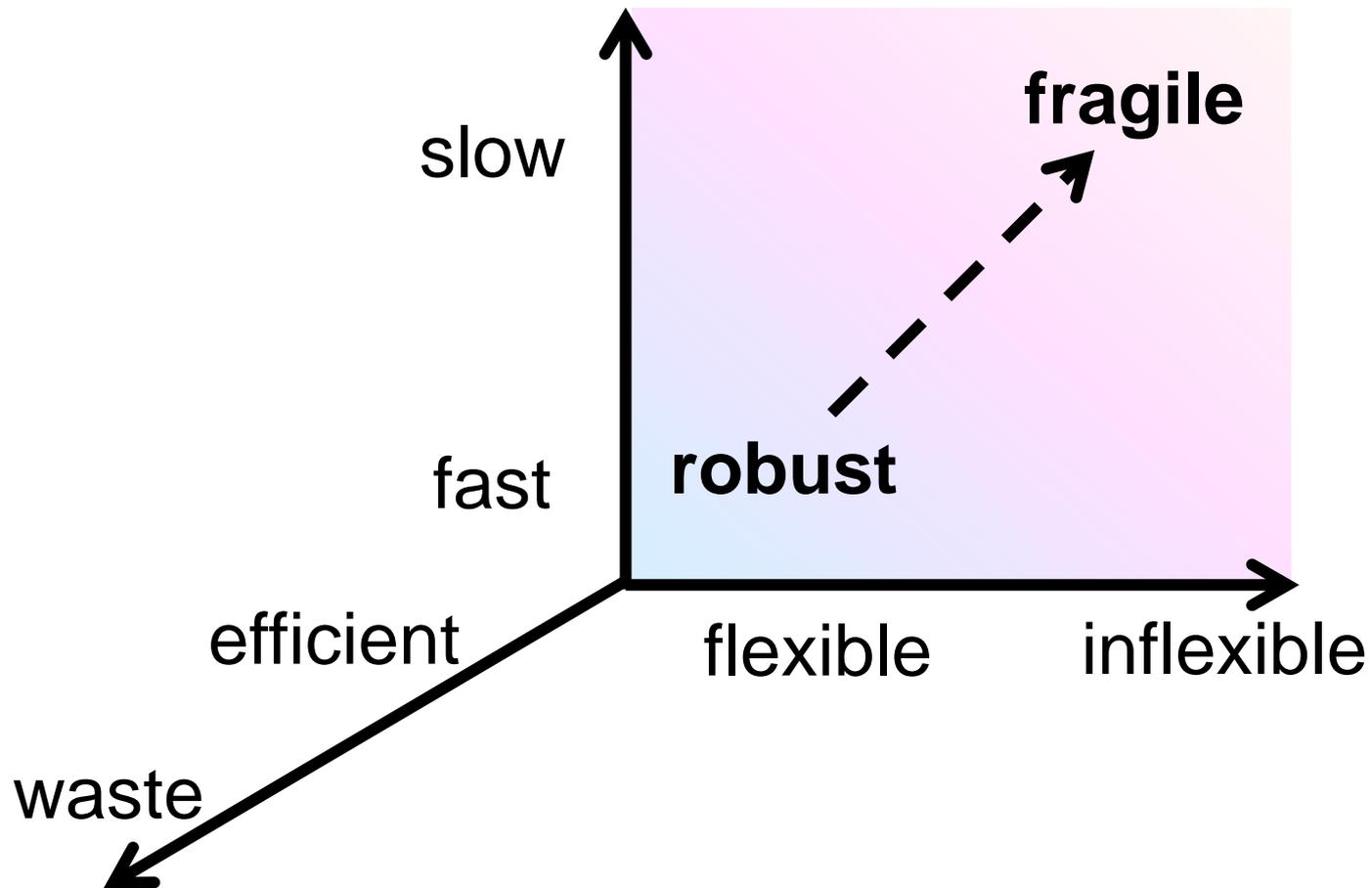
- False
- Unhealthy

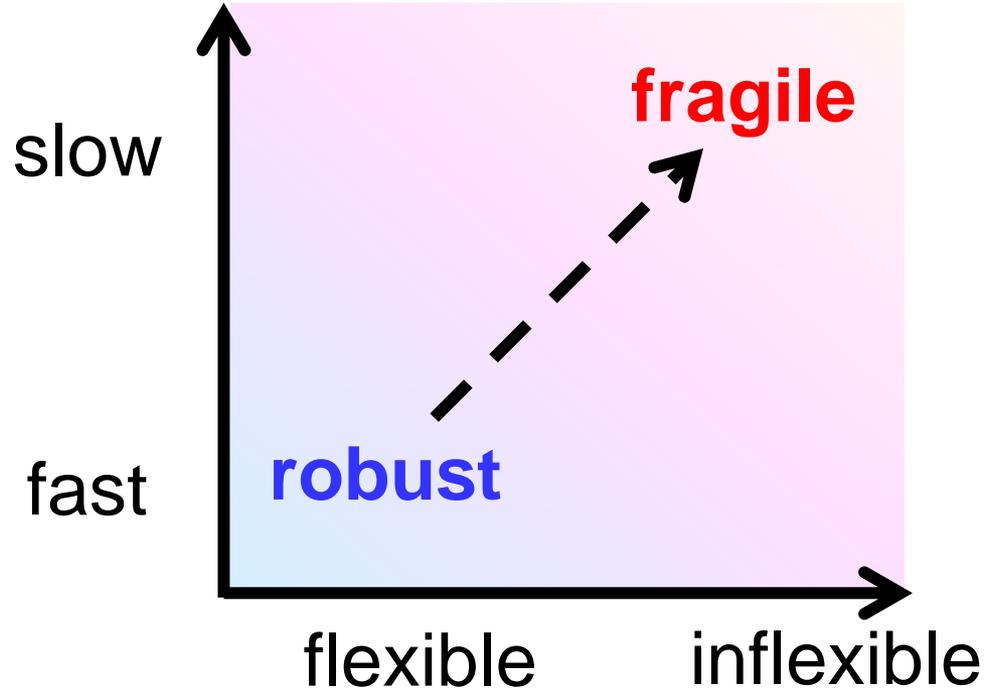
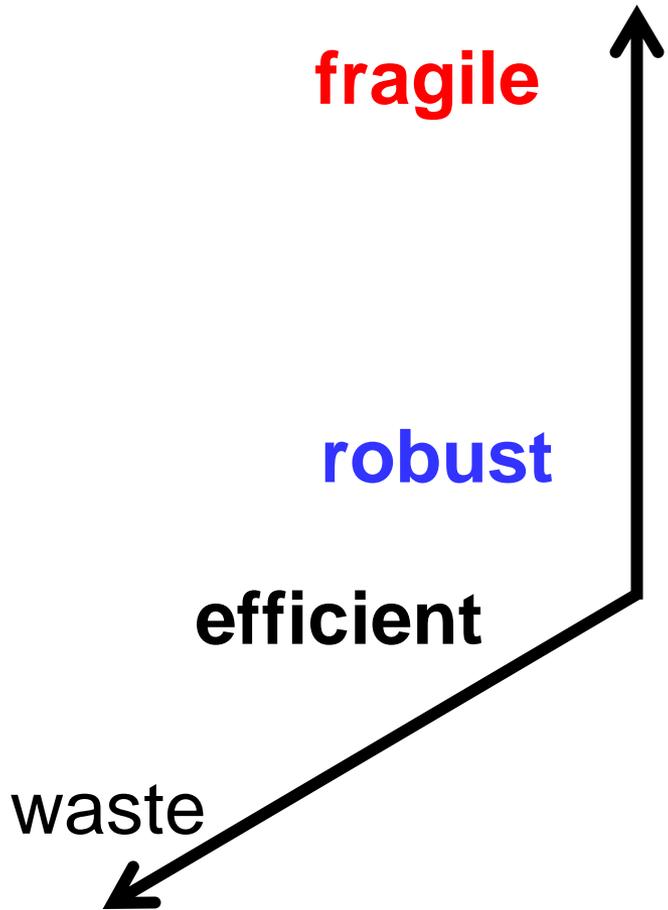


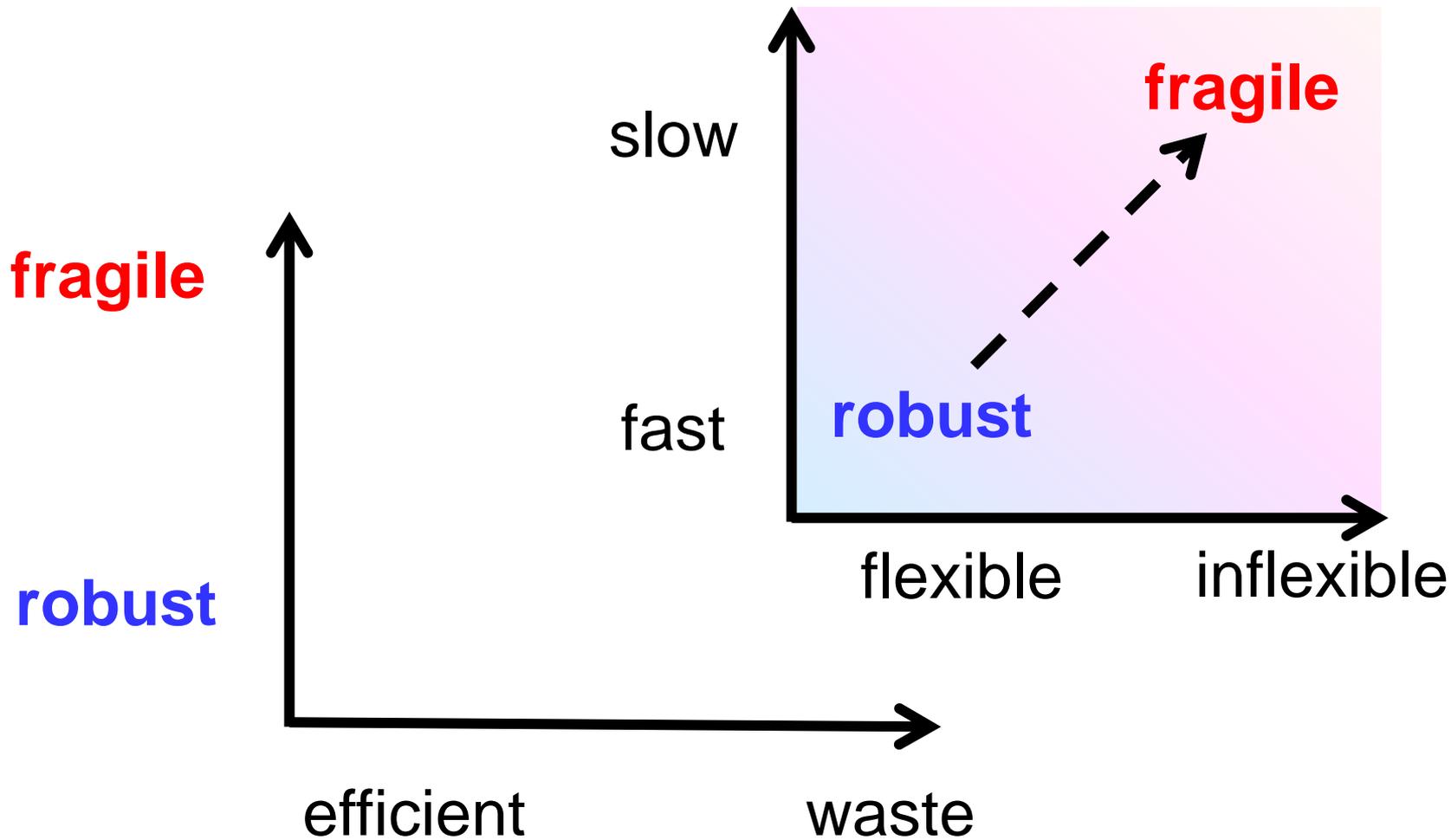
**Our
greatest
fragility?**

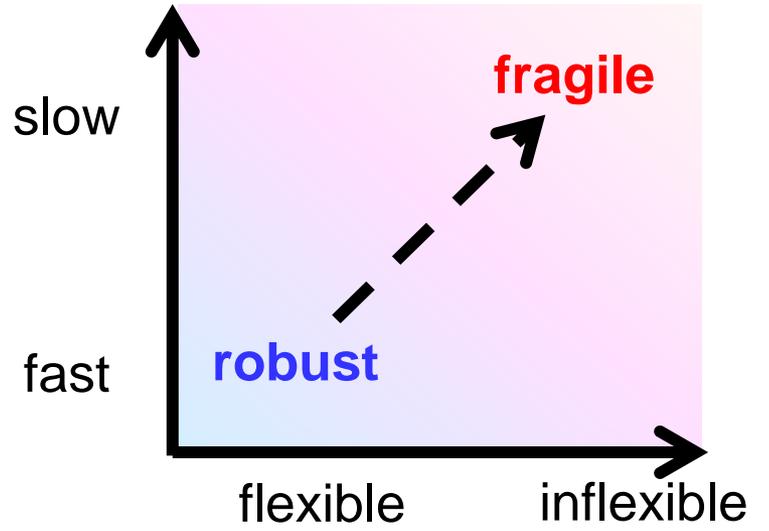
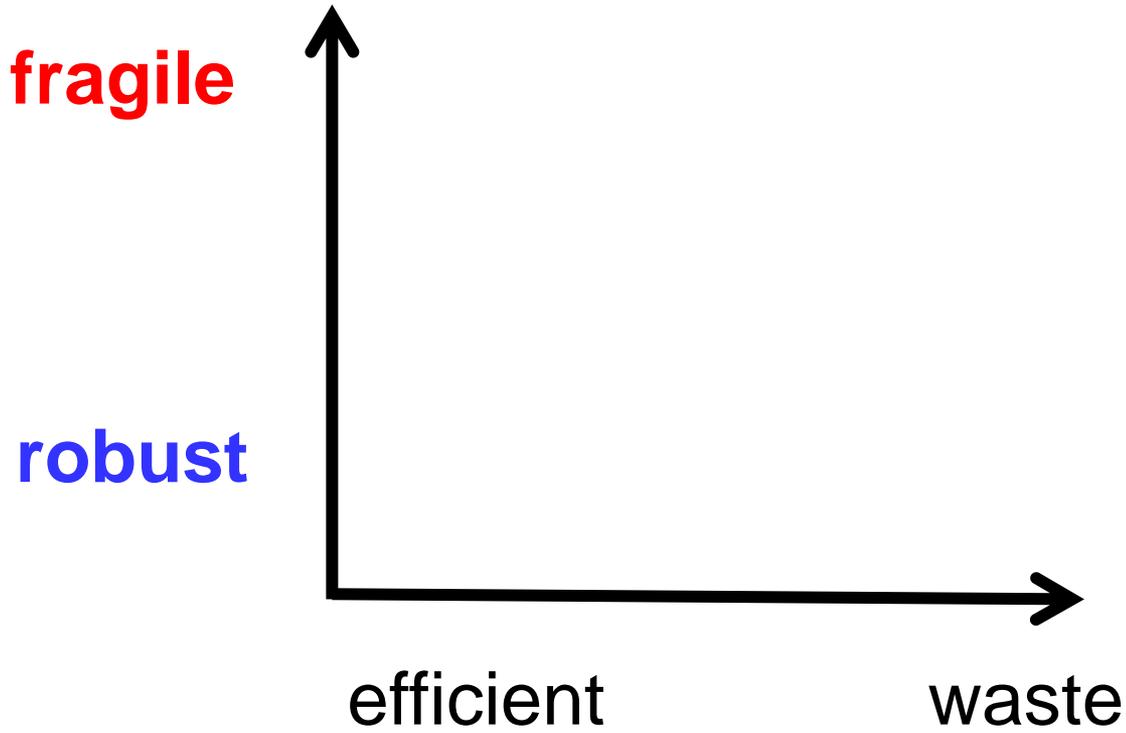












fragile



Function=
Movement

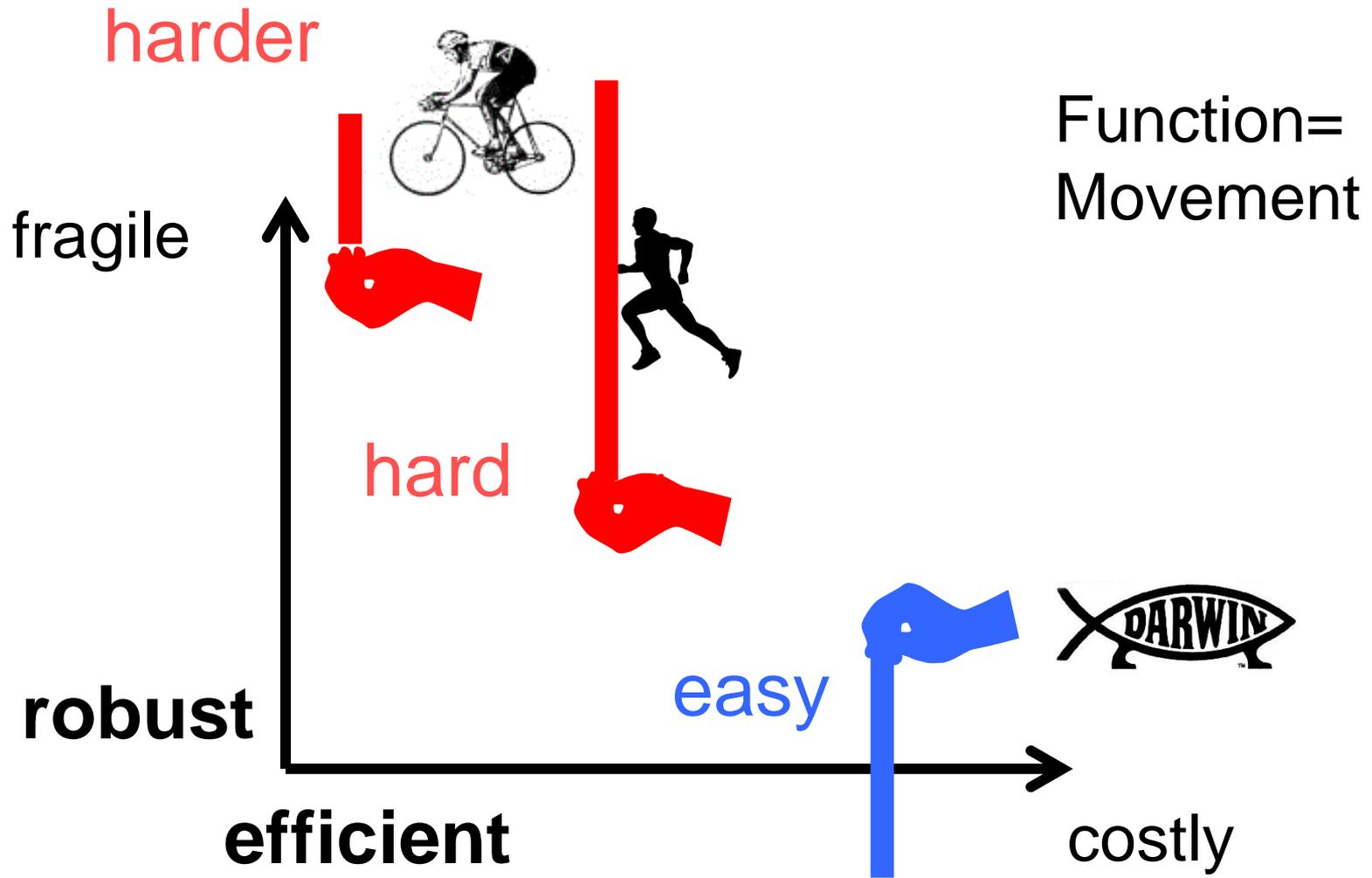
robust



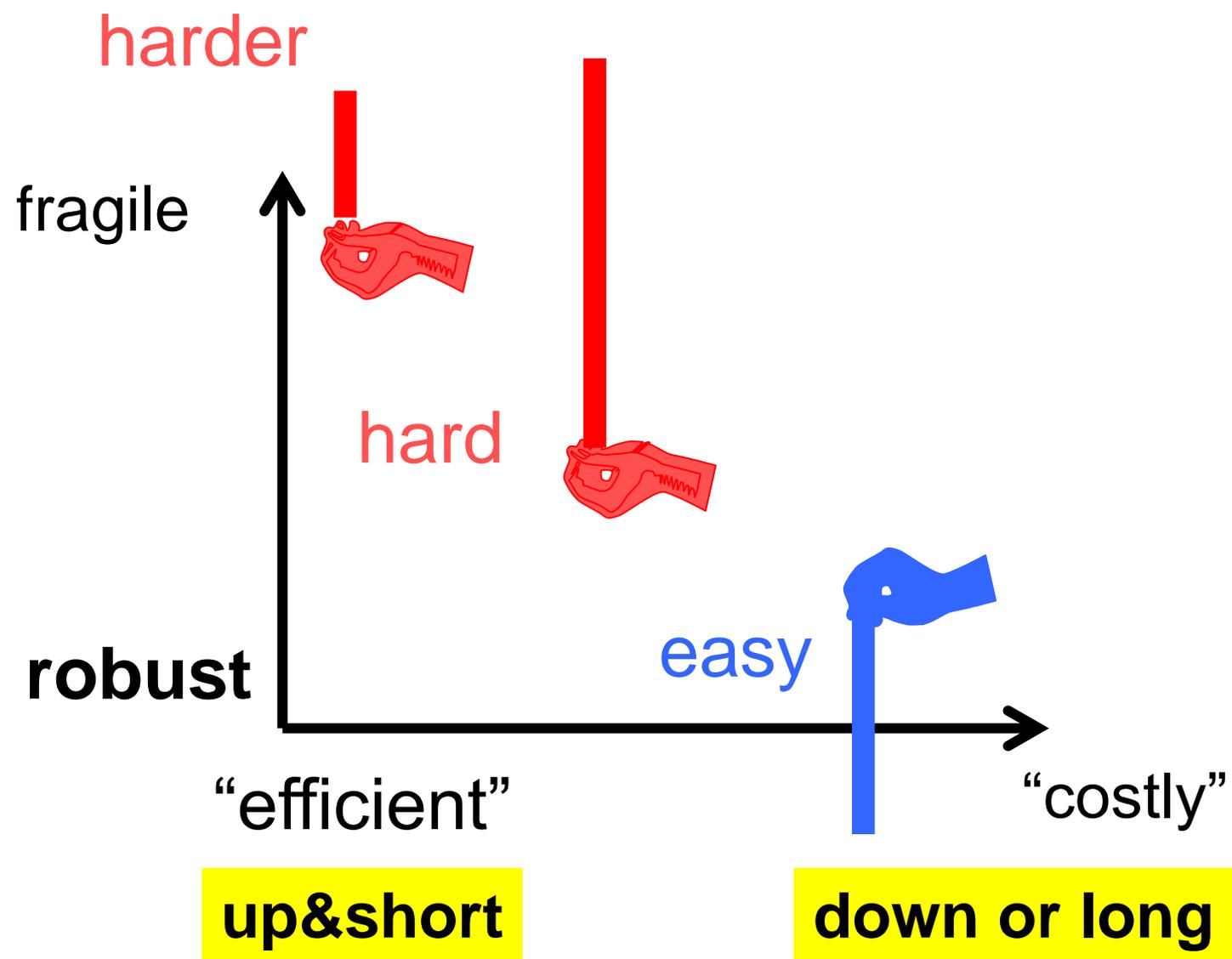
efficient

costly

A convenient cartoon



cartoon demo

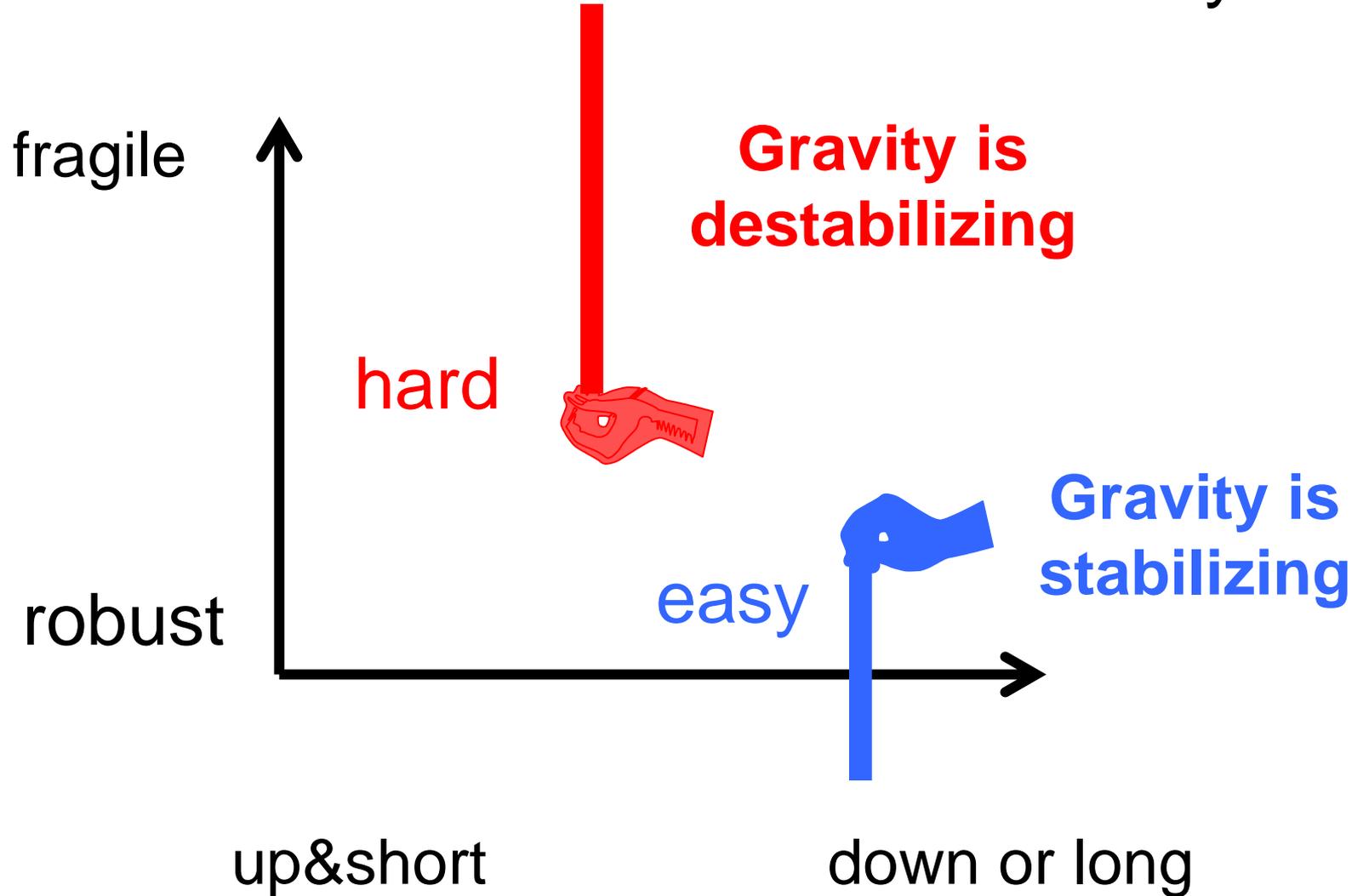


up&short

down or long

Universal laws?

Law #1 : Mechanics
Law #2 : Gravity



Efficiency/instability/layers/feedback

- New efficiencies but also instabilities
- New distributed/layered/complex/active control

• Sustainable infrastructure? (e.g. smartgrids)

• Money/finance/buyists/etc

• Industry/education

• Society/agriculture/weapons/etc

• **Bipedalism**

• Maternal care

• Warm blood

• Flight

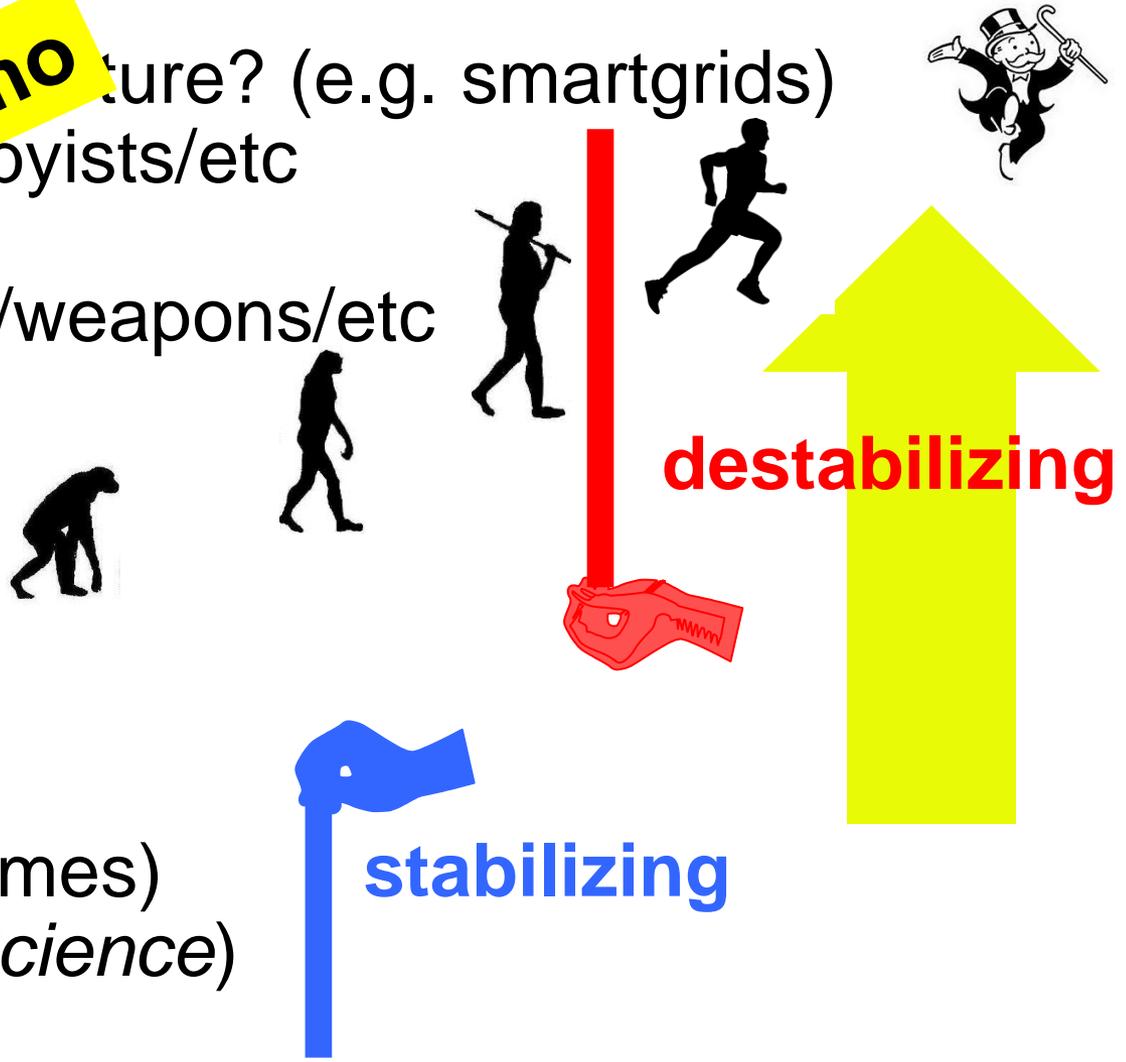
• Mitochondria

• Oxygen

• Translation (ribosomes)

• Glycolysis (2011 *Science*)

cartoon demo



**More
unstable**

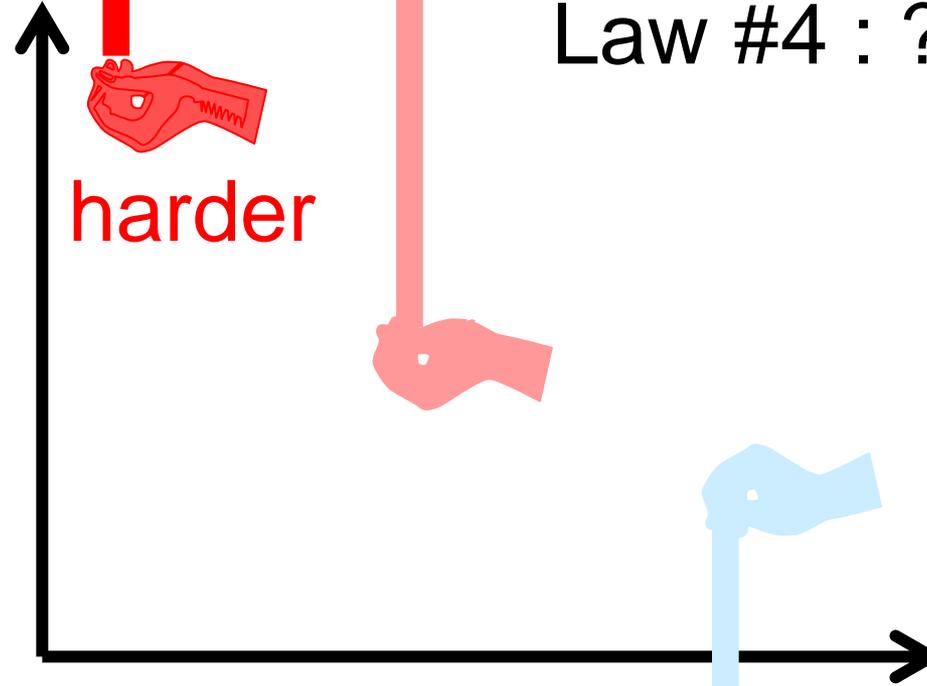
Law #1 : Mechanics

Law #2 : Gravity

Law #3 : ??

Law #4 : ??

fragile



harder

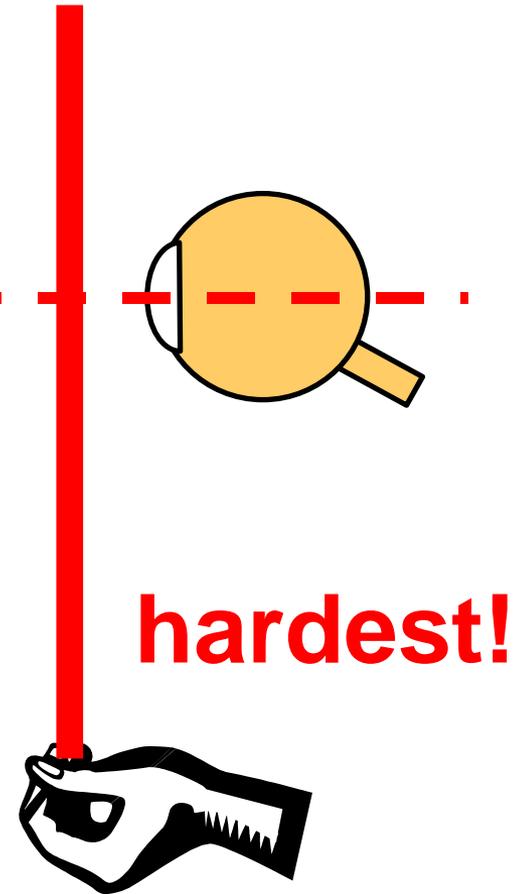
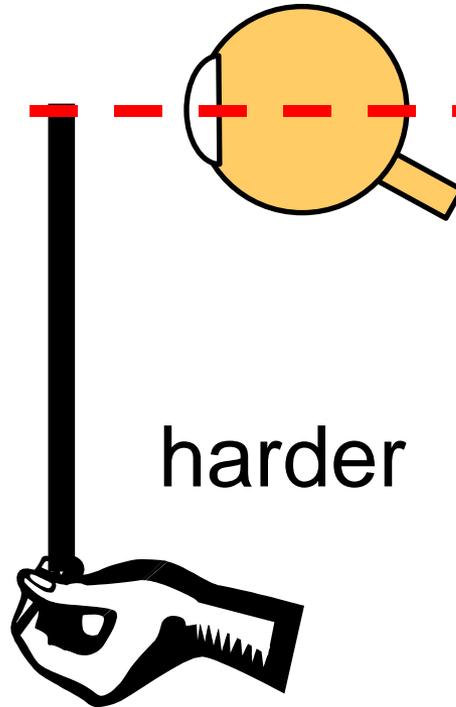
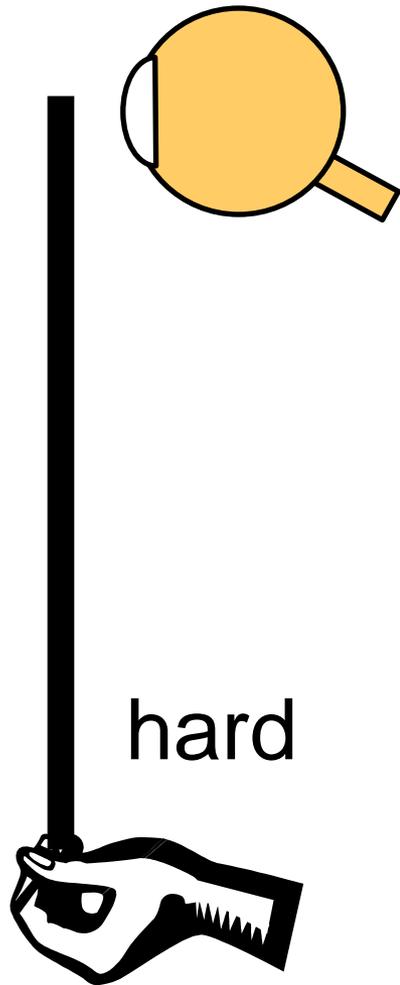
robust

up&short

down or long

What is *sensed* matters.

Why?!?

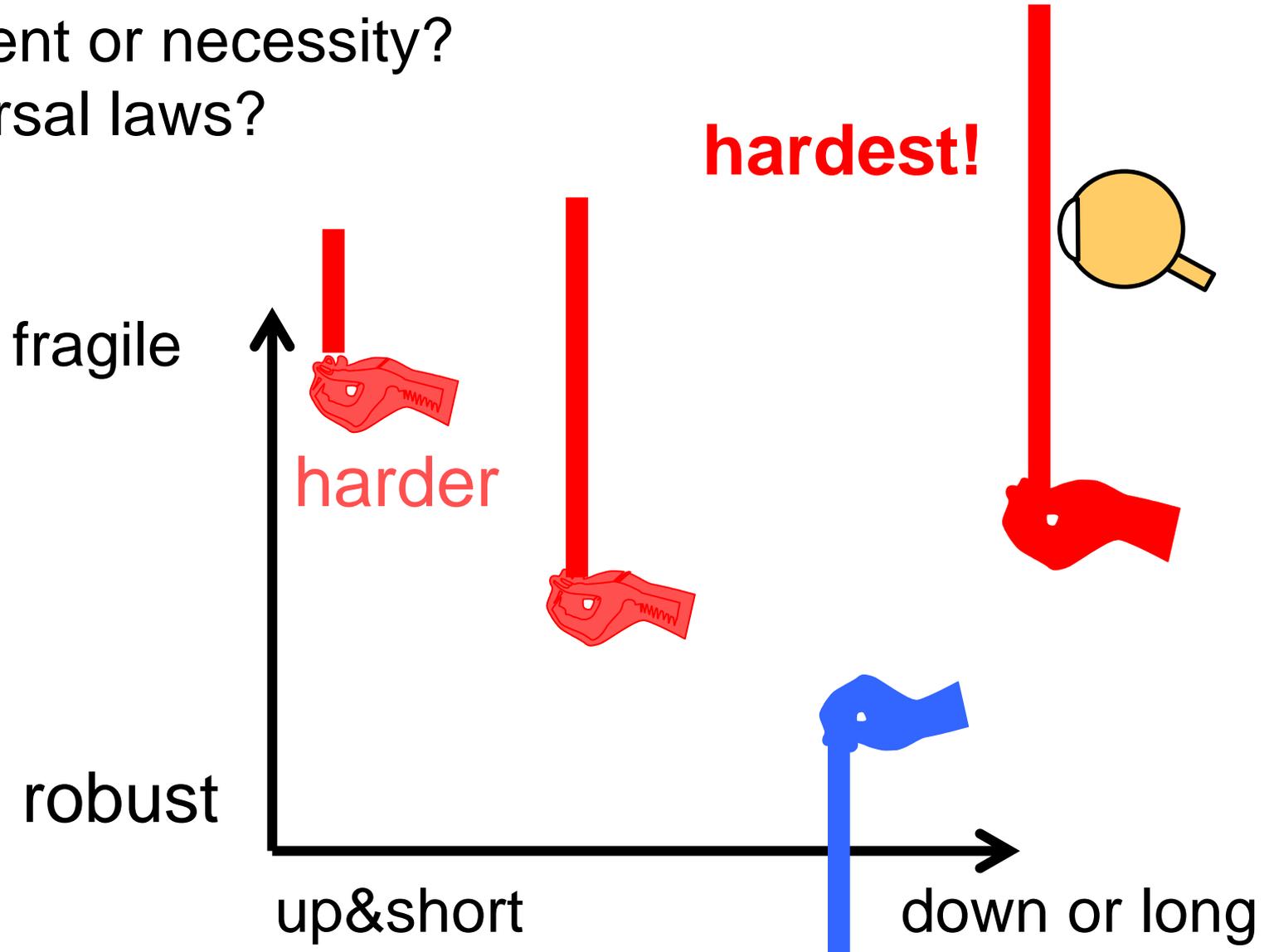


Why?

Easy to *prove* using simple models.

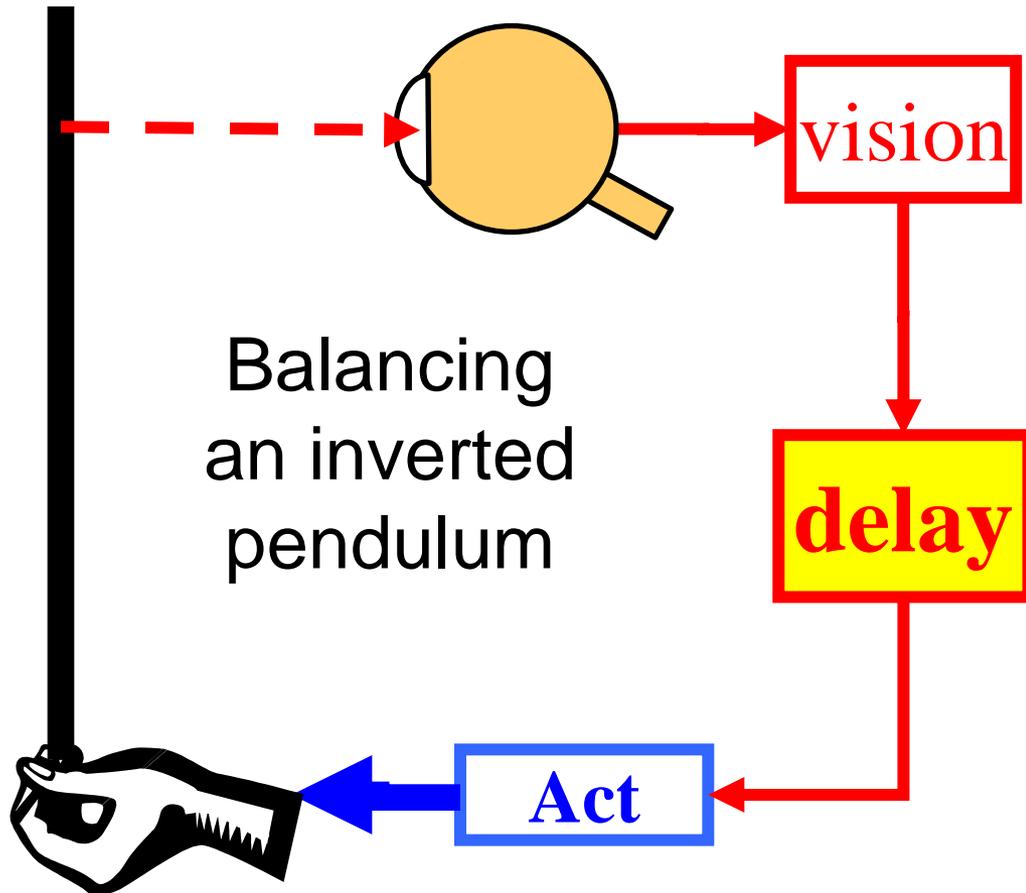
Why?

Accident or necessity?
Universal laws?



Some
minimal
math
details

Four Universal laws =



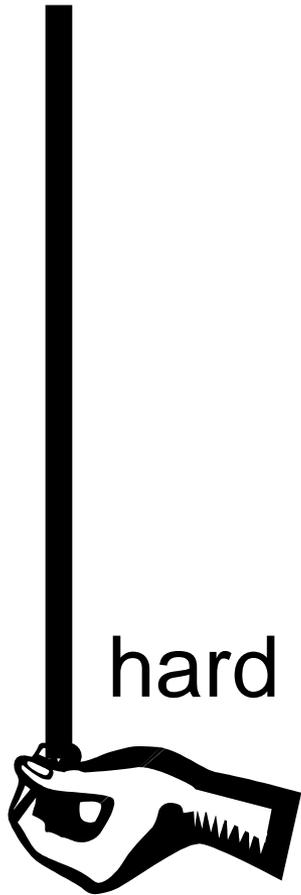
Mechanics+
Gravity +
Light +

$$\|T\|_{\infty} \geq \exp(p\tau) \left| \frac{z+p}{z-p} \right|$$

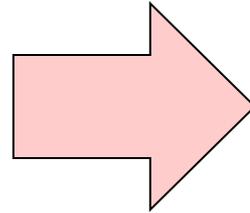
+ Neuroscience

Law #1 : Mechanics

Law #2 : Gravity



1d motion



$$(M + m) \ddot{x} + ml \ddot{\theta} = u$$

$$\ddot{x} + l \ddot{\theta} + g \theta = 0$$

$$y = x + l_0 \theta$$

linearize

$$(M + m) \ddot{x} + ml (\ddot{\theta} \cos \theta - \dot{\theta}^2 \sin \theta) = u$$

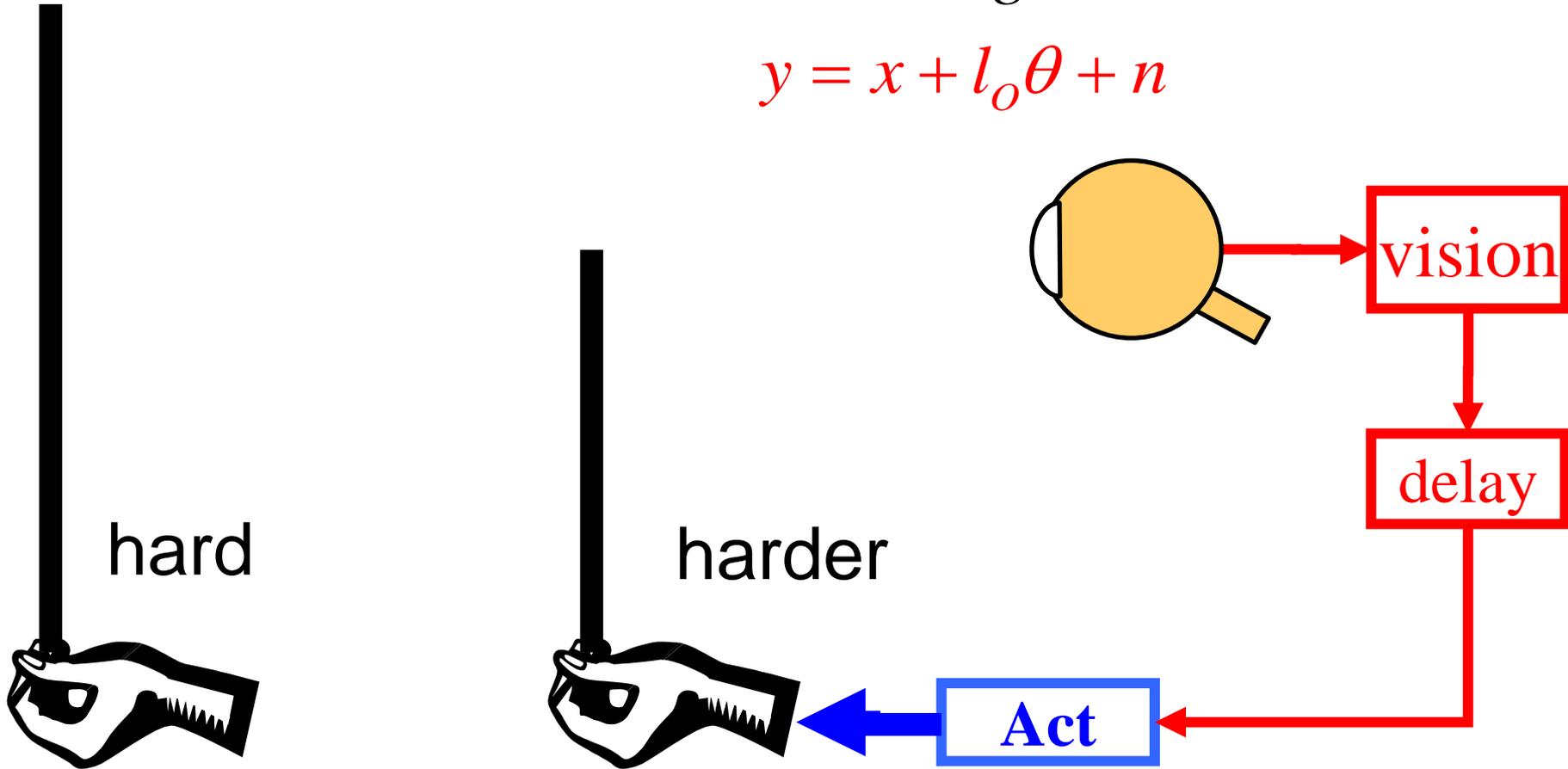
$$\ddot{x} \cos \theta + l \ddot{\theta} + g \sin \theta = 0$$

$$y = x + l_0 \sin \theta$$

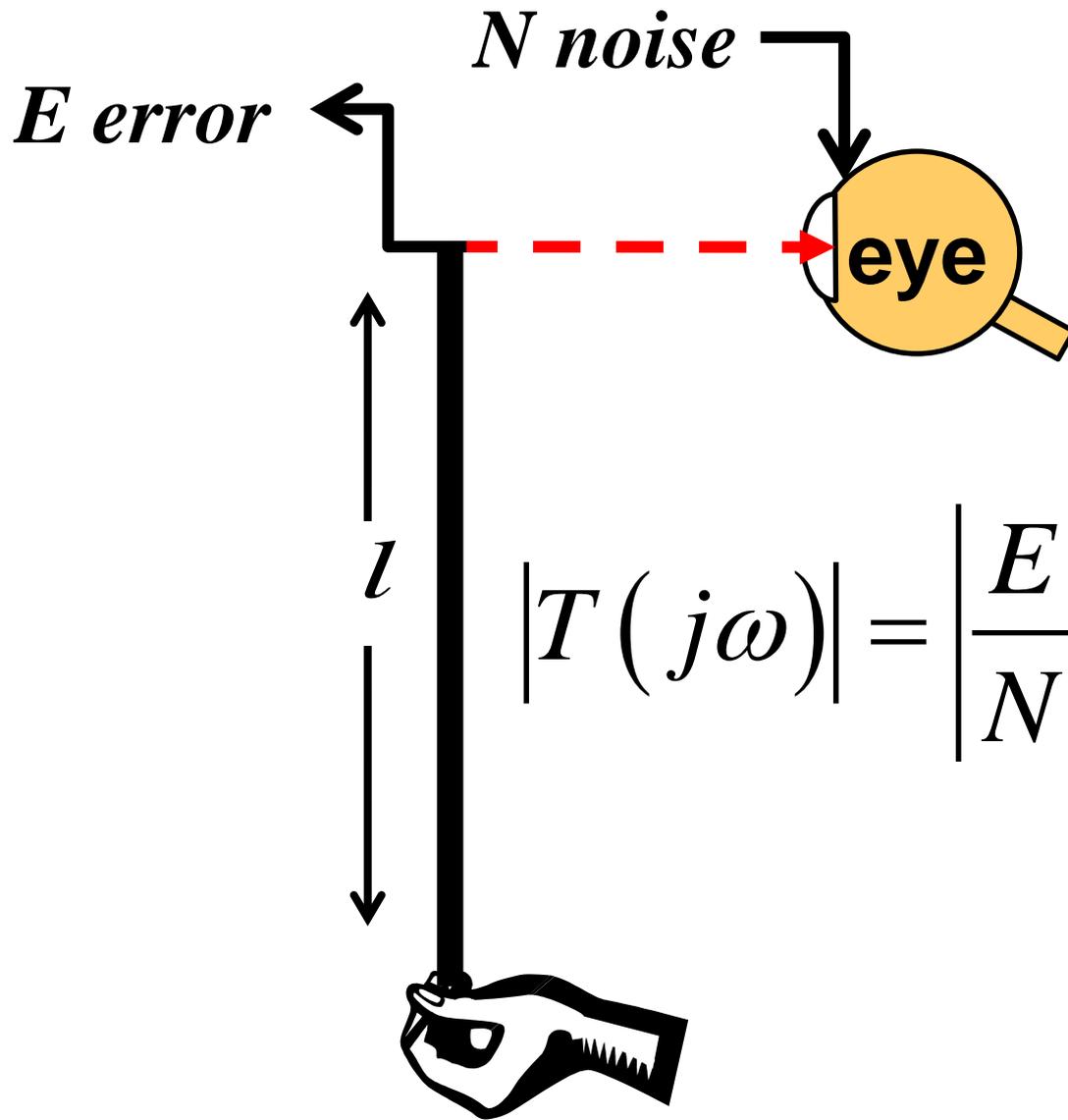
Law #3 : Light $(M + m) \ddot{x} + ml\ddot{\theta} = u$

$$\ddot{x} + l\ddot{\theta} \pm g\theta = 0$$

$$y = x + l_0\theta + n$$

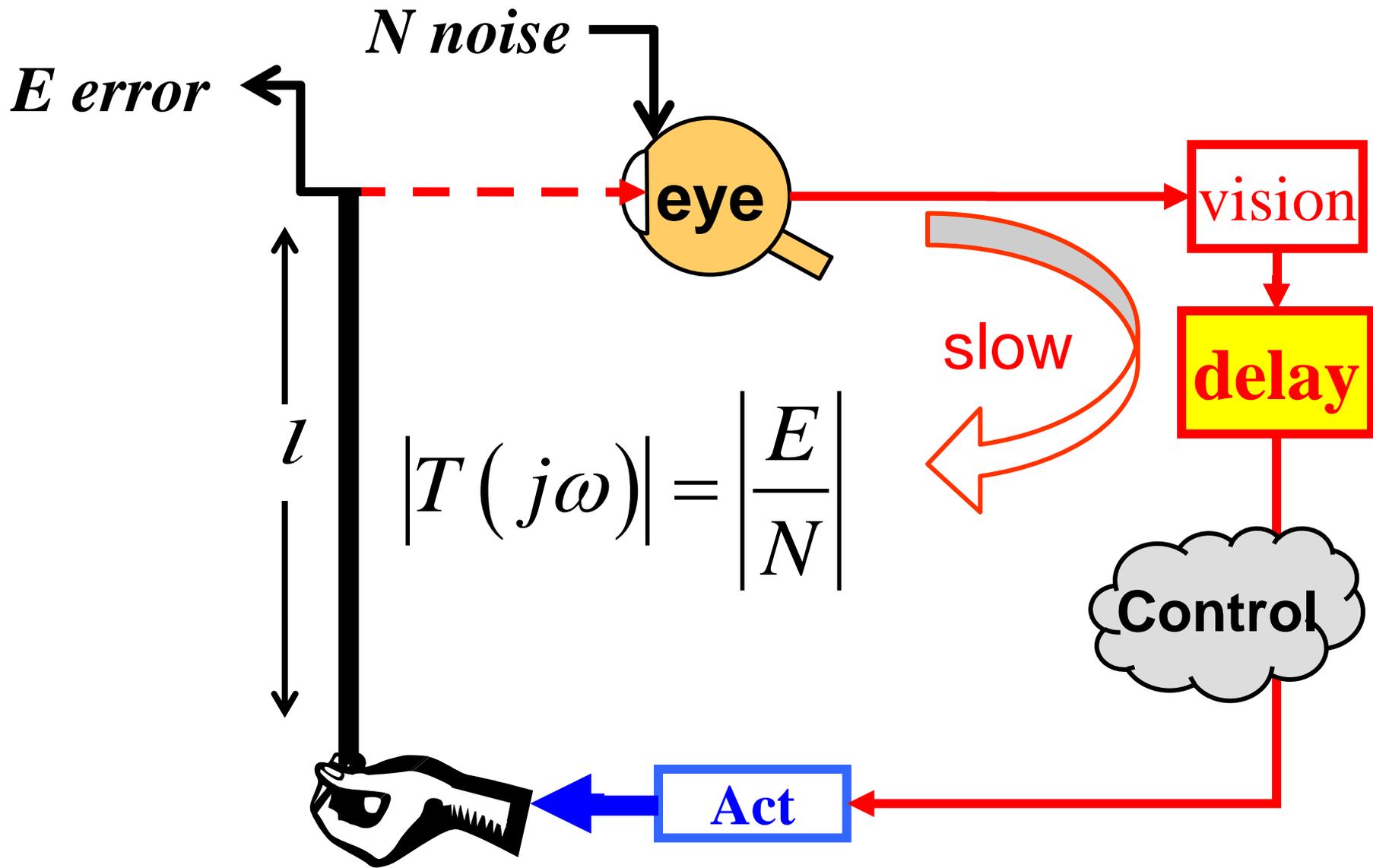


Easy to *prove* using simple models.



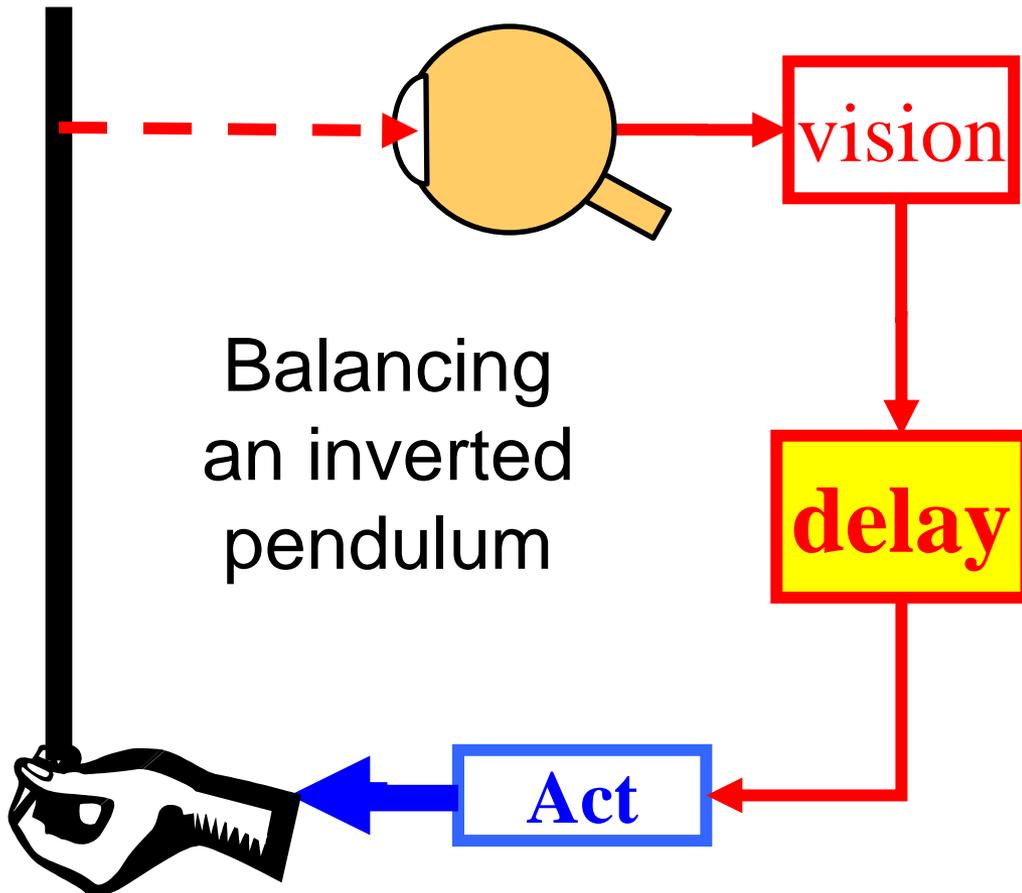
$$\|T\|_{\infty} \geq ?$$

Frequency
domain



Universal laws

Mechanics+
Gravity +
Light +



$$\|T\|_{\infty} \geq \exp(p\tau)$$

$$p \propto \sqrt{\frac{1}{l}}$$

$$\tau \approx .3s$$

$$\exp\left(\int \ln |T|\right) @ \exp\left(\frac{1}{\pi} \int_0^{\infty} \ln |T(j\omega)| \left(\frac{p}{p^2 + \omega^2}\right) d\omega\right)$$

$$\|T\|_{\infty} = \sup_{\omega} |T(j\omega)|$$

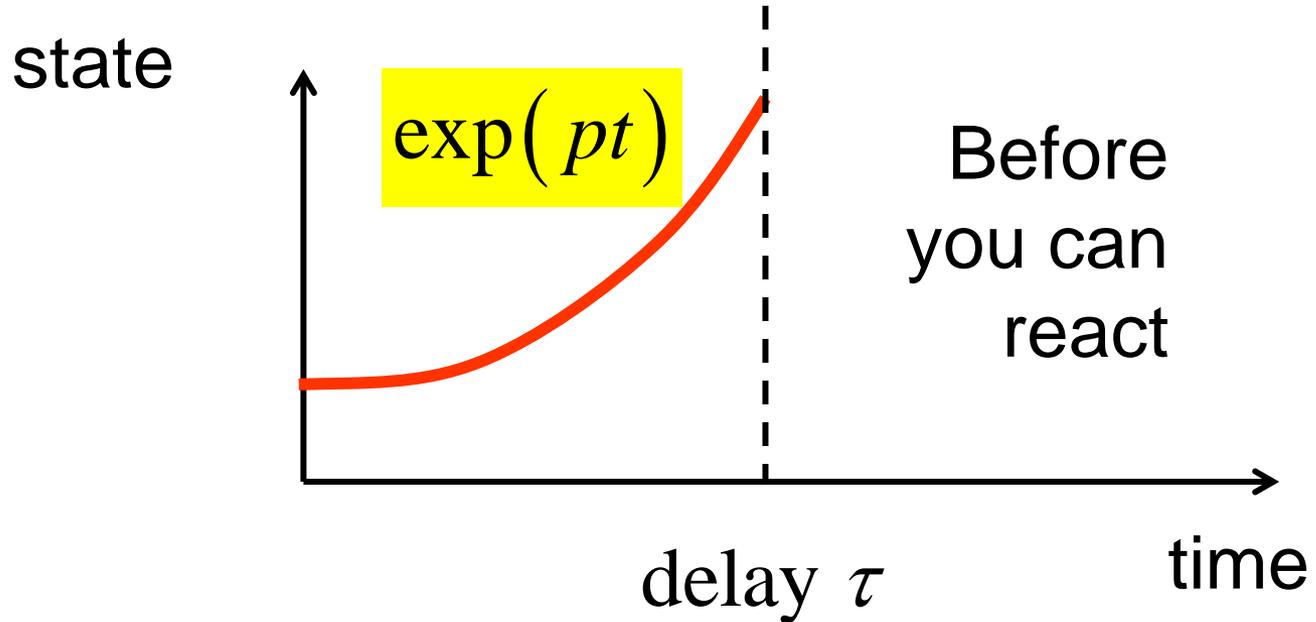
Amplification (noise to error)

Entropy rate

Energy (L2)

$$\left. \begin{array}{l} \exp\left(\int \ln |T|\right) \\ \|T\|_{\infty} \end{array} \right\} \geq \exp(p\tau)$$

intuition

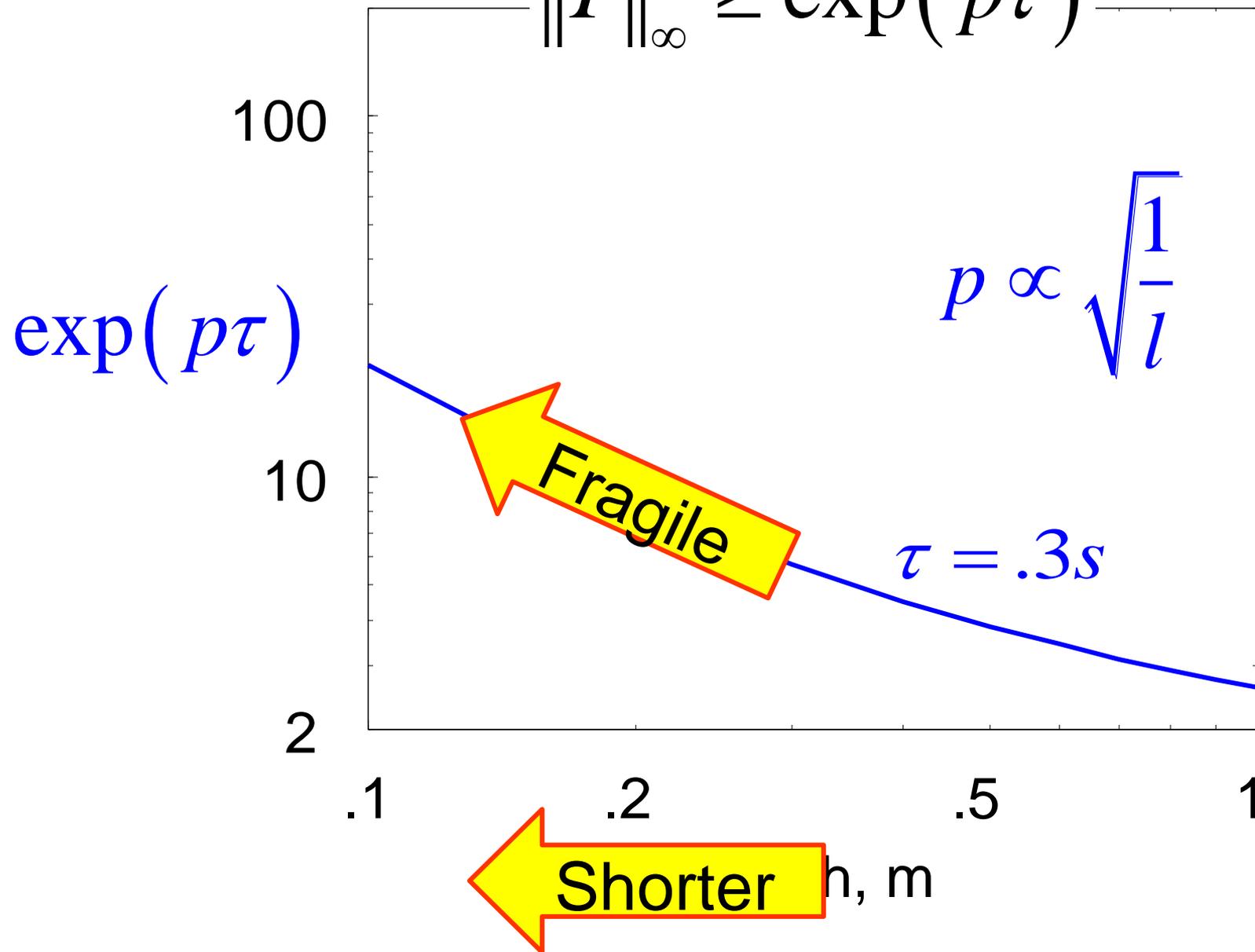


Entropy rate

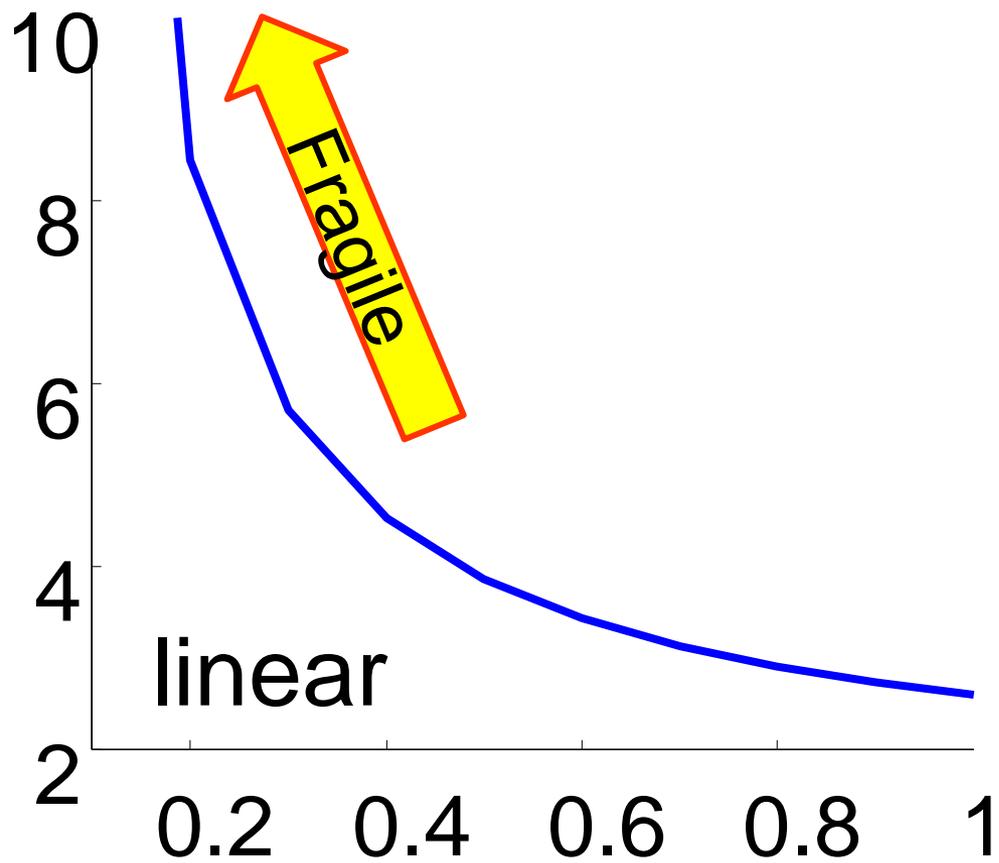
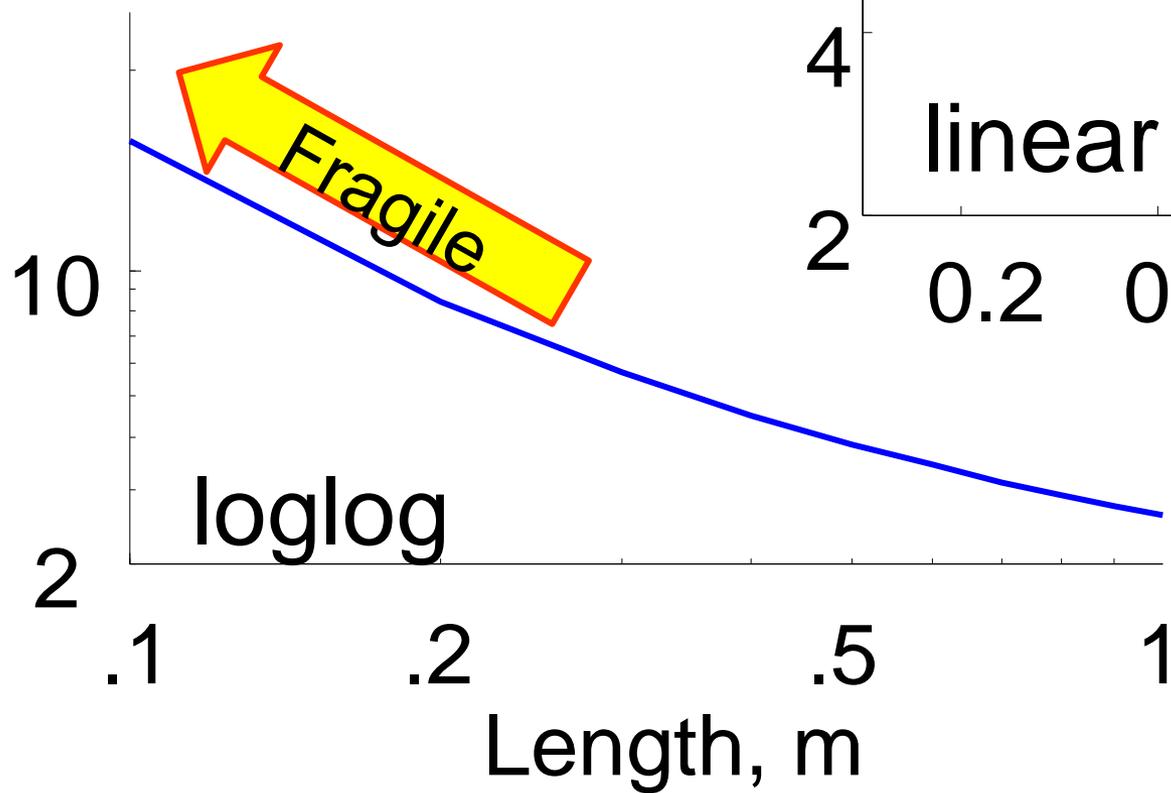
Energy (L2)

$$\left. \exp\left(\int \ln |T|\right) \right\|_{\|T\|_{\infty}} \geq \exp(p\tau)$$

$$\|T\|_{\infty} \geq \exp(p\tau)$$

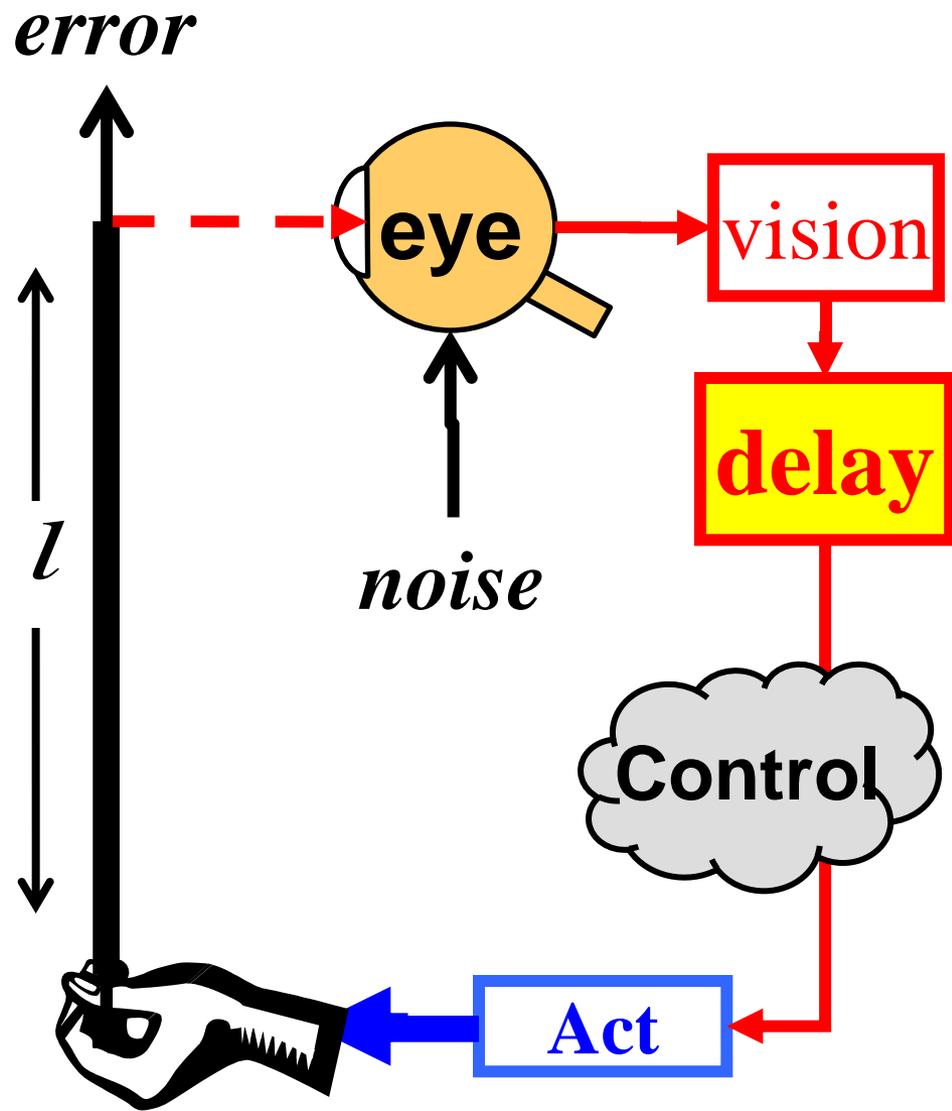
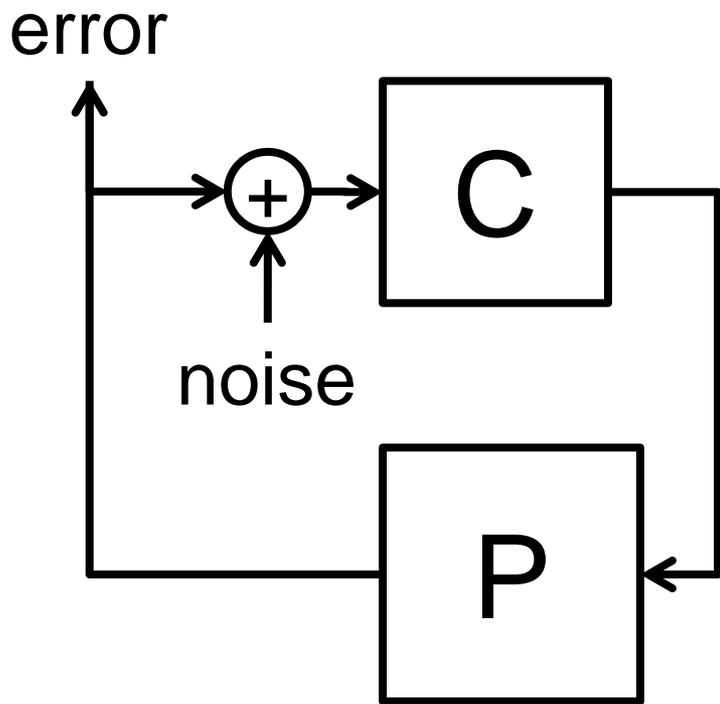


$$\exp(p\tau)$$



Also
exponential
in delay!

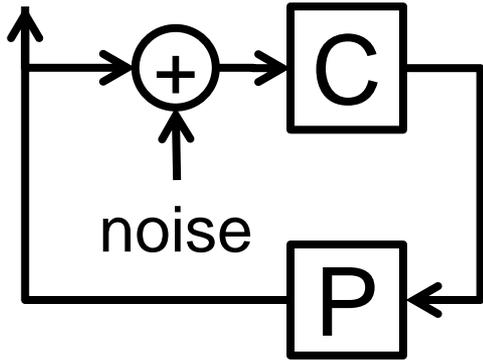
$$|T(j\omega)| = \left| \frac{E}{N} \right|$$



Proof? Easy

$$\|T\|_{\infty} = \sup_{\omega} |T(j\omega)| = \sup \left\{ |T(s)| \mid \operatorname{Re}(s) \geq 0 \right\}$$

error



Max modulus

$$T(s) = M(s)\Theta(s) \quad |\Theta(j\omega)| = 1$$

$$\Theta(s) = \exp(-\tau s)$$

$$P(p) = \infty \Rightarrow T(p) = 1$$

$$\Rightarrow M(p) = \Theta(p)^{-1}$$

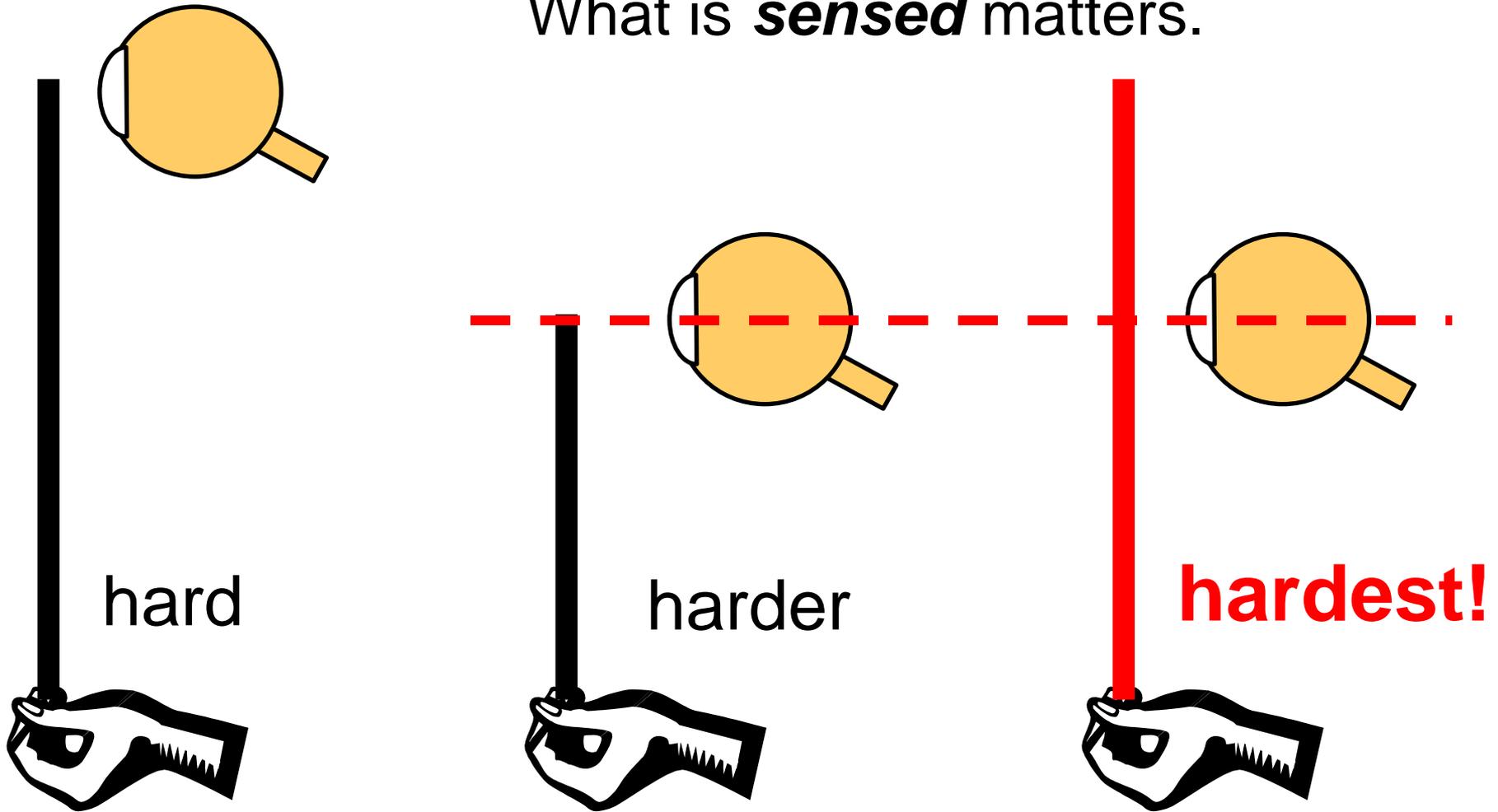
$$|T(j\omega)| = \left| \frac{E}{N} \right|$$

$$P(s) = P_M(s) \exp(-\tau s) \Rightarrow$$

$$\|T\|_{\infty} = \|M\|_{\infty} \geq |M(p)| \geq |\Theta(p)^{-1}| \geq \exp(\tau p)$$

$$\Rightarrow \|T\|_{\infty} \geq \exp(\tau p)$$

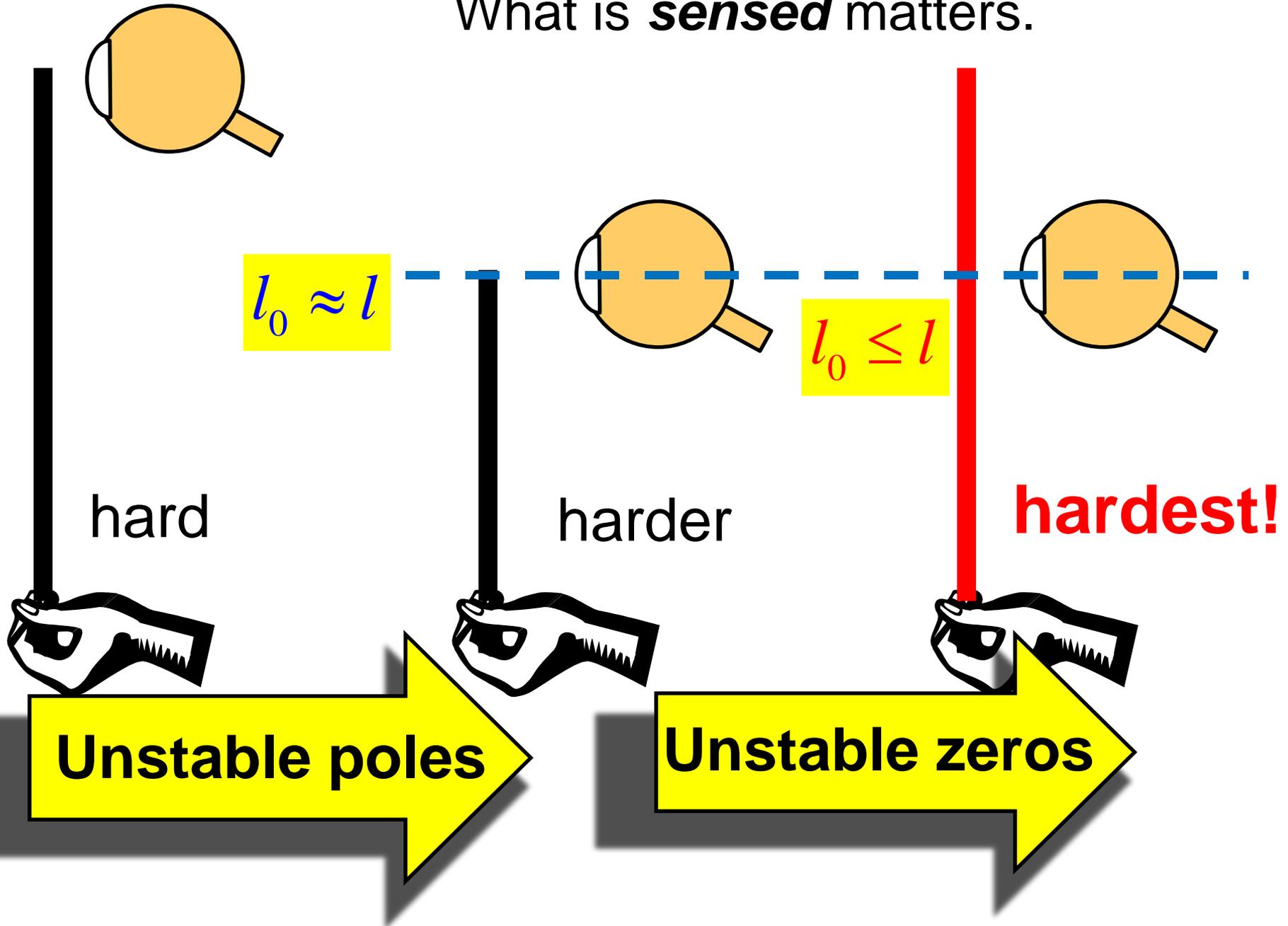
What is *sensed* matters.



Why?

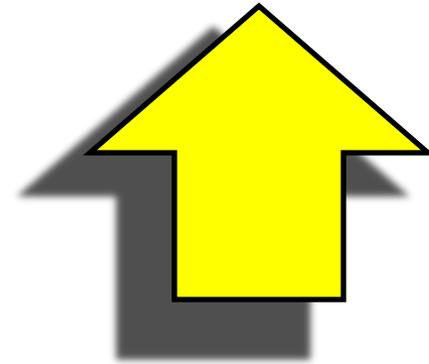
Easy to *prove* using simple models.

What is *sensed* matters.



Fragility two ways (Bode* and Zames):

$$\left. \exp\left(\int \ln |T|\right) \right\|_{\|T\|_{\infty}} \geq \exp(p\tau) \left| \frac{z+p}{z-p} \right|$$

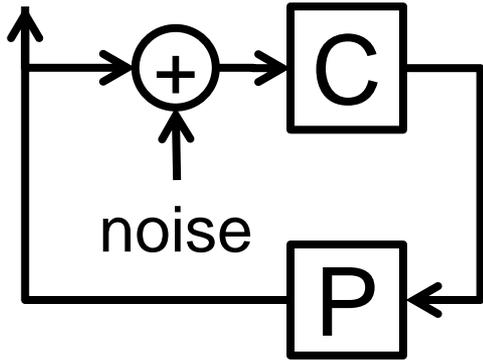


Unstable zeros

Proof?

$$\|T\|_{\infty} = \sup_{\omega} |T(j\omega)| = \sup \left\{ |T(s)| \mid \operatorname{Re}(s) \geq 0 \right\}$$

error



$$T(s) = M(s)\Theta(s) \quad |\Theta(j\omega)| = 1$$

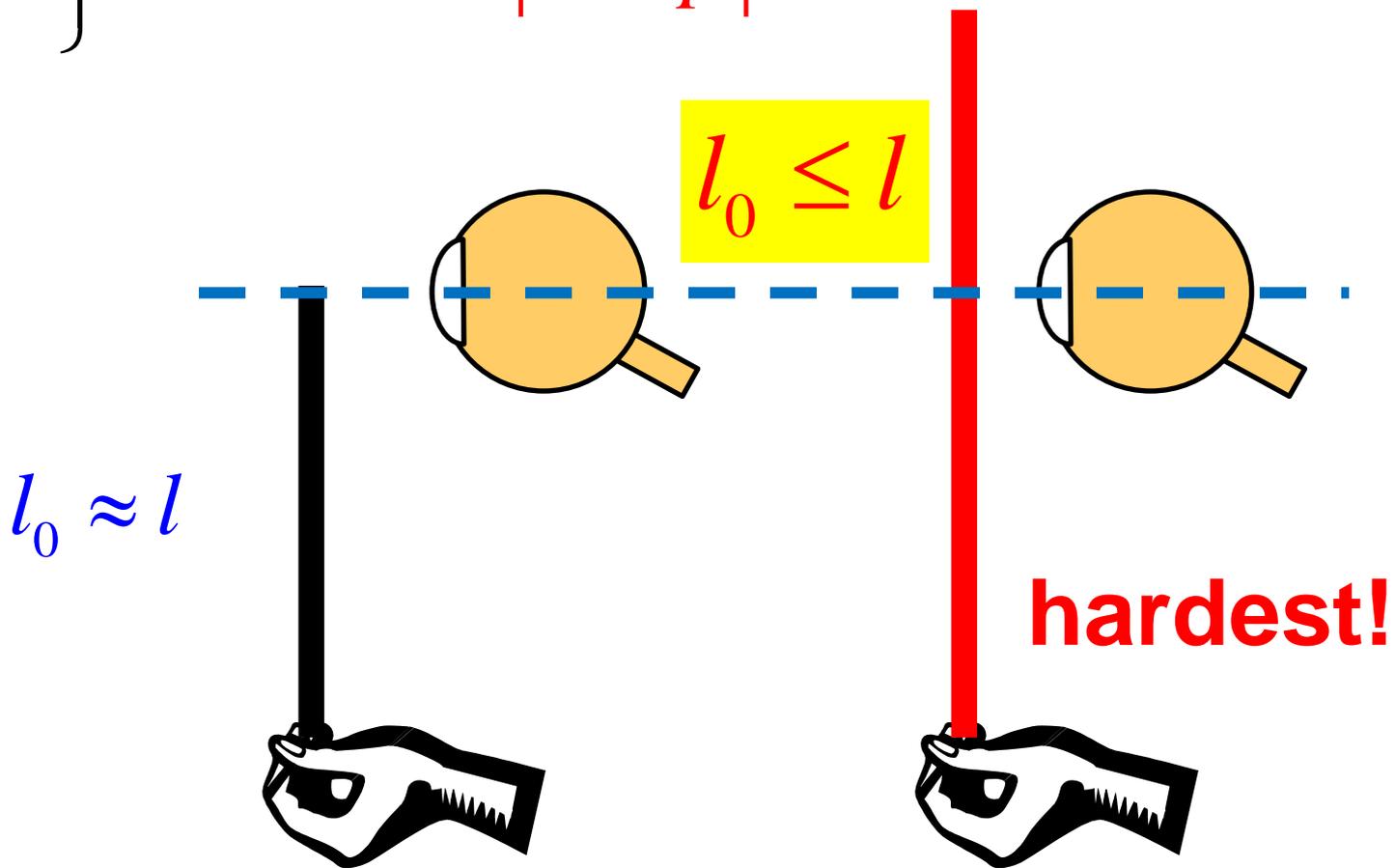
$$\Theta(s) = \exp(-\tau s) \frac{s - z}{s + z}$$

$$P(s) = P_M(s) \left[\exp(-\tau s) \frac{s - z}{s + z} \right] \Rightarrow$$

$$\|T\|_{\infty} = \|M\|_{\infty} \geq |M(p)| \geq |\Theta(p)^{-1}| \geq \exp(\tau p) \left| \frac{z + p}{z - p} \right|$$

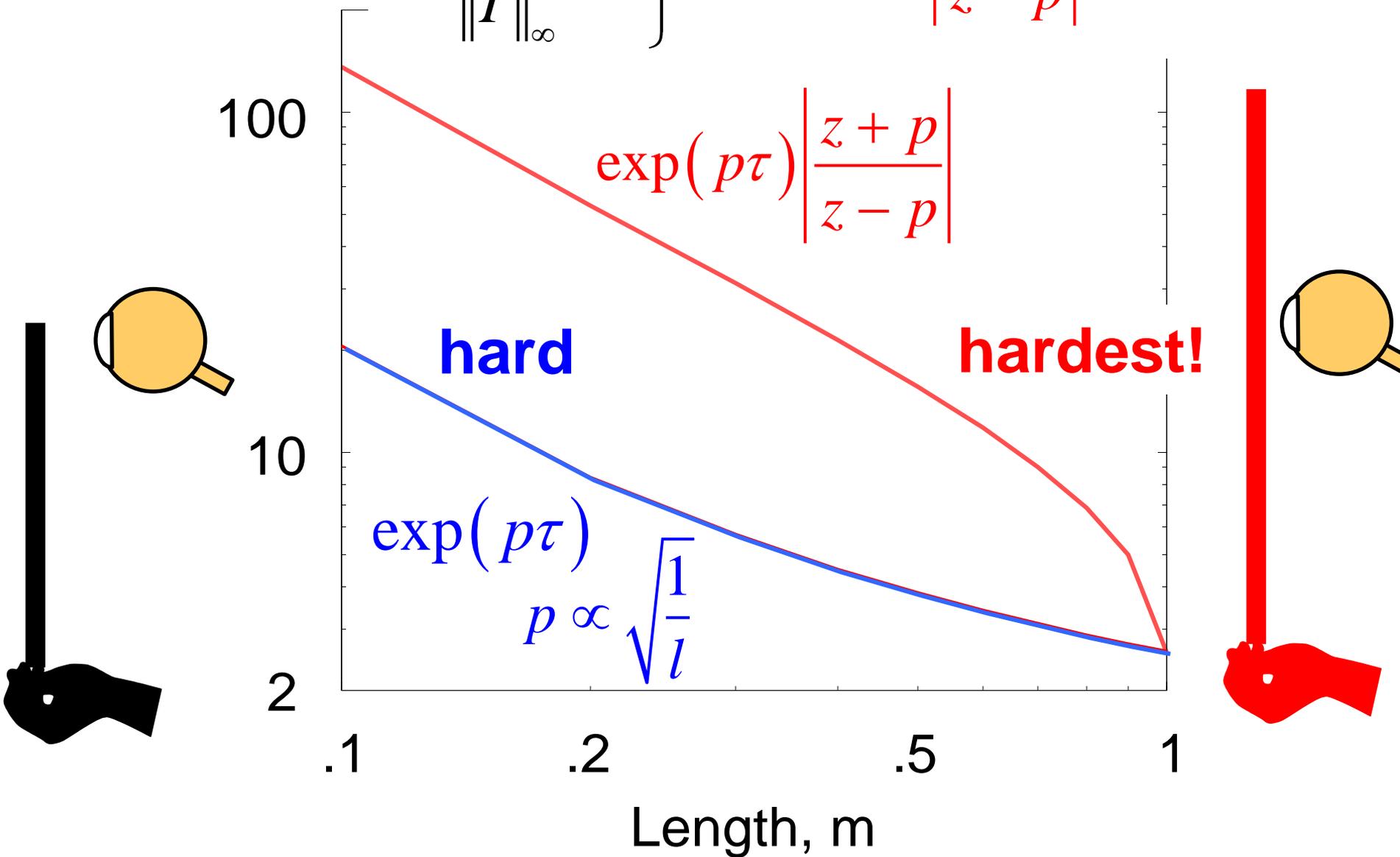
$$\Rightarrow \|T\|_{\infty} \geq \exp(\tau p) \left| \frac{z + p}{z - p} \right|$$

$$\left. \exp\left(\int \ln |T|\right) \right\|_{\infty} \geq \exp(p\tau) \left| \frac{z+p}{z-p} \right| \geq \exp(p\tau)$$

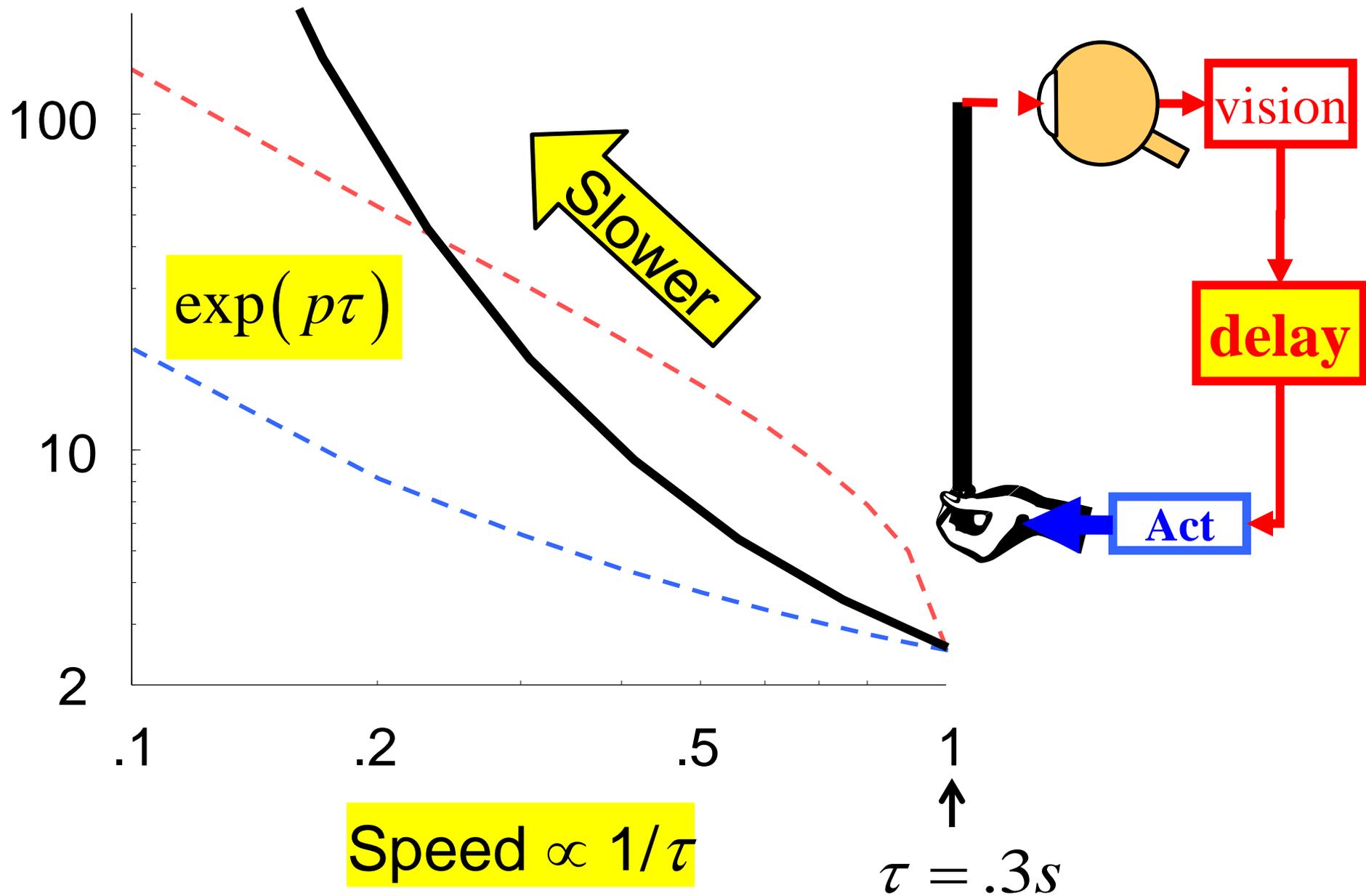


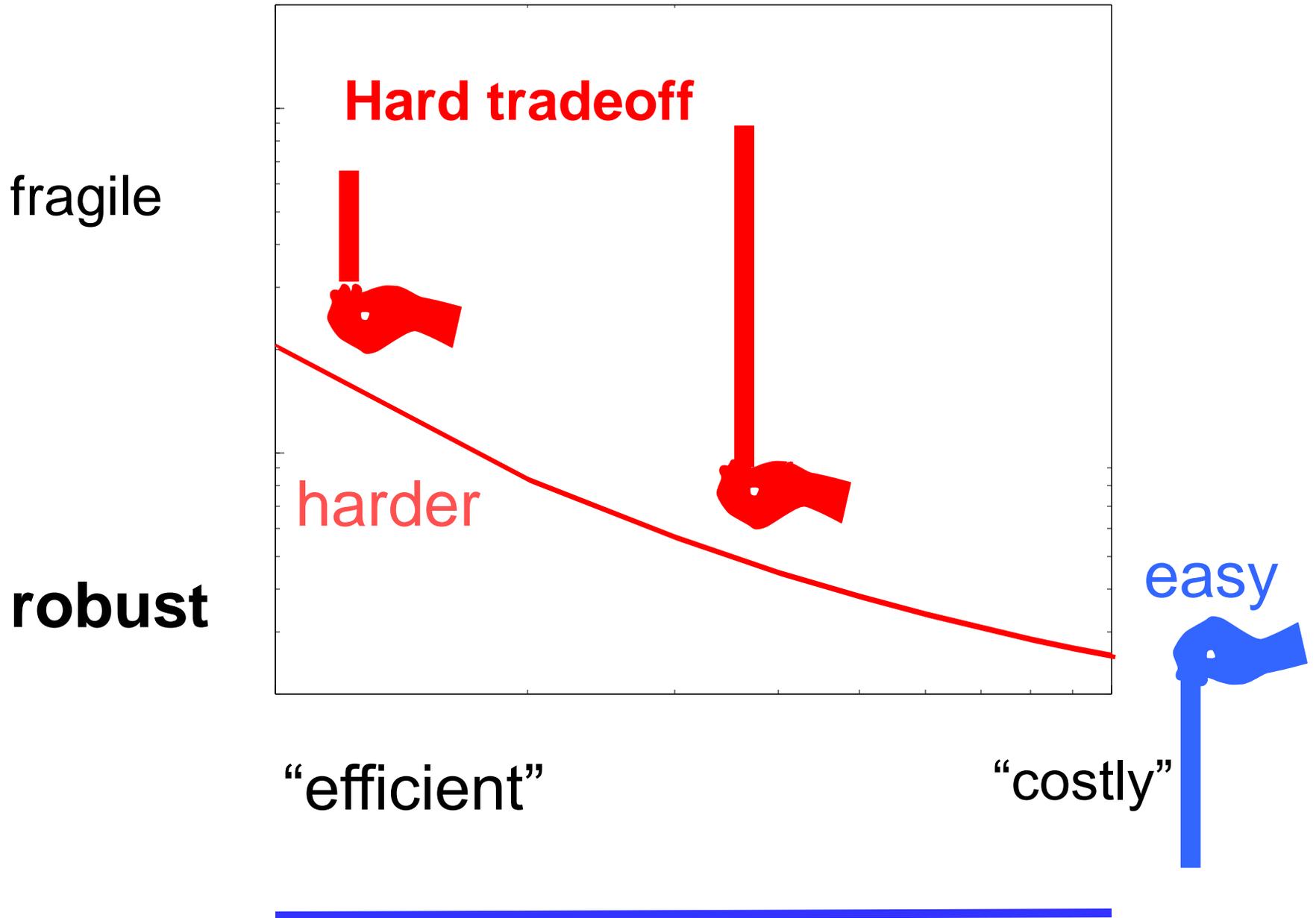
$$\tau = .3s$$

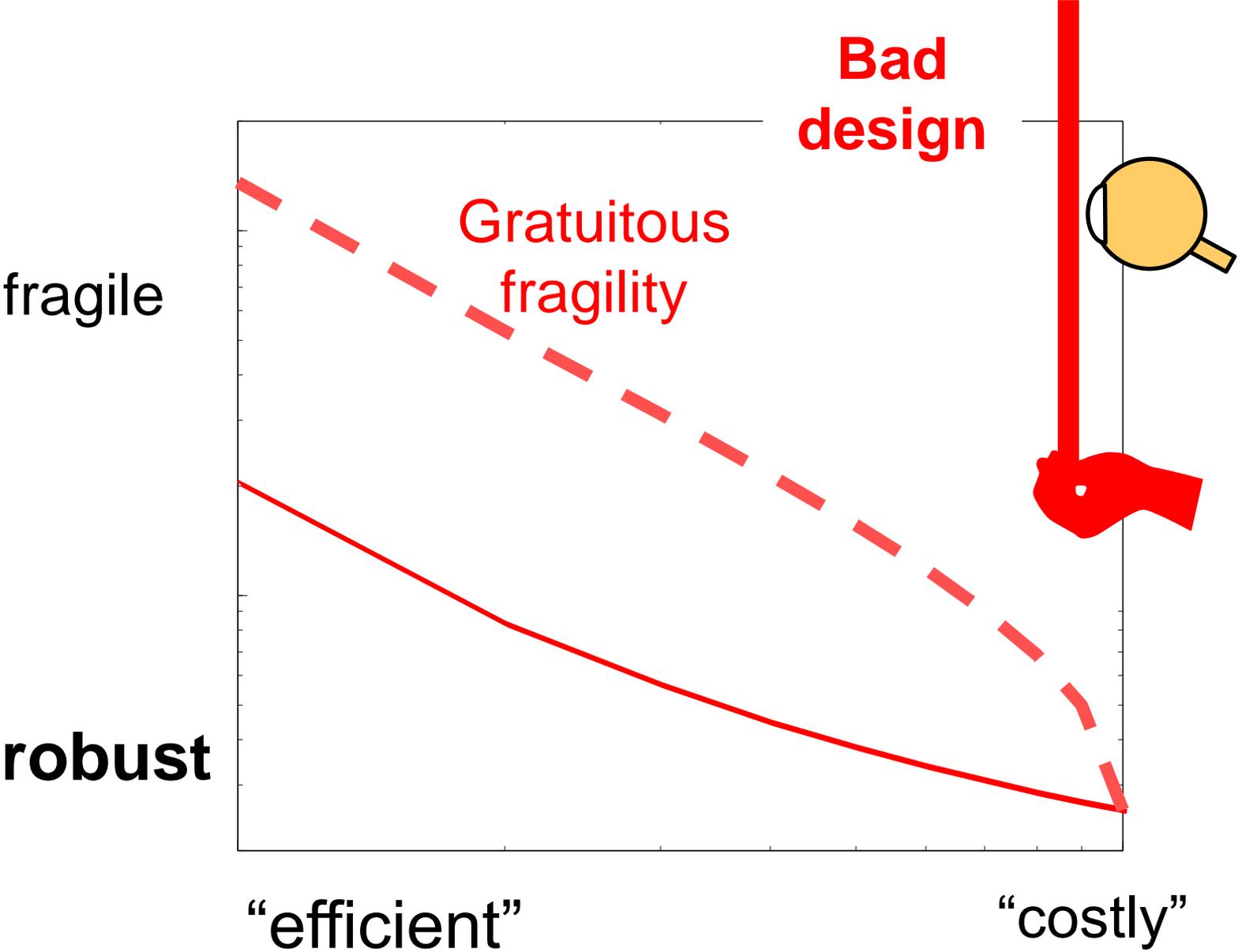
$$\left. \begin{array}{l} \exp\left(\int \ln|T|\right) \\ \|T\|_\infty \end{array} \right\} \geq \exp(p\tau) \left| \frac{z+p}{z-p} \right| \geq \exp(p\tau)$$



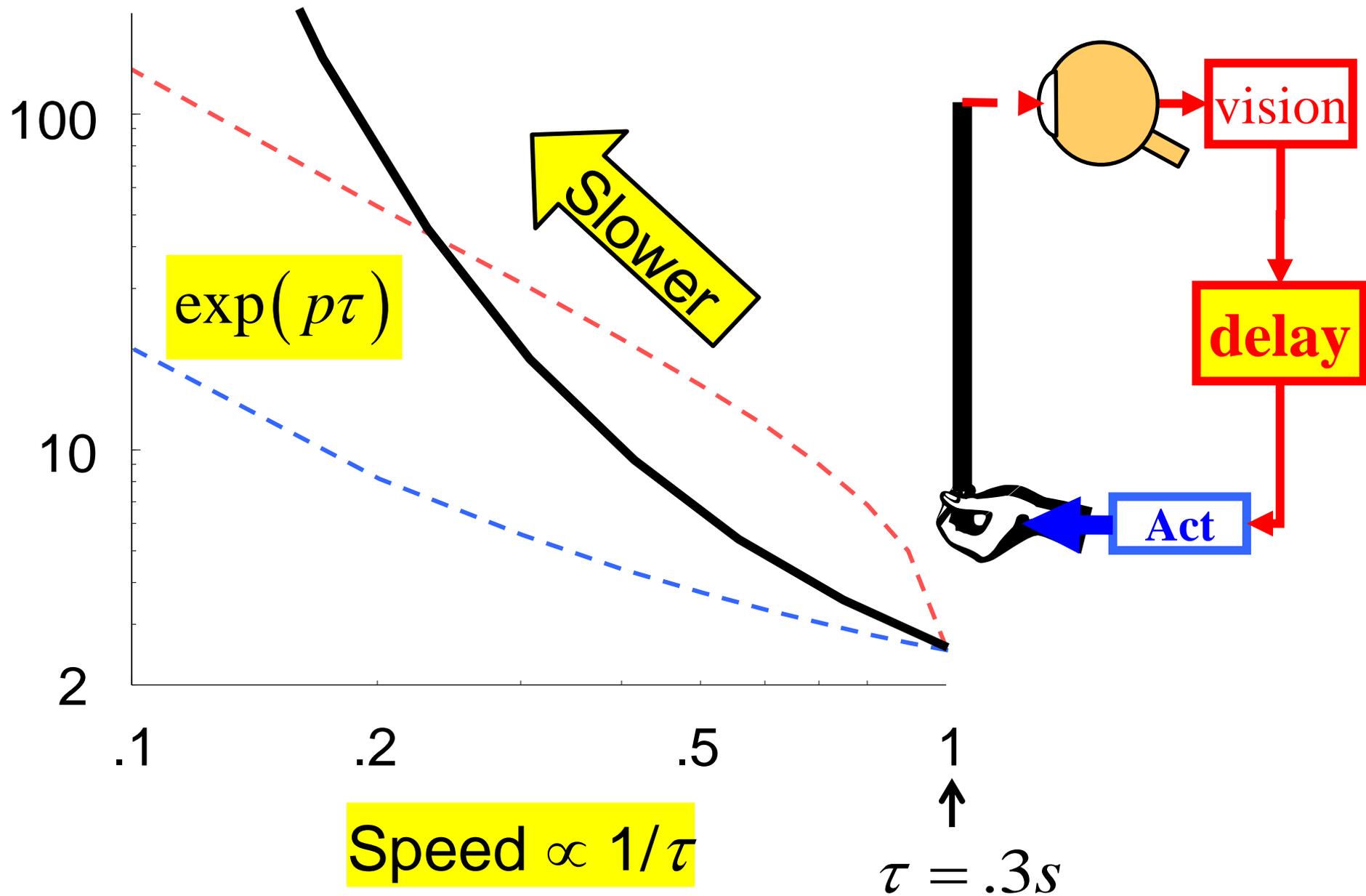
Vary delay?







Delay kills.

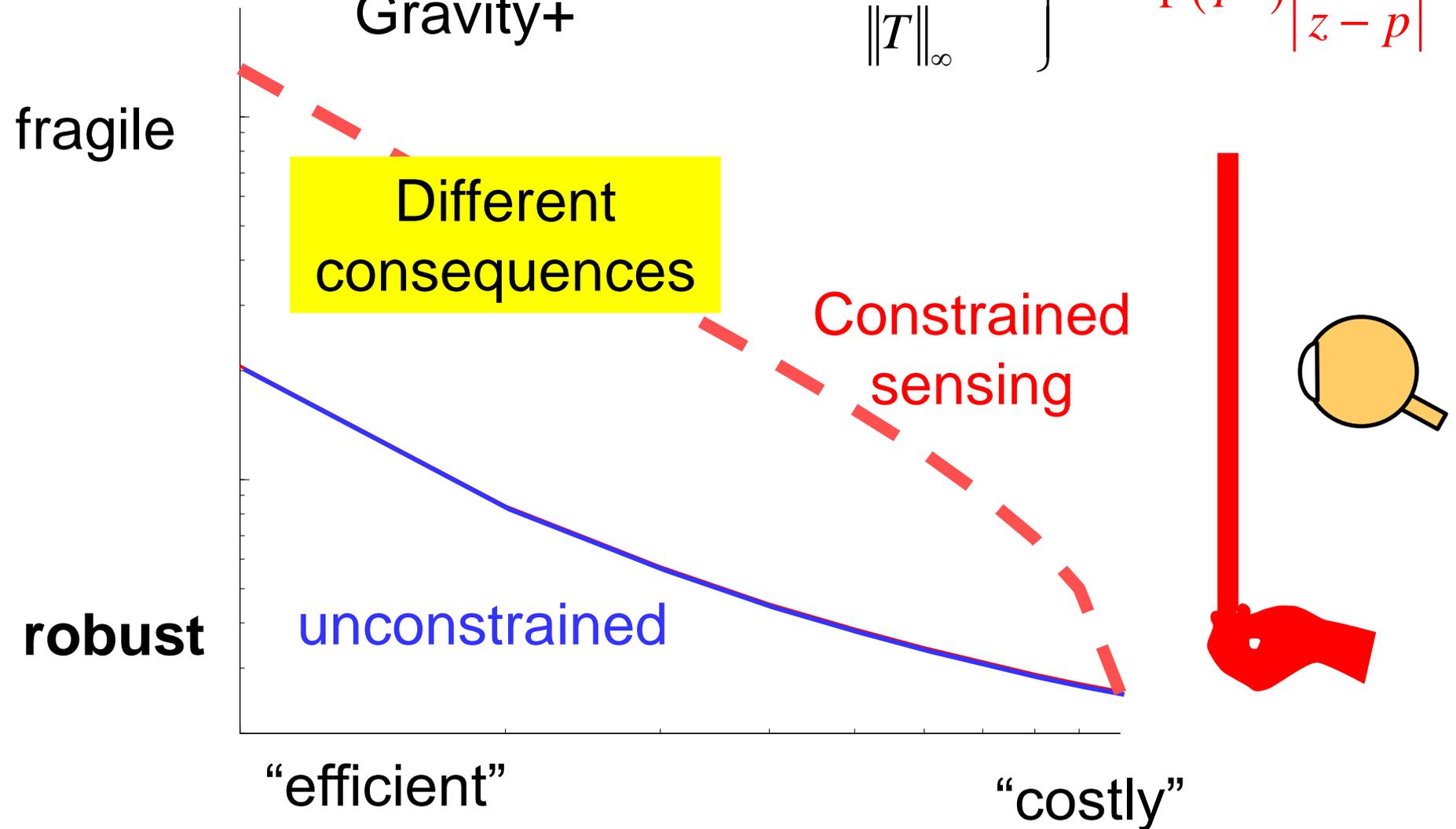


The nature of “laws”

Same constraints:

Mechanics+
Gravity+

$$\left. \begin{array}{l} \exp\left(\int \ln|T|\right) \\ \|T\|_{\infty} \end{array} \right\} \geq \exp(p\tau) \left| \frac{z+p}{z-p} \right|$$



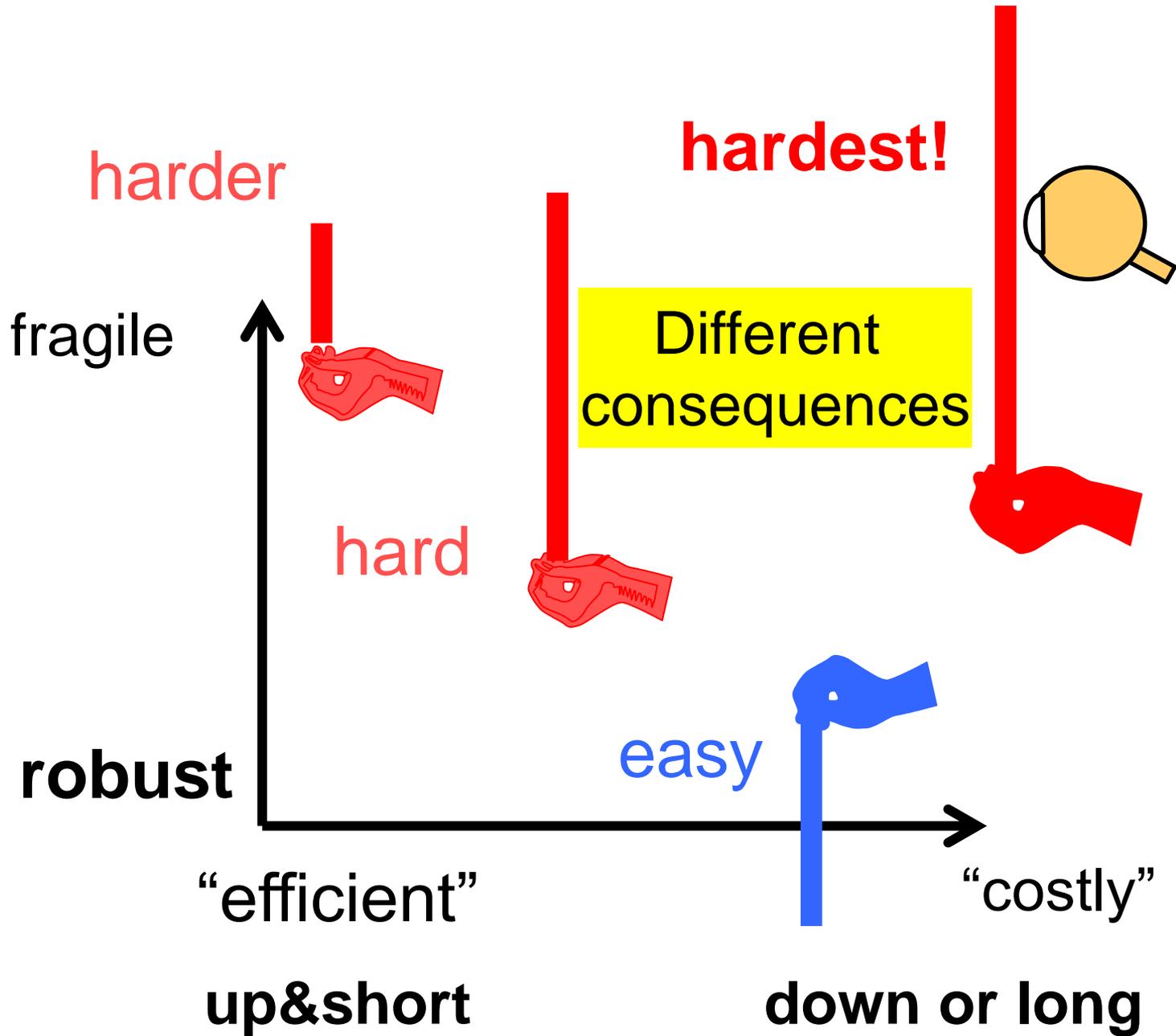
Different
consequences

Constrained
sensing

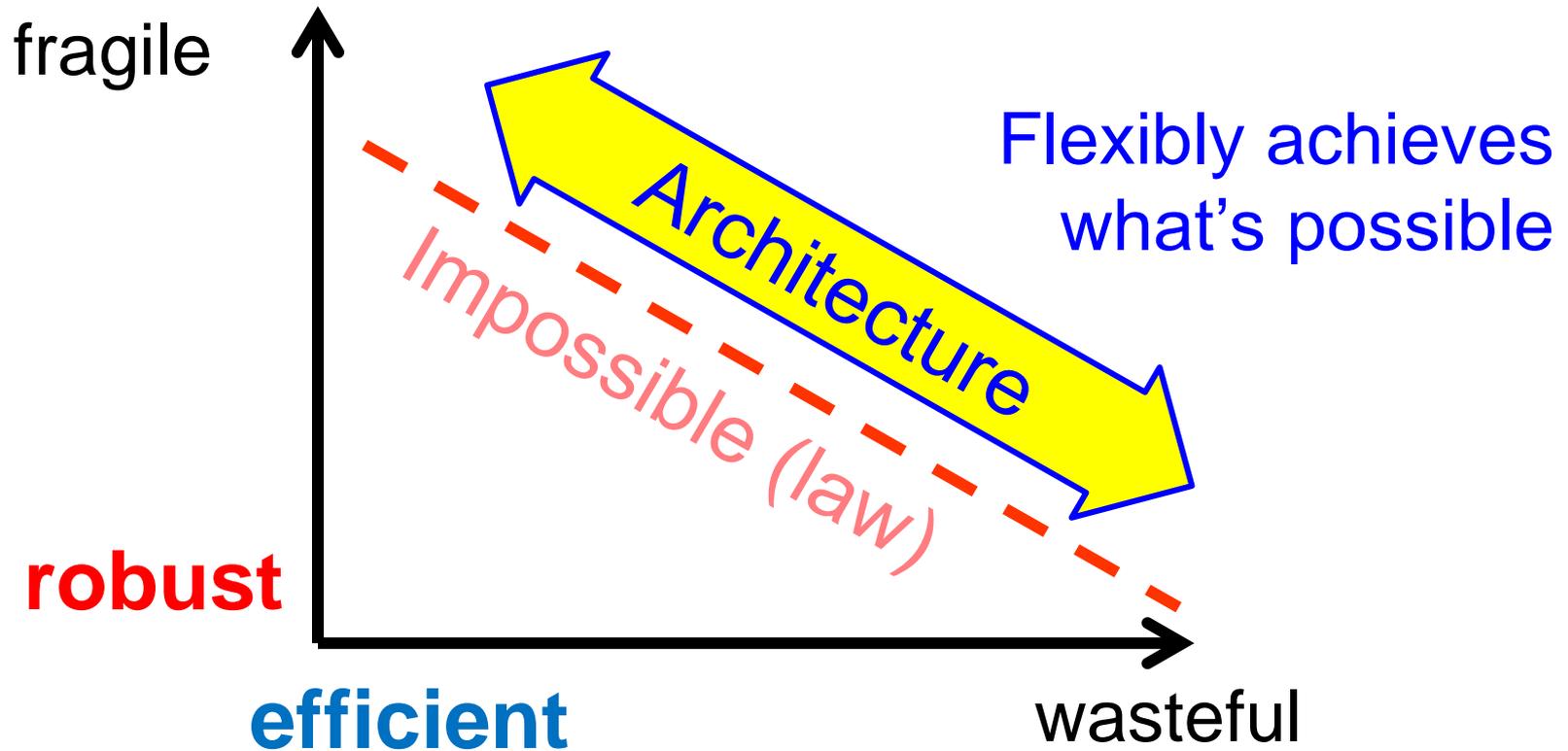
unconstrained

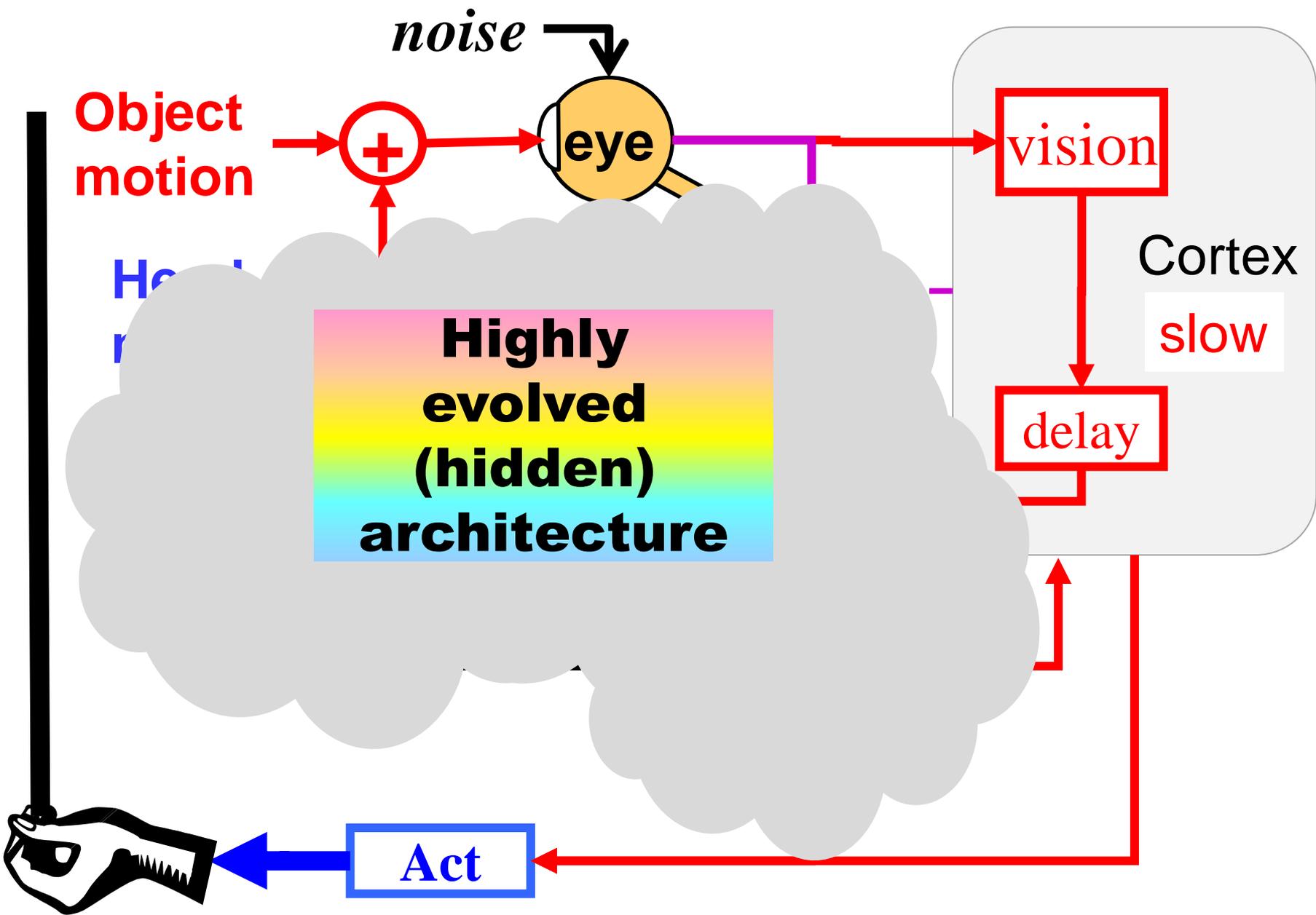
“efficient”

“costly”

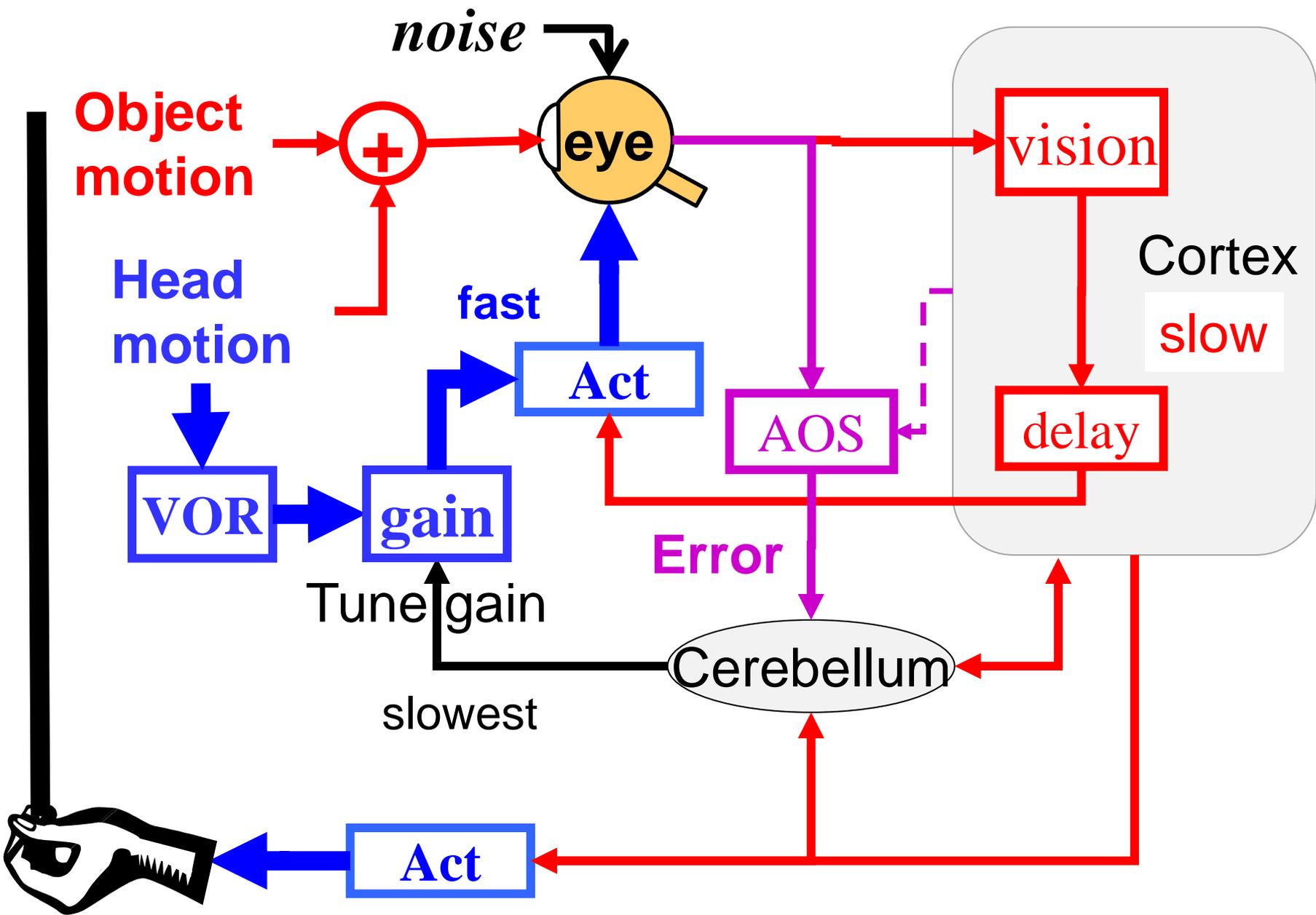


Universal laws and architectures

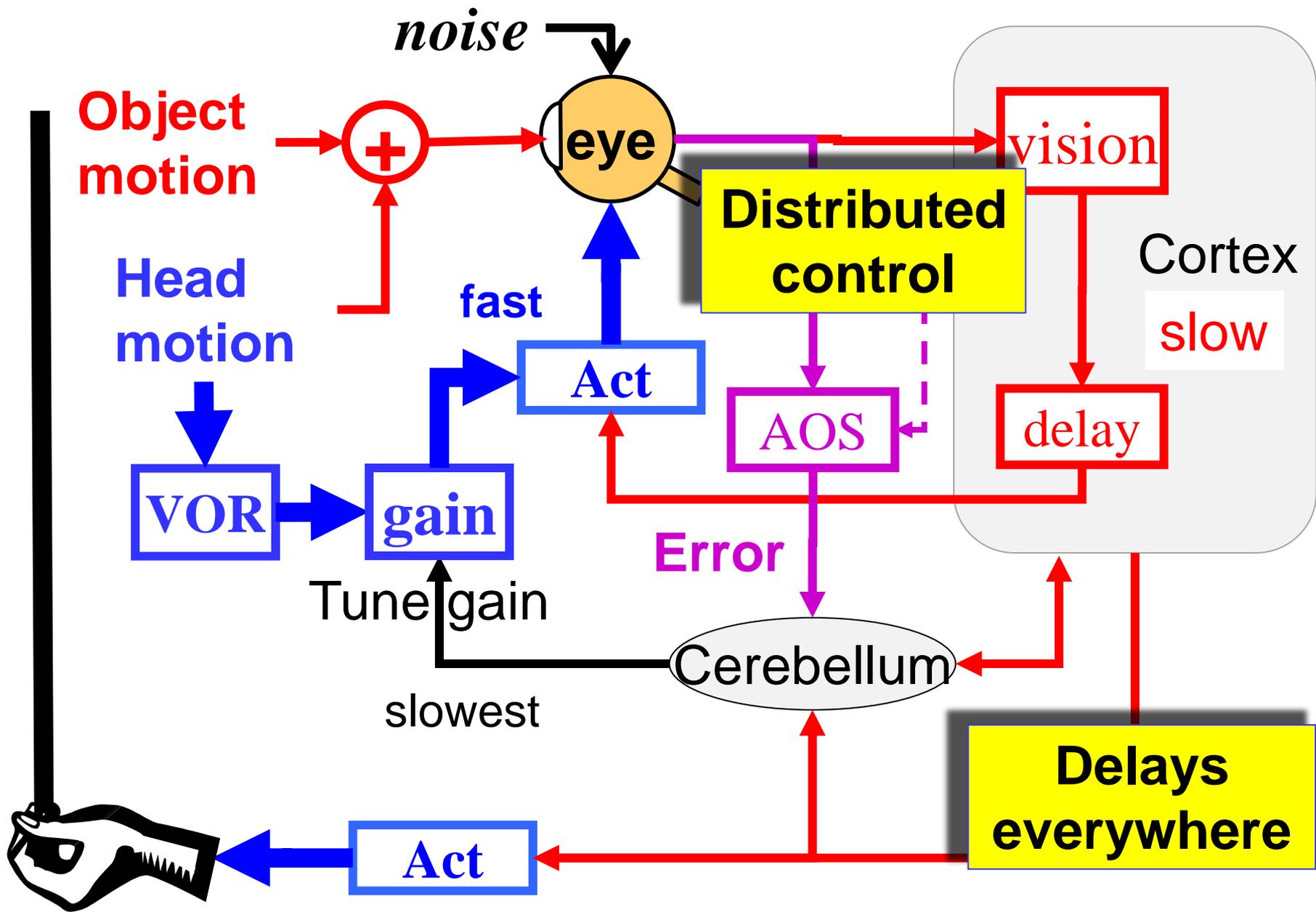




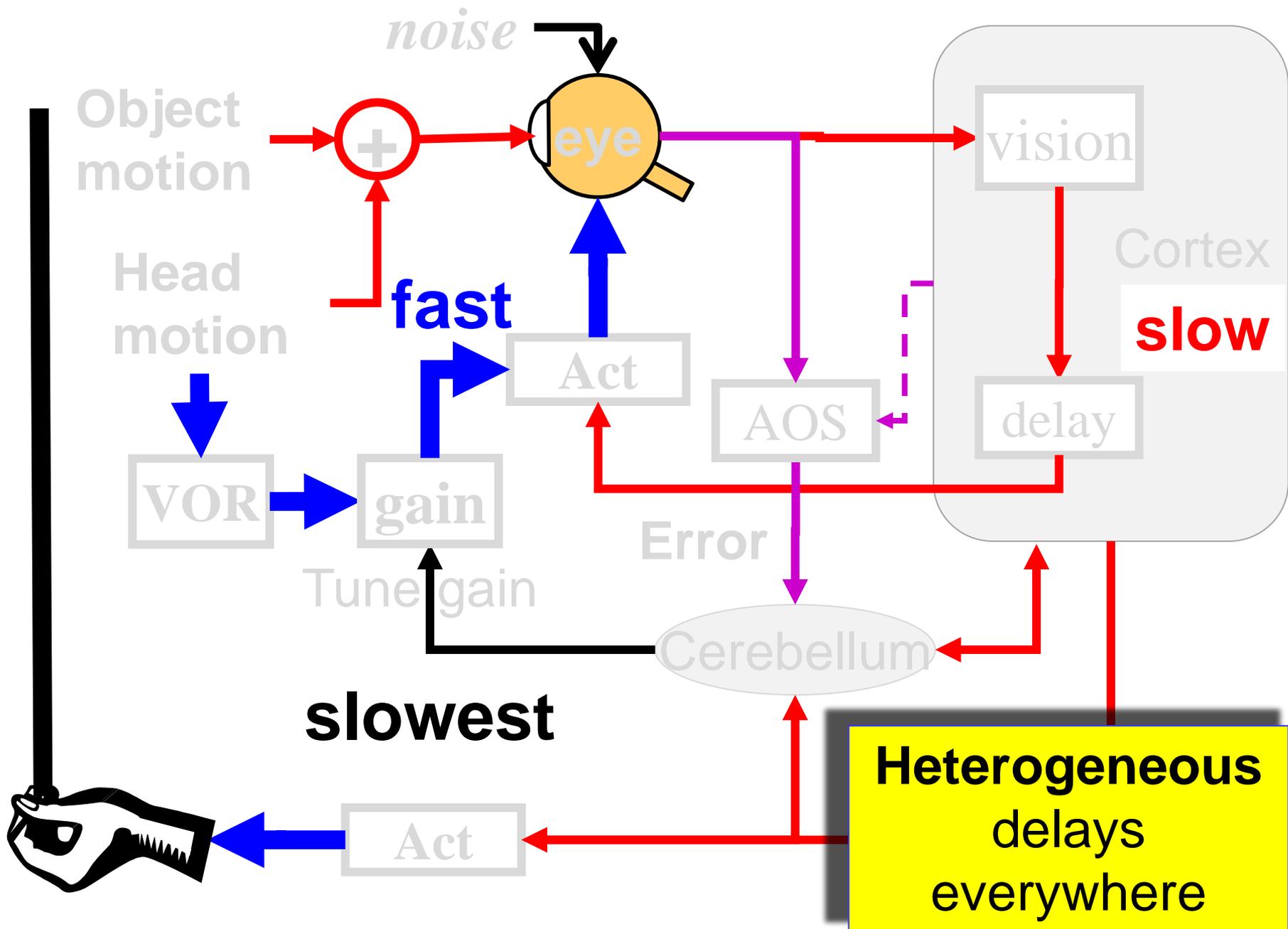
AOS = Accessory Optical system



AOS = Accessory Optical system



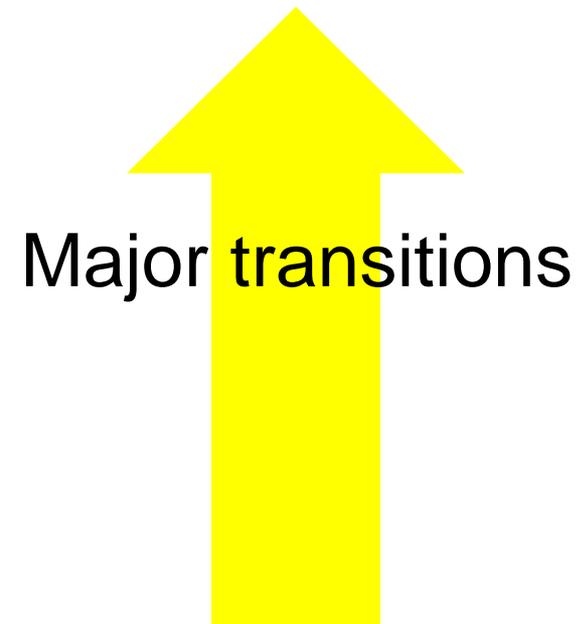
AOS = Accessory Optical system



Efficiency/instability/layers/feedback

How universal? Very.

- Sustainable infrastructure? (e.g. smartgrids)
- Money/finance/lobbyists/etc
- Industrialization
- Society/agriculture/weapons/etc
- Bipedalism
- Maternal care
- Warm blood
- Flight
- Mitochondria
- Oxygen
- Translation (ribosomes)
- **Glycolysis (2011 Science)**



Glycolytic Oscillations and Limits on Robust Efficiency

Fiona A. Chandra,^{1*} Gentian Buzi,² John C. Doyle²

Both engineering and evolution are constrained by trade-offs between efficiency and robustness, but theory that formalizes this fact is limited. For a simple two-state model of glycolysis, we explicitly derive analytic equations for hard trade-offs between robustness and efficiency with oscillations as an inevitable side effect. The model describes how the trade-offs arise from individual parameters, including the interplay of feedback control with autocatalysis of network products necessary to power and catalyze intermediate reactions. We then use control theory to prove that the essential features of these hard trade-off “laws” are universal and fundamental, in that they depend minimally on the details of this system and generalize to the robust efficiency of any autocatalytic network. The theory also suggests worst-case conditions that are consistent with initial experiments.

Chandra, Buzi, and Doyle

Insight
Accessible
Verifiable

UG biochem, math, control theory

the cell's use of ATP. In glycolysis, two ATP molecules are consumed upstream and four are produced downstream, which normalizes to $q = 1$ (each y molecule produces two downstream) with kinetic exponent $a = 1$. To highlight essential trade-offs with the simplest possible analysis, we normalize the concentration such that the unperturbed ($\delta = 0$) steady states are $\bar{y} = 1$ and $\bar{x} = 1/k$ [the system can have one additional steady state, which is unstable when $(1, 1/k)$ is stable]. [See the supporting online material (SOM) part I]. The basal rate of the PFK reaction and the consumption rate have been normalized to 1 (the 2 in the numerator and feedback coefficients of the reactions come from these normalizations). Our results hold for more general systems as discussed below and in SOM, but the analysis



Glycolytic oscillations

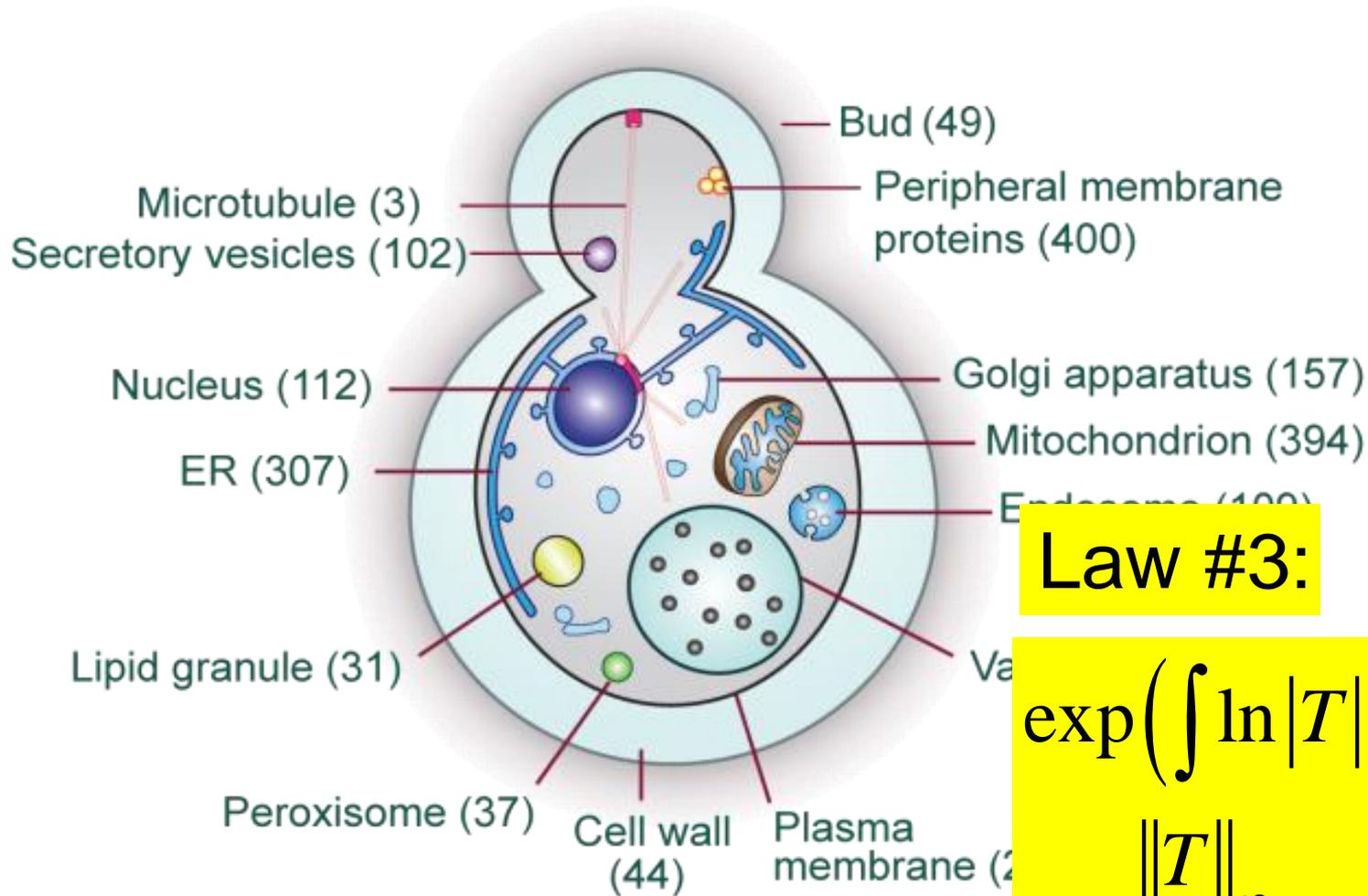
- Exhaustively studied
 - Extensive experiments and data
 - Detailed models and simulations
 - Great! But all just deepen the mystery
- Perfectly illustrates “conservation law”
- Without which? Bewilderment.

$$\left. \exp\left(\int \ln |T|\right) \right\|_{\|T\|_{\infty}} \geq \left| \frac{z+p}{z-p} \right|$$

Law #1 : Chemistry (vs mechanics)

Law #2 : Autocatalysis (vs gravity)

(\rightarrow RHP p and z)



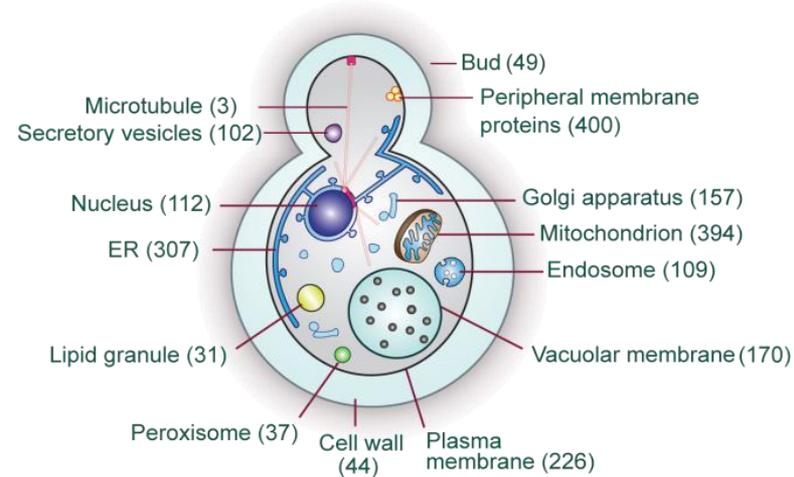
Law #3:

$$\left. \exp\left(\int \ln |T|\right) \right\|_{\infty} \geq \left| \frac{z+p}{z-p} \right|$$

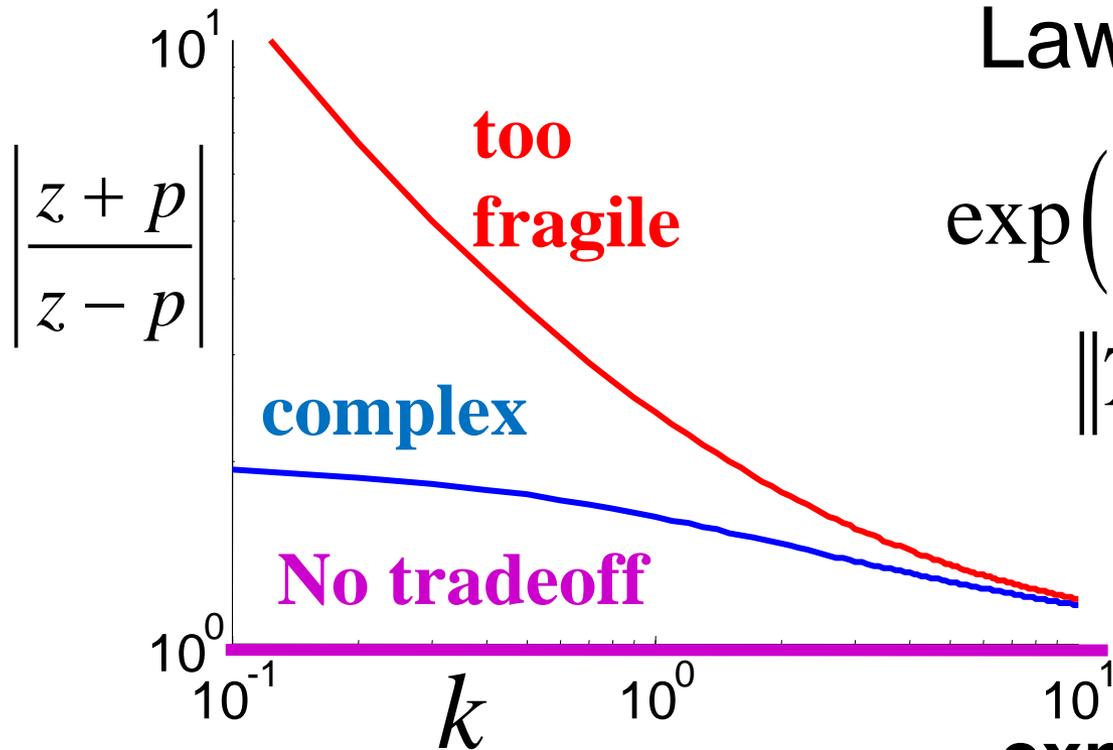
Law #1 : Chemistry

Law #2 : Autocatalysis

(\rightarrow RHP p and z)



fragile

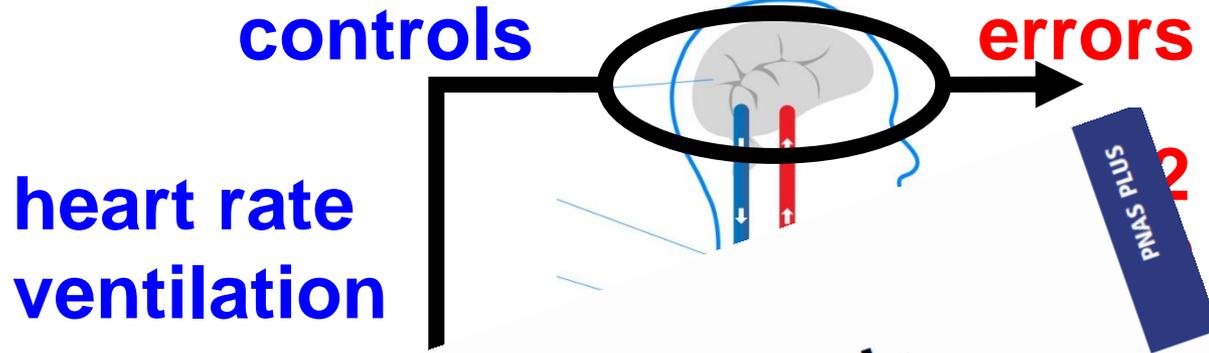


Law #3:

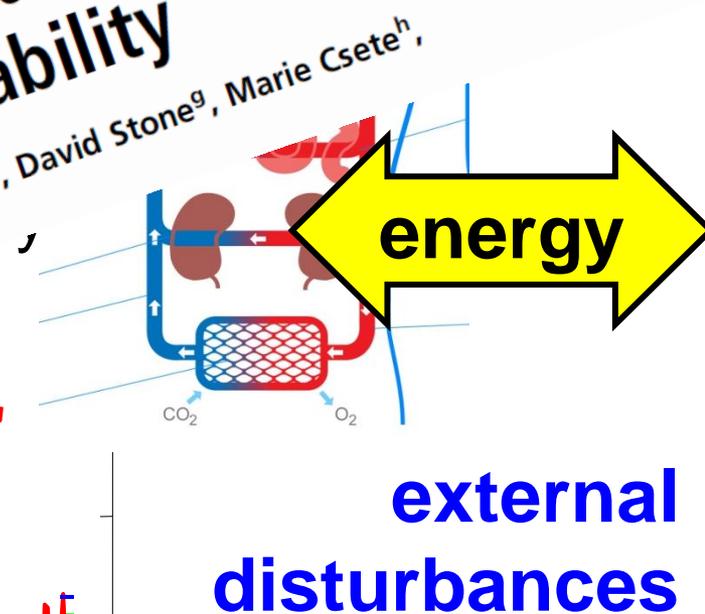
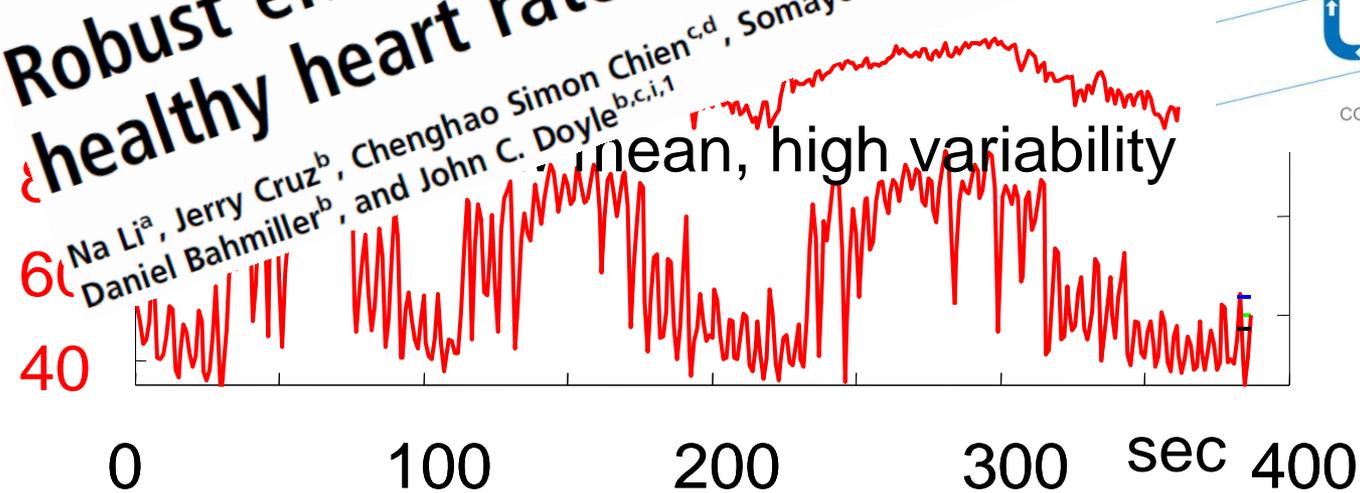
$$\left. \begin{array}{l} \exp\left(\int \ln |T|\right) \\ \|T\|_{\infty} \end{array} \right\} \geq \left| \frac{z+p}{z-p} \right|$$

expensive

Homeostasis and HRV



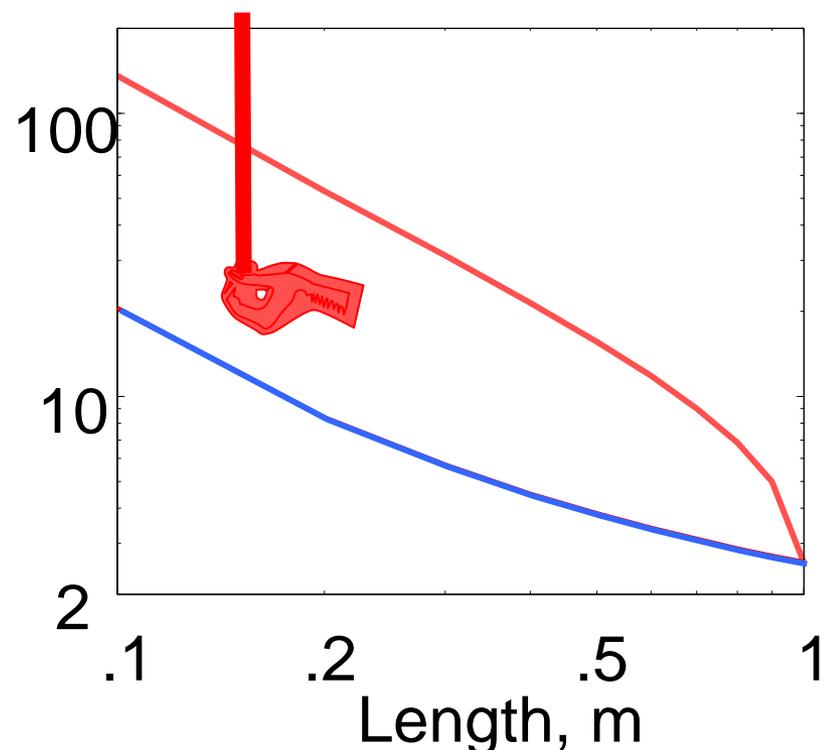
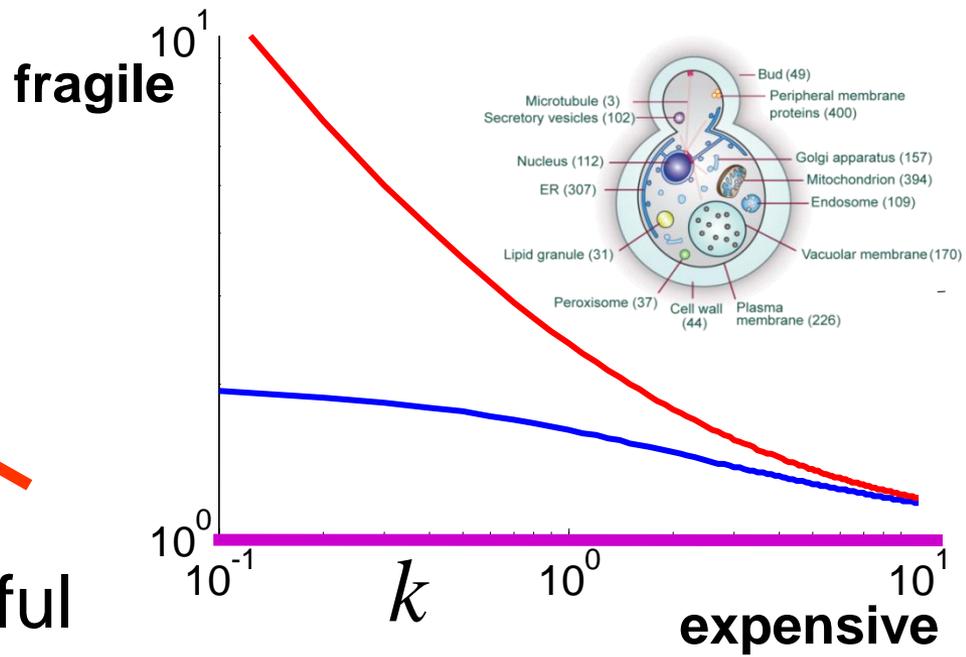
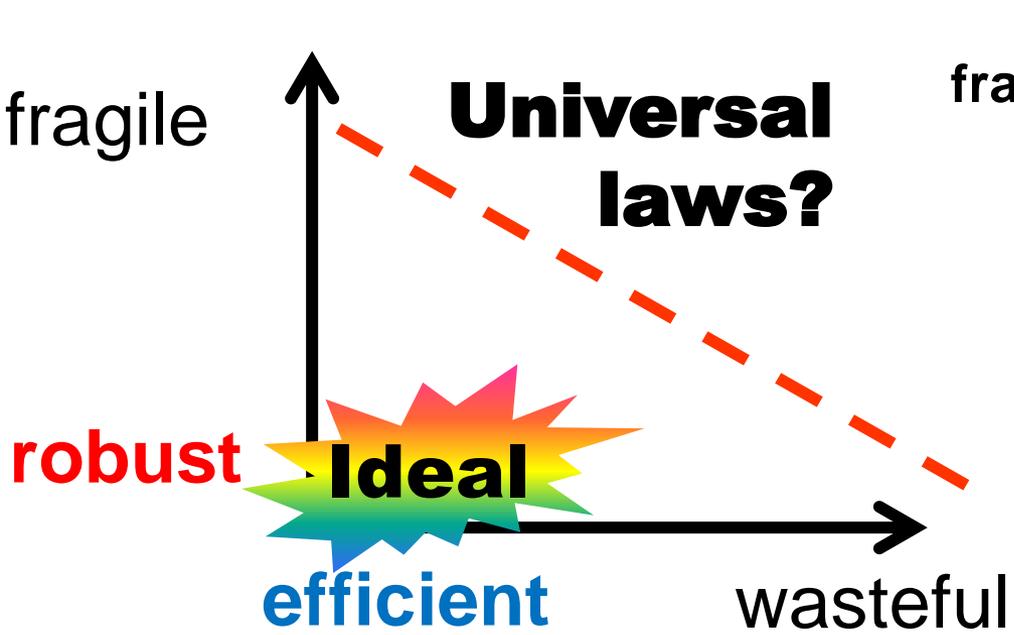
Robust efficiency and actuator saturation explain healthy heart rate control and variability



Na Li^a, Jerry Cruz^b, Chenghao Simon Chien^{c,d}, Somayeh Sojoudi^e, Ben Recht^f, David Stone^g, Marie Csete^h, Daniel Bahmiller^b, and John C. Doyle^{b,c,i,1}

mean, high variability

external disturbances



$$\left. \exp\left(\int \ln |T|\right) \right\|_{\|T\|_{\infty}} \geq \exp(p\tau) \left| \frac{z+p}{z-p} \right|$$

Compute

**Comms for
Comp/Cntrl/Bio**

Info Thry

Optimization

Statistics

Theory

Control, OR

**Orthophysics
(Eng/Bio/Math)**

Physics

Localized (distributed) control

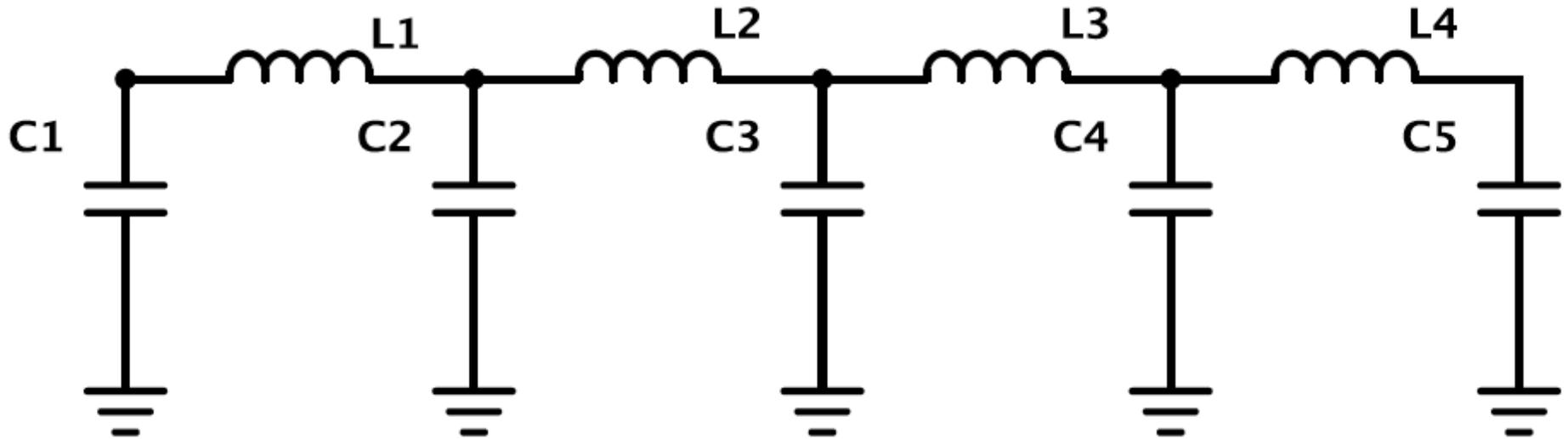
- Localizable control: Wang, Matni, You and Doyle ACC '14
- Localized LQR control: Wang, Matni, and Doyle CDC'14

Another *extremely* toy model

- Concretely illustrate important new ideas
- Minimal complexity otherwise
- Familiar, intuitive circuit dynamics
- Emphasize role of delays

- Instability mechanism is artificial
- Comparable to biological instabilities
- ... but (so far) rare in tech infrastructure

- LC circuit
- Each node = grounded capacitance
- Each link = inductance



System Model

- Assuming each L and each C has unit value, the dynamics of the system are

$$\dot{x}(t) = Ax(t)$$
$$A = \begin{bmatrix} 0 & M \\ -M' & 0 \end{bmatrix}$$

where $x(t)$ is states of node voltage and link current, M is the incidence matrix of the circuit graph.

(Will reorder for plotting later.)

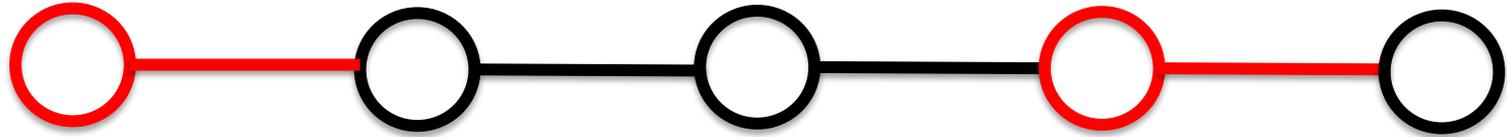
Discrete Time System Model

- A first order (Euler) approximation is

$$Ad = \begin{bmatrix} I & step * M \\ -step * M' & I \end{bmatrix}$$

- With step = 0.2, the maximum eigenvalue of Ad is 1.0768
- Artificially create a **very** unstable system
- Only biology is systematically this unstable, so far.

Simplified diagram (2 states per node)



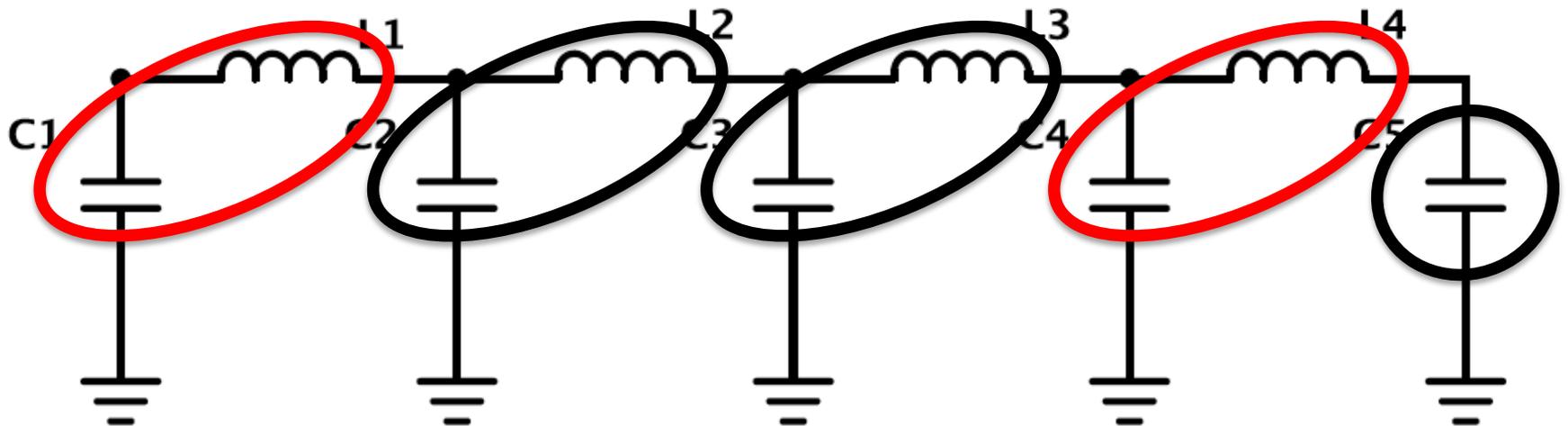
**Actuated
and sensed**

**Only
sensed**

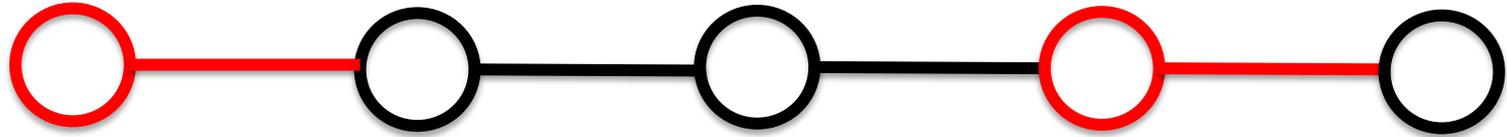
**Only
sensed**

**Actuated
and sensed**

**Only
sensed**



Simplified diagram (2 states per node)



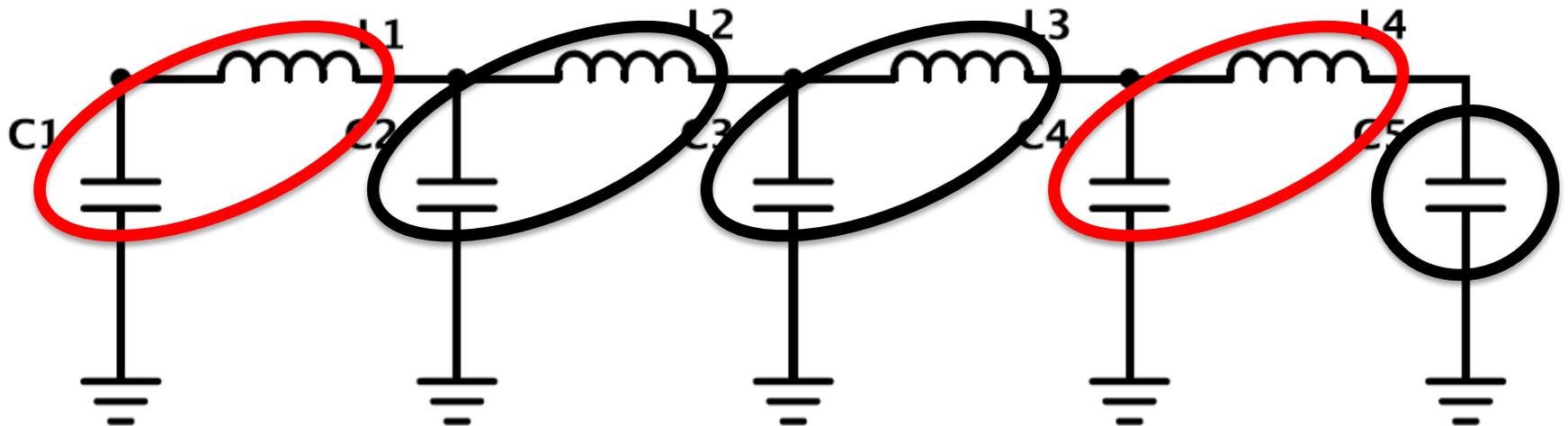
**Actuated
and sensed**

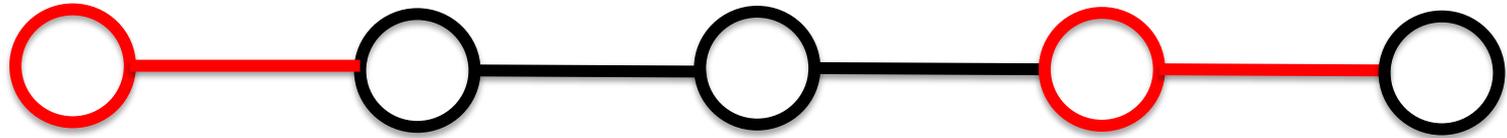
**Only
sensed**

**Only
sensed**

**Actuated
and sensed**

**Only
sensed**





**Actuated
and sensed**

**Only
sensed**

Simplified diagram (2 states per node)



**Actuated
and sensed**

**Only
sensed**

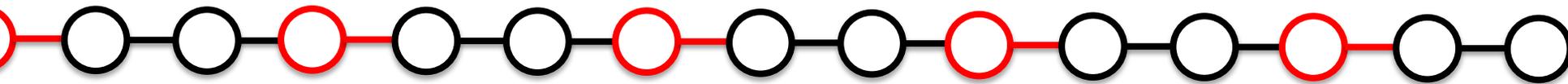
Expensive?

0. Physical

1. Actuation

Nominally each has delay 1.

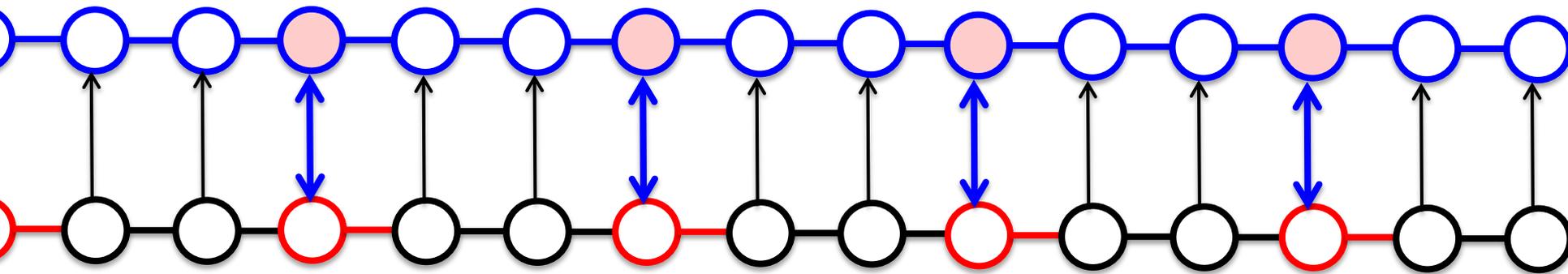
Simplified diagram (2 states per node)



**Actuated
and sensed**

**Only
sensed**

Controller



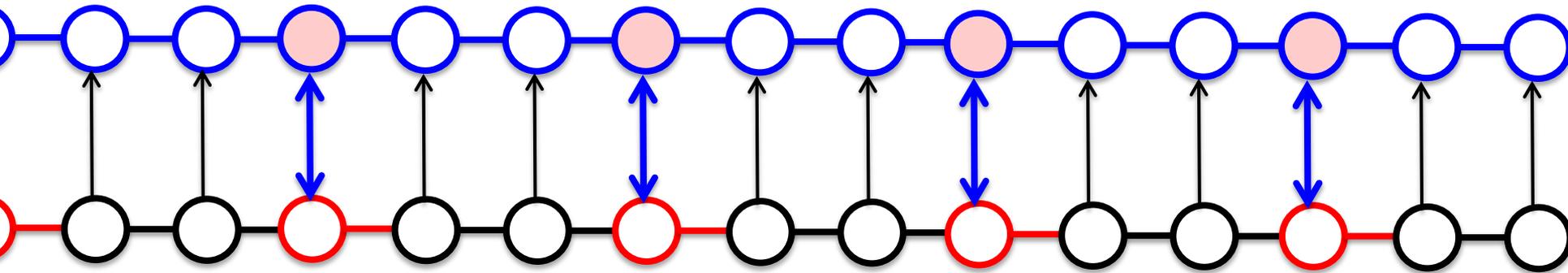
Physical plant

Expensive?

0. Physical
1. Actuation
2. Comms speed
3. Comp speed
4. Sensing

...

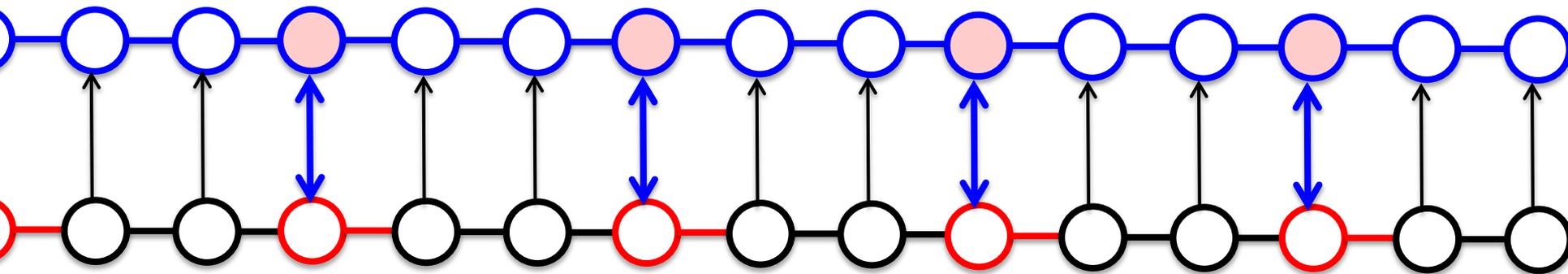
Sense, comm/comp, *act.*



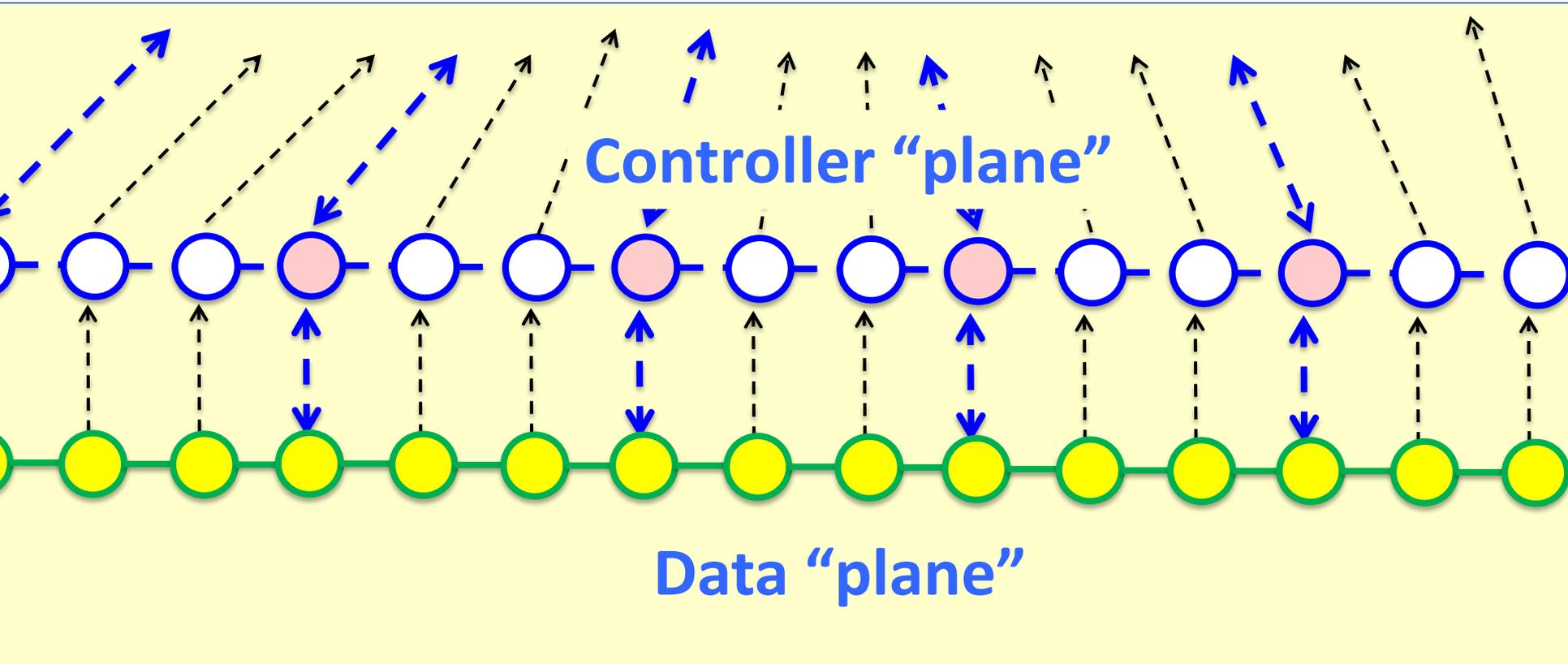
**Actuated
and sensed**

**Only
sensed**

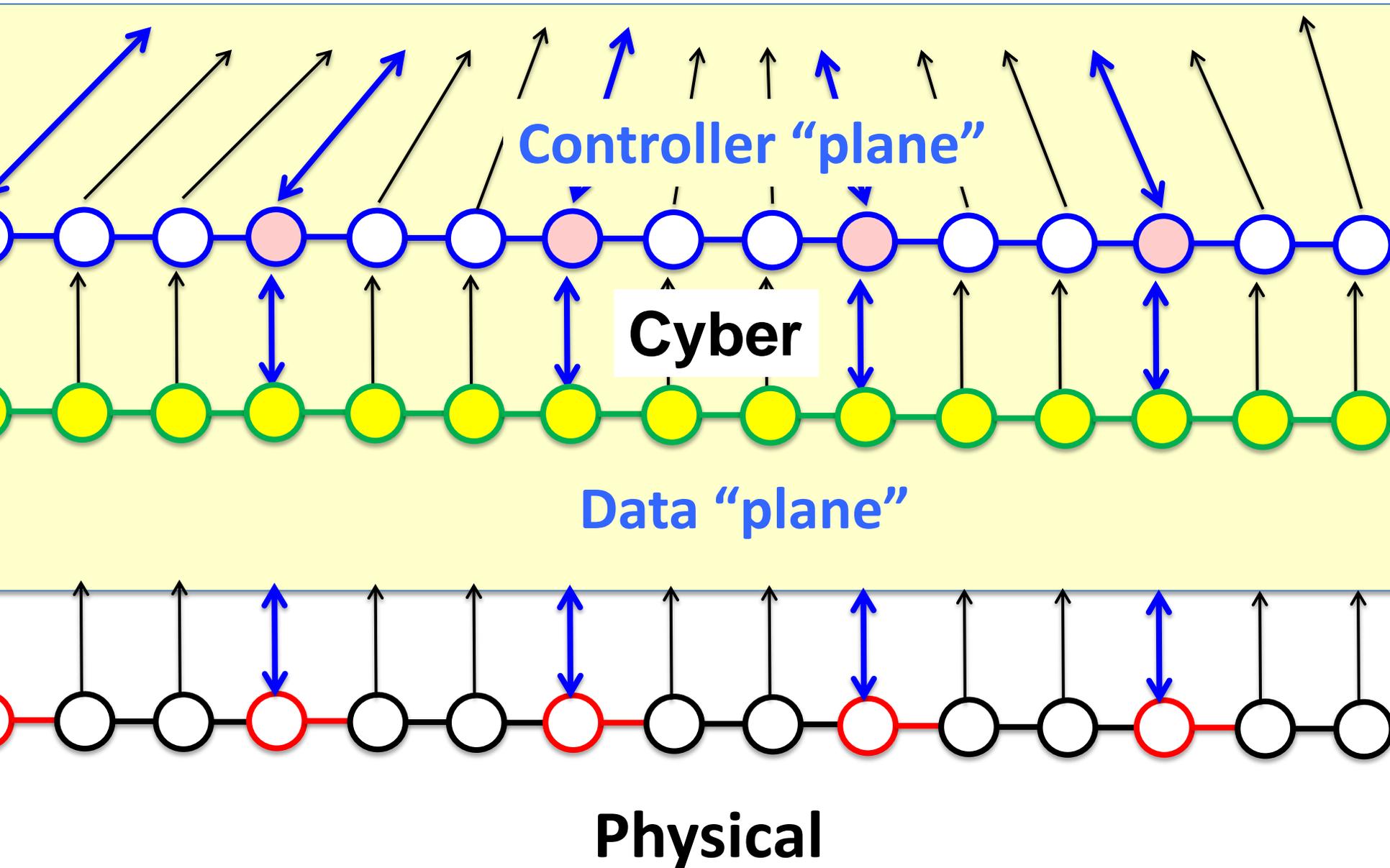
Controller



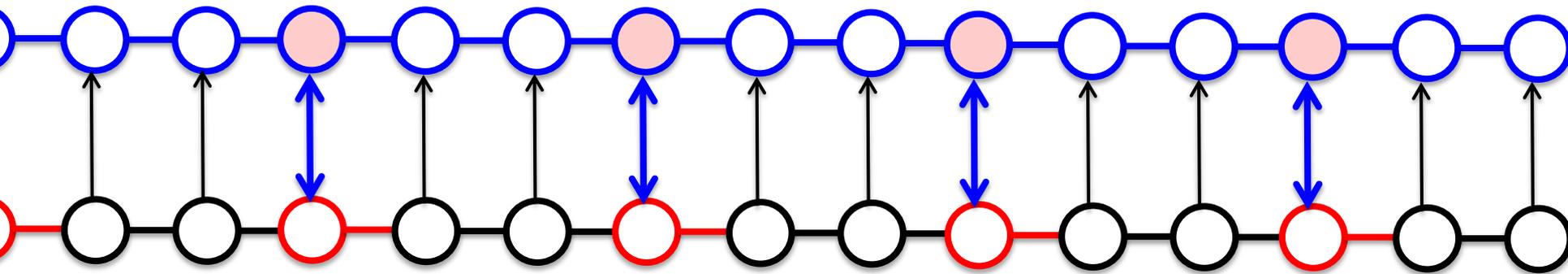
Physical plant



SDN/ODP



Sense, comm/comp, *act.*
Nominally each has delay 1.



**Actuated
and sensed**

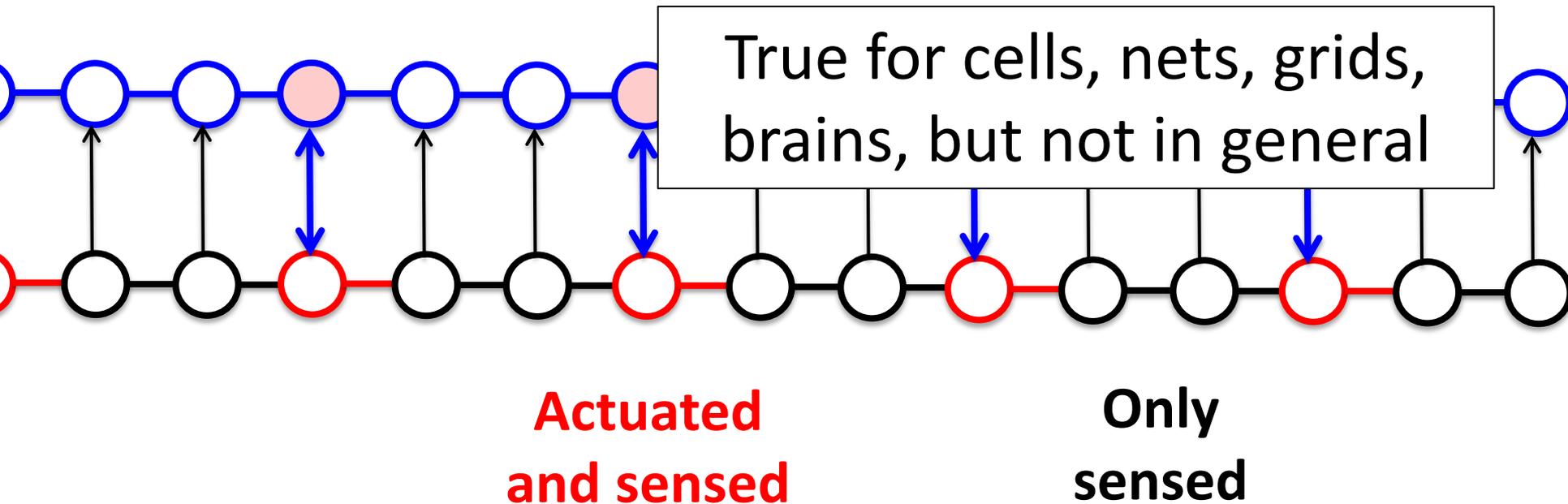
**Only
sensed**

Expensive:

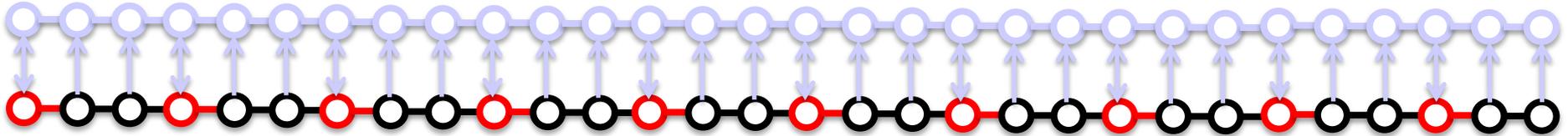
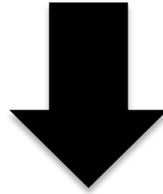
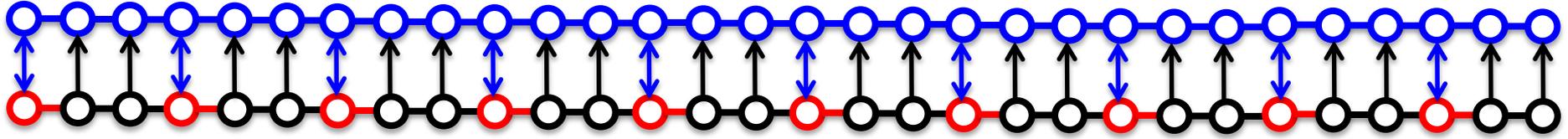
- physical plant
- passive stability
- actuation
- low delay (comms and comp)

Cheap:

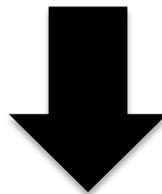
- comms bandwidth
- compute memory
- sensing

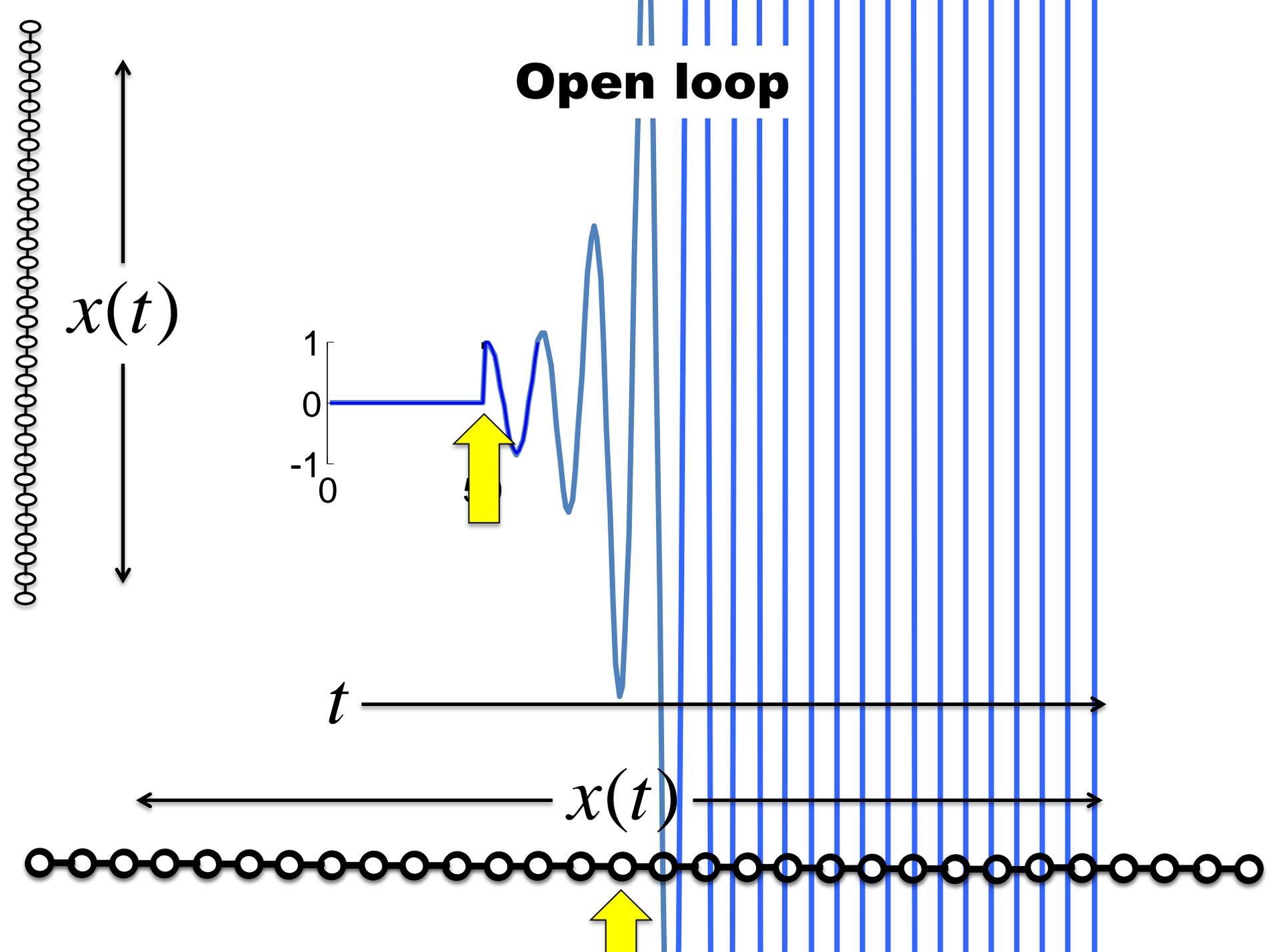


Open loop dynamics



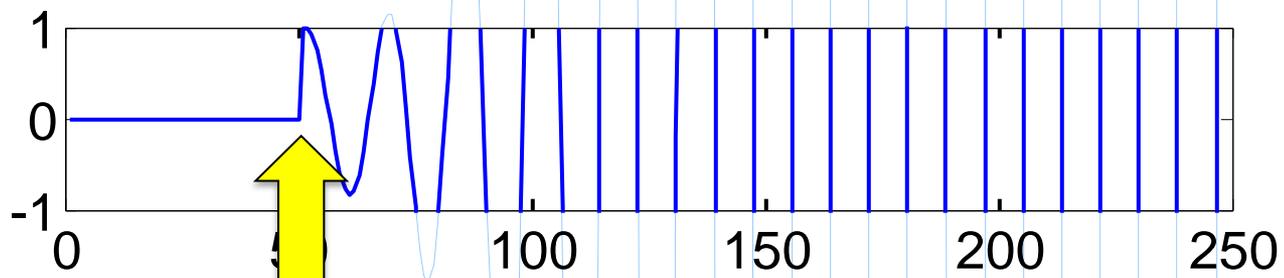
Simplified diagram



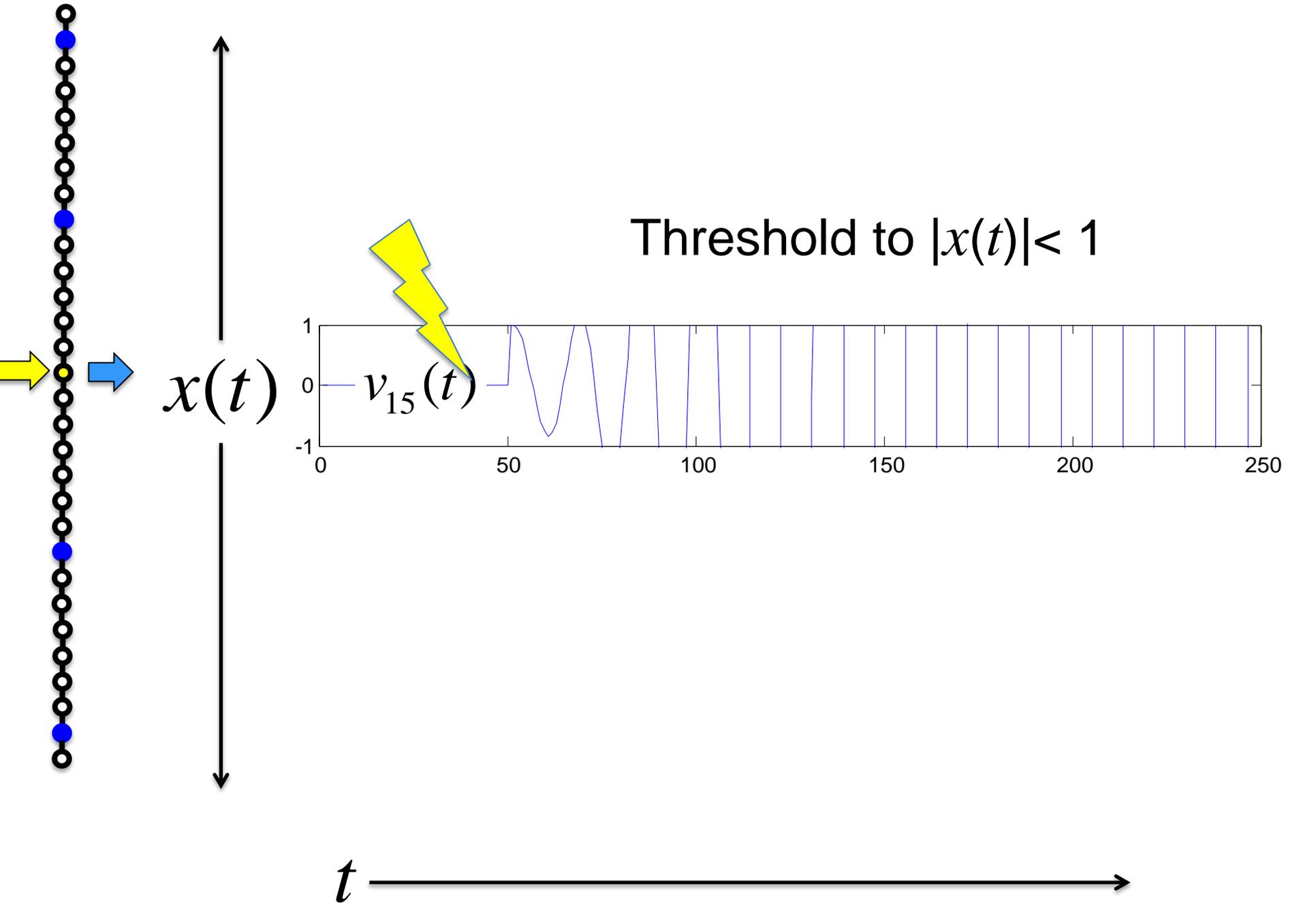


Open loop

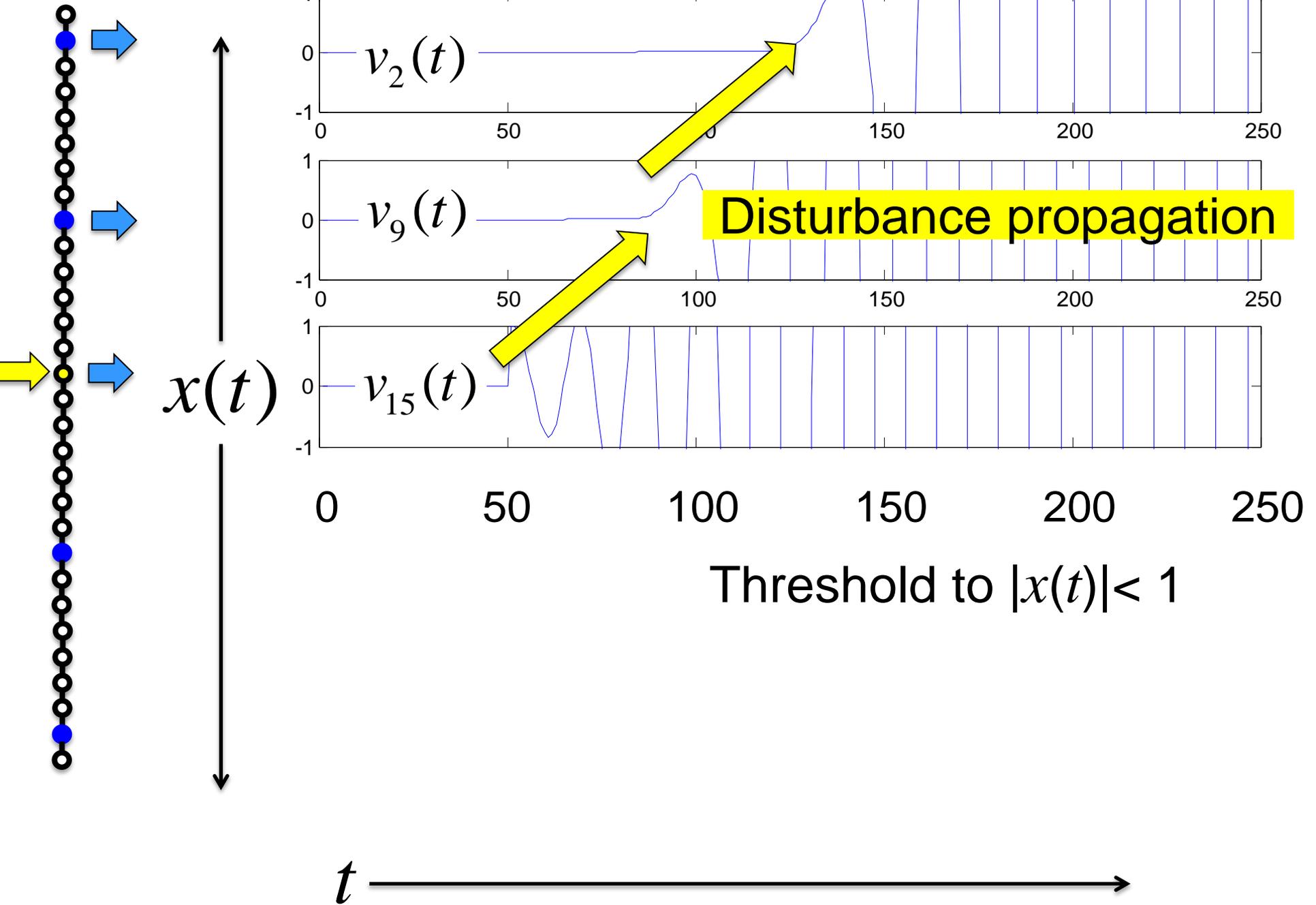
$x(t)$



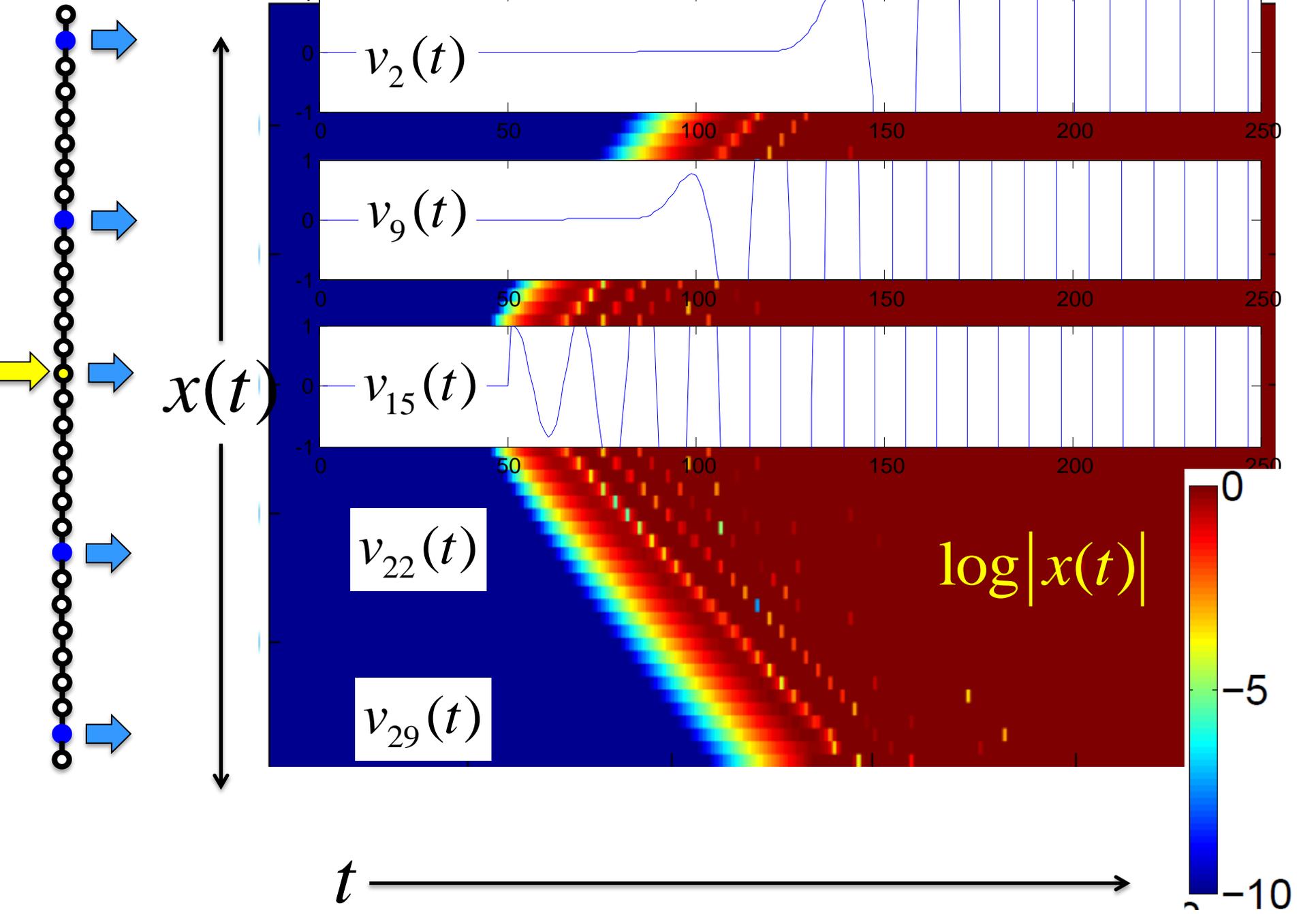
t

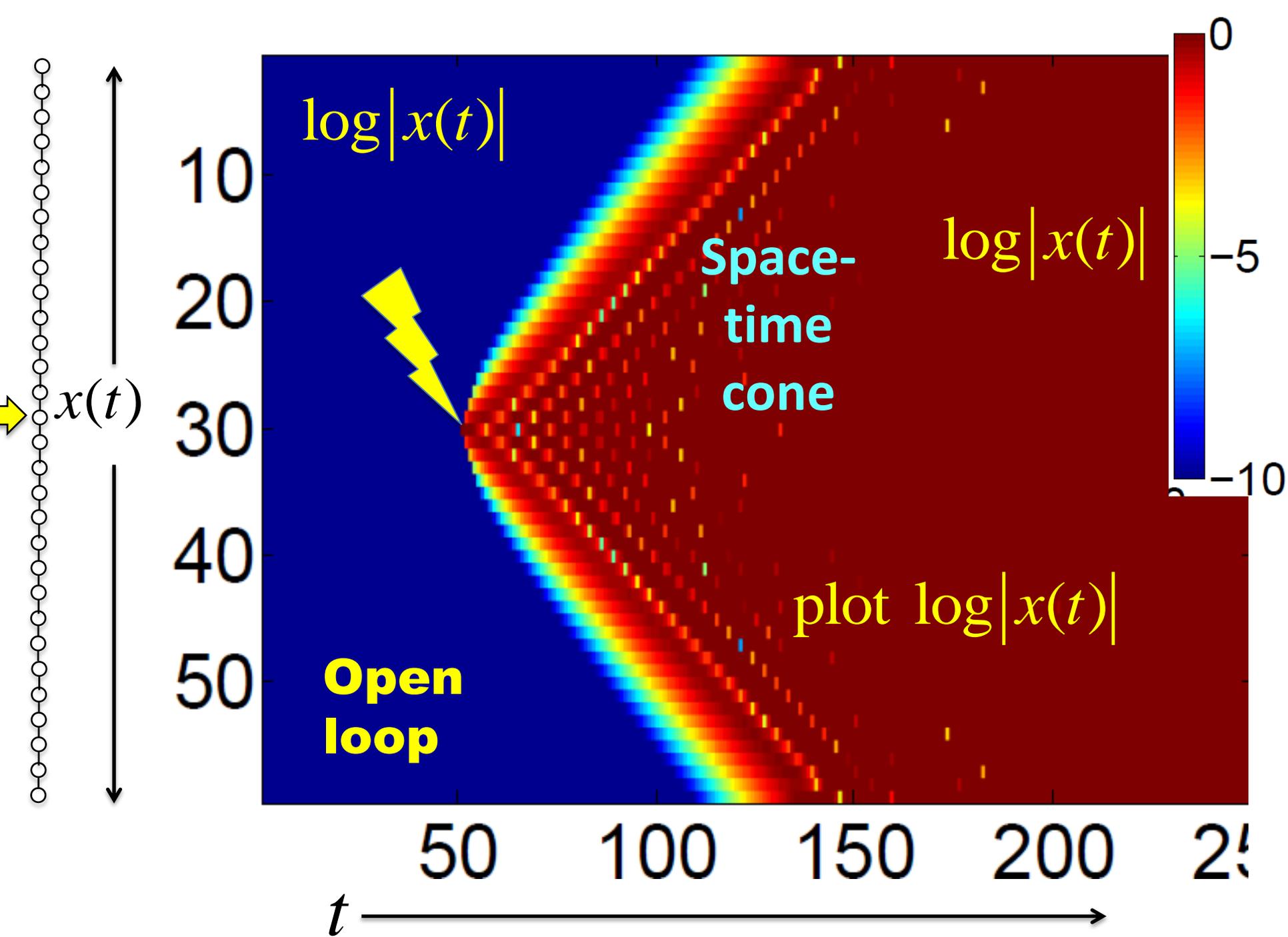


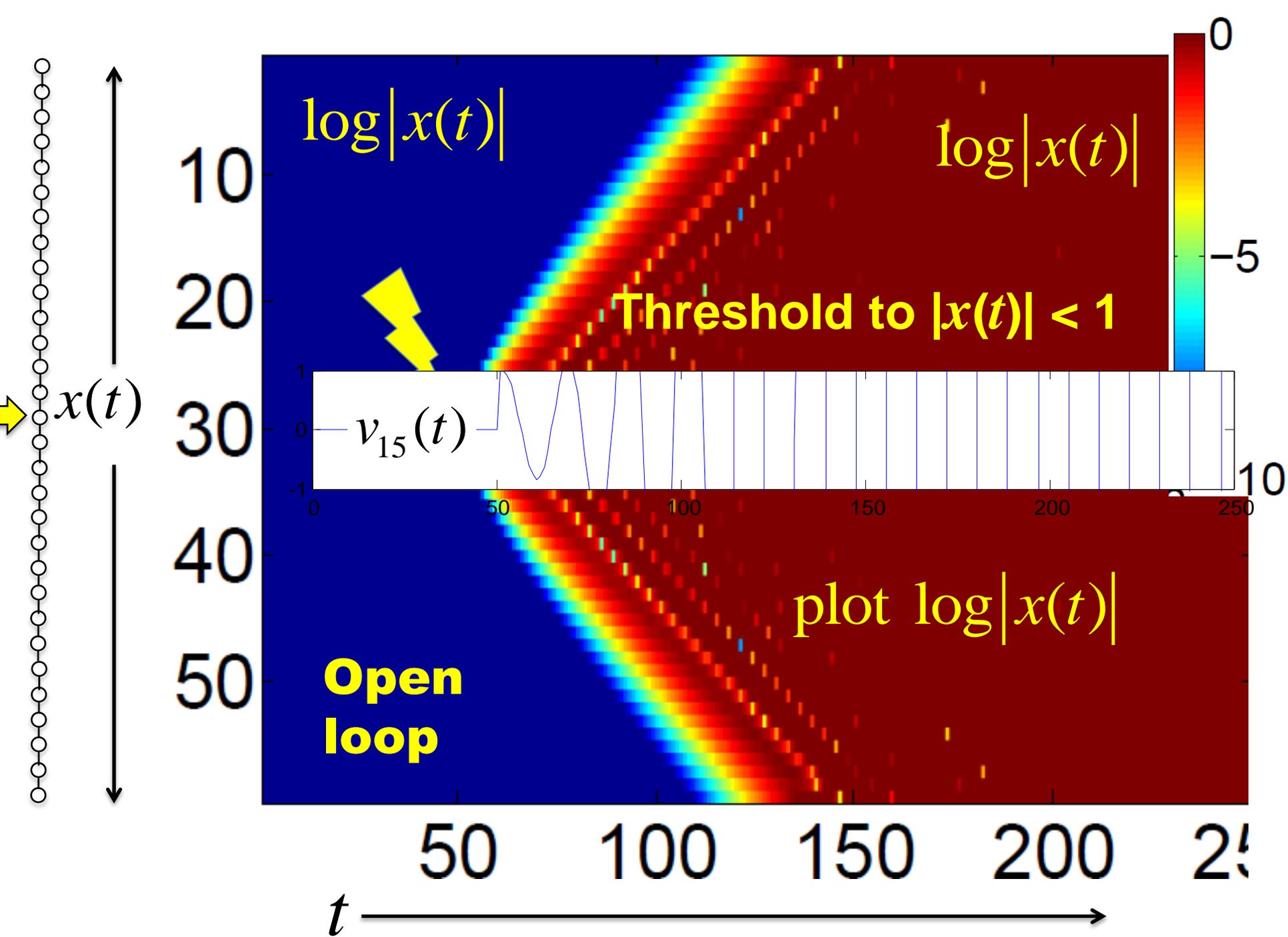
Open loop

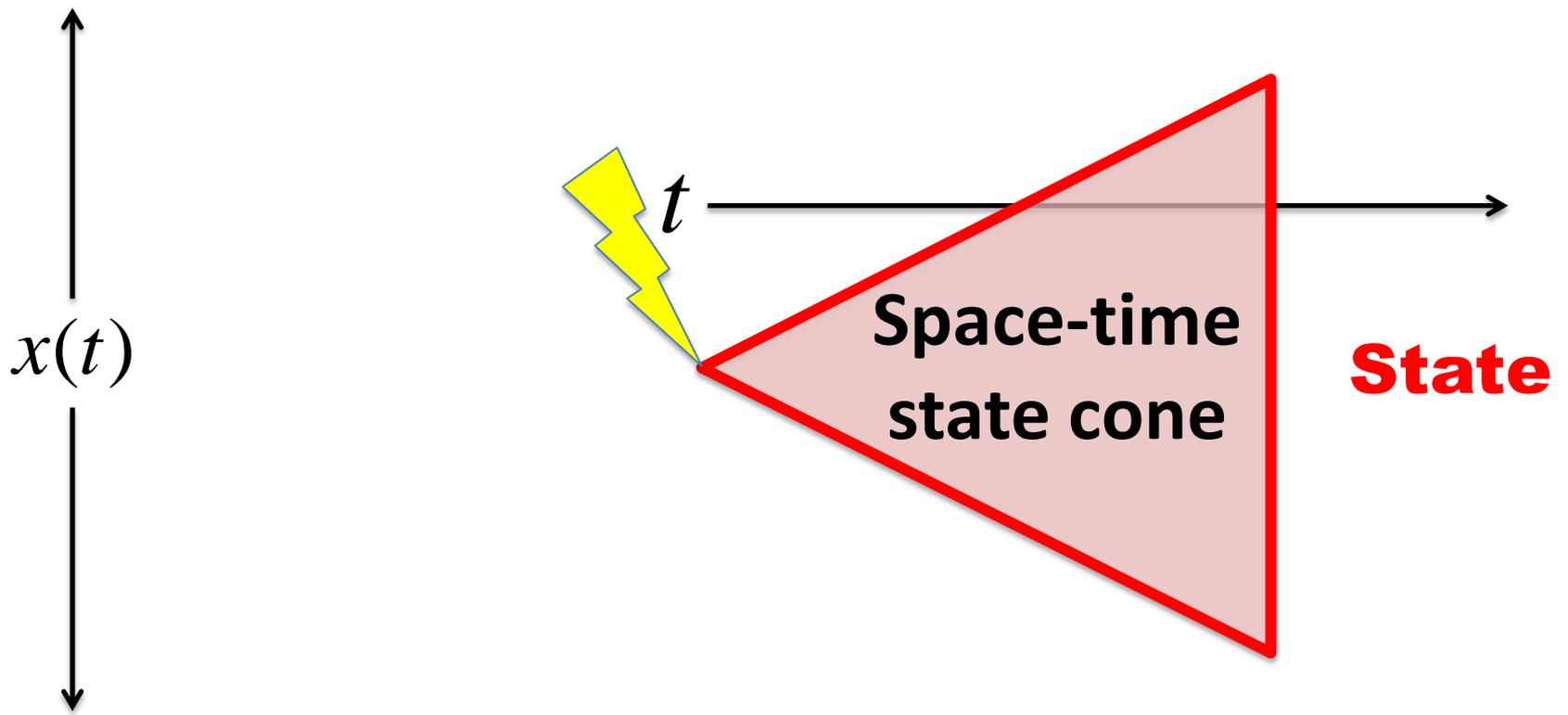


Open loop









$$\begin{bmatrix} x[T] \\ \vdots \\ x[1] \end{bmatrix} = \begin{bmatrix} A^T \\ \vdots \\ A \end{bmatrix} x[0]$$

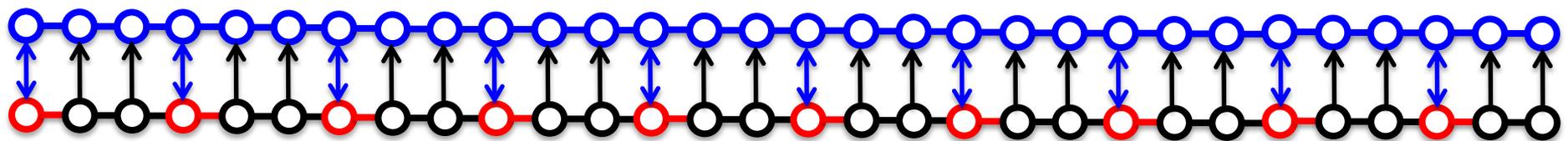
Controller Design

Critical Issues

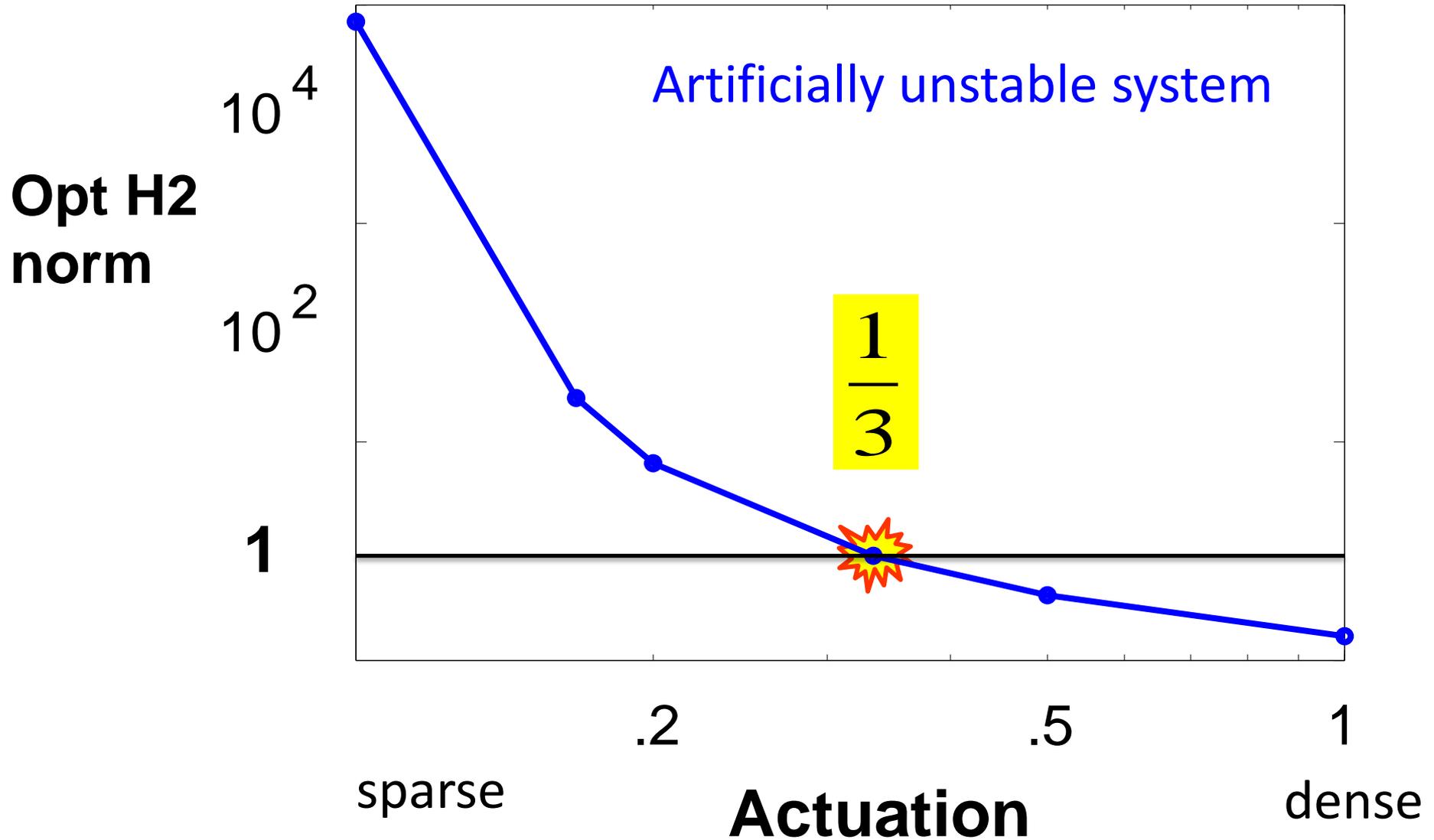
1. Transient LQ (H2) cost: $\Sigma(x'x+u'u)$
2. Actuator Density
3. Communication (vs plant) Speed
4. Locality/Scalability (Computation)
5. Time/space horizon

Actuator Density

- **Standard** (centralized) optimal H2 control
- No delay (initially)
- Defer other issues (∞ comm, comp, sense)
- Objective: $\min \sum (x'x + u'u)$
- Actuator density = # actuators / # states
- Trade-off: actuator density vs norm
- Example: 30 C, 29 L

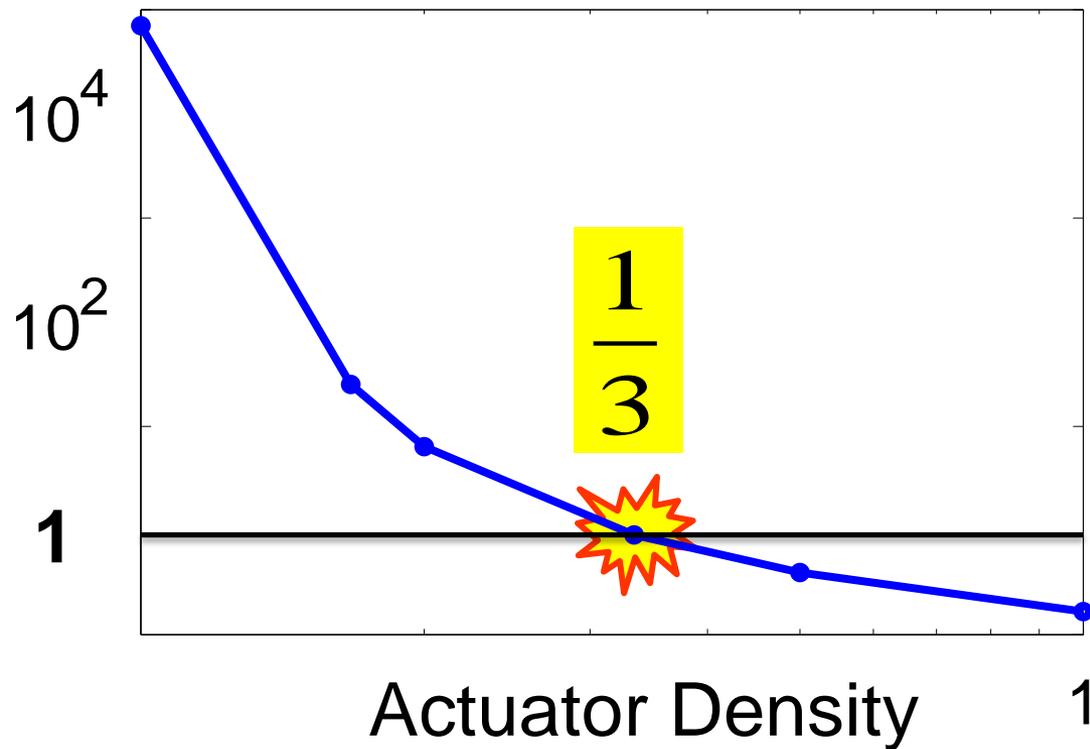


Norm - Actuator Density (normalized)

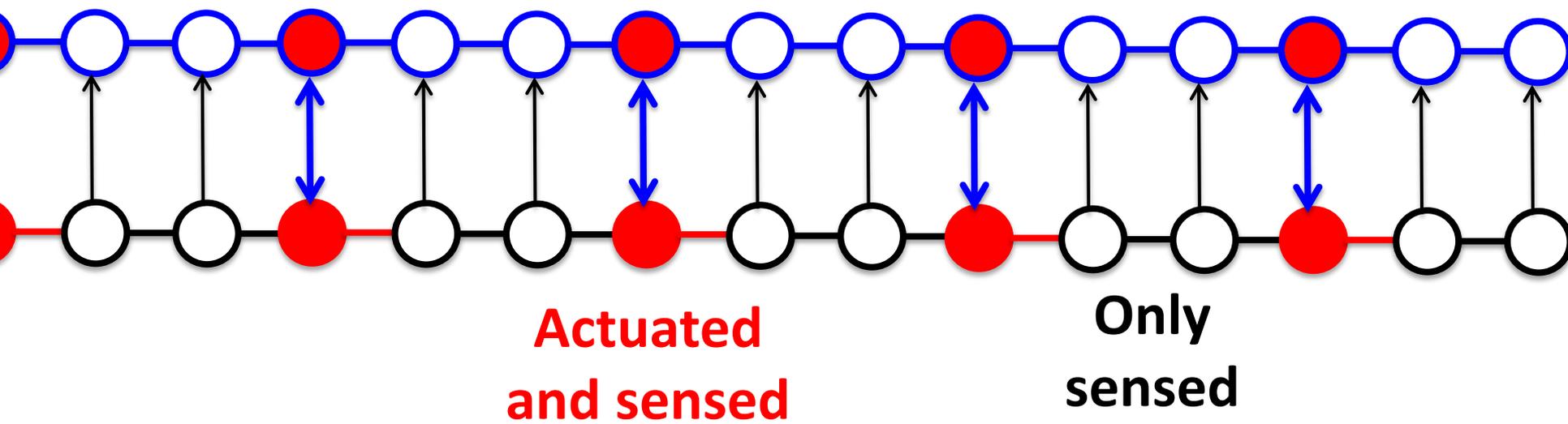


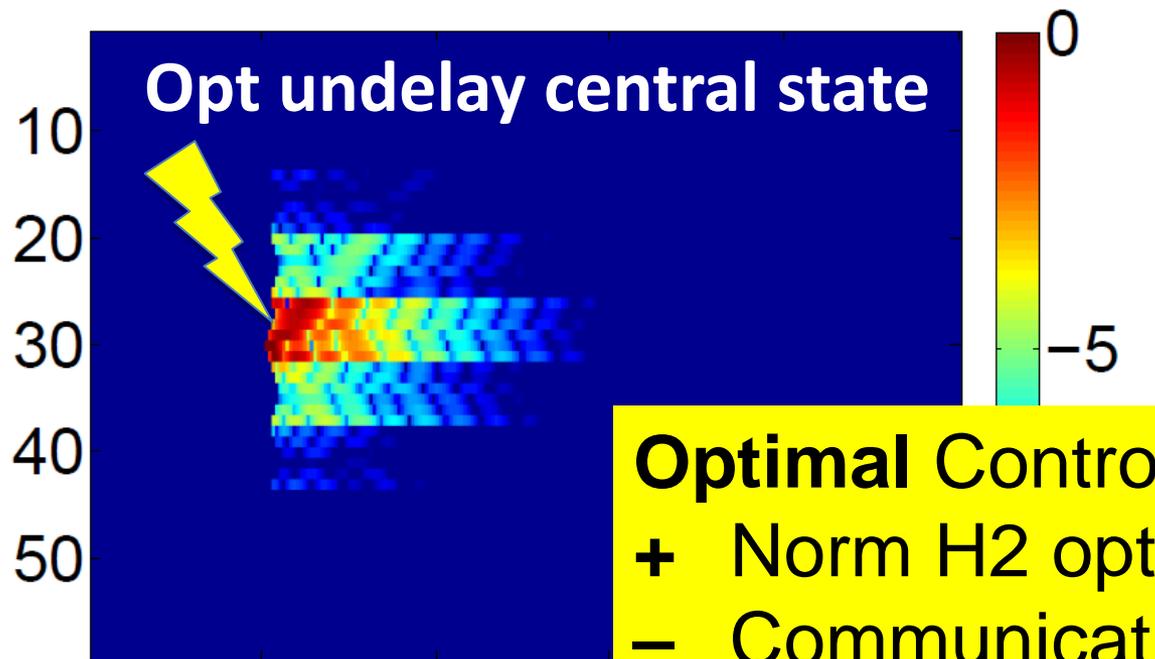
Standard control
(circa 1970)

norm



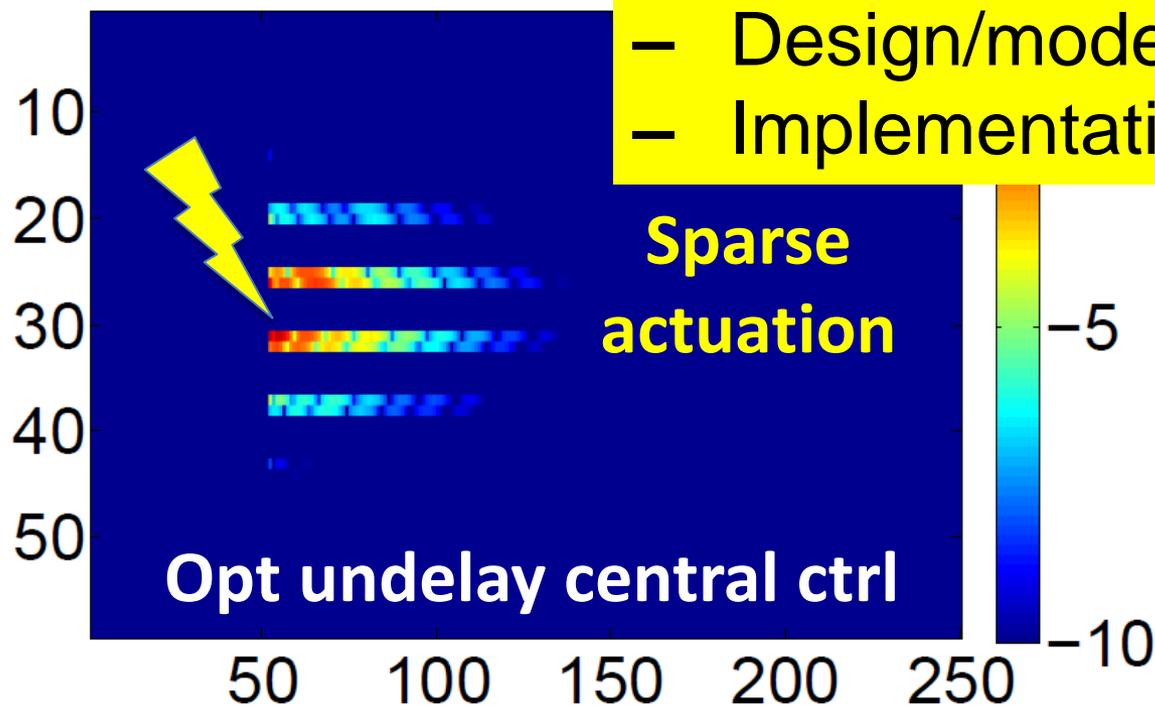
∞ Comm speed = 0 delay

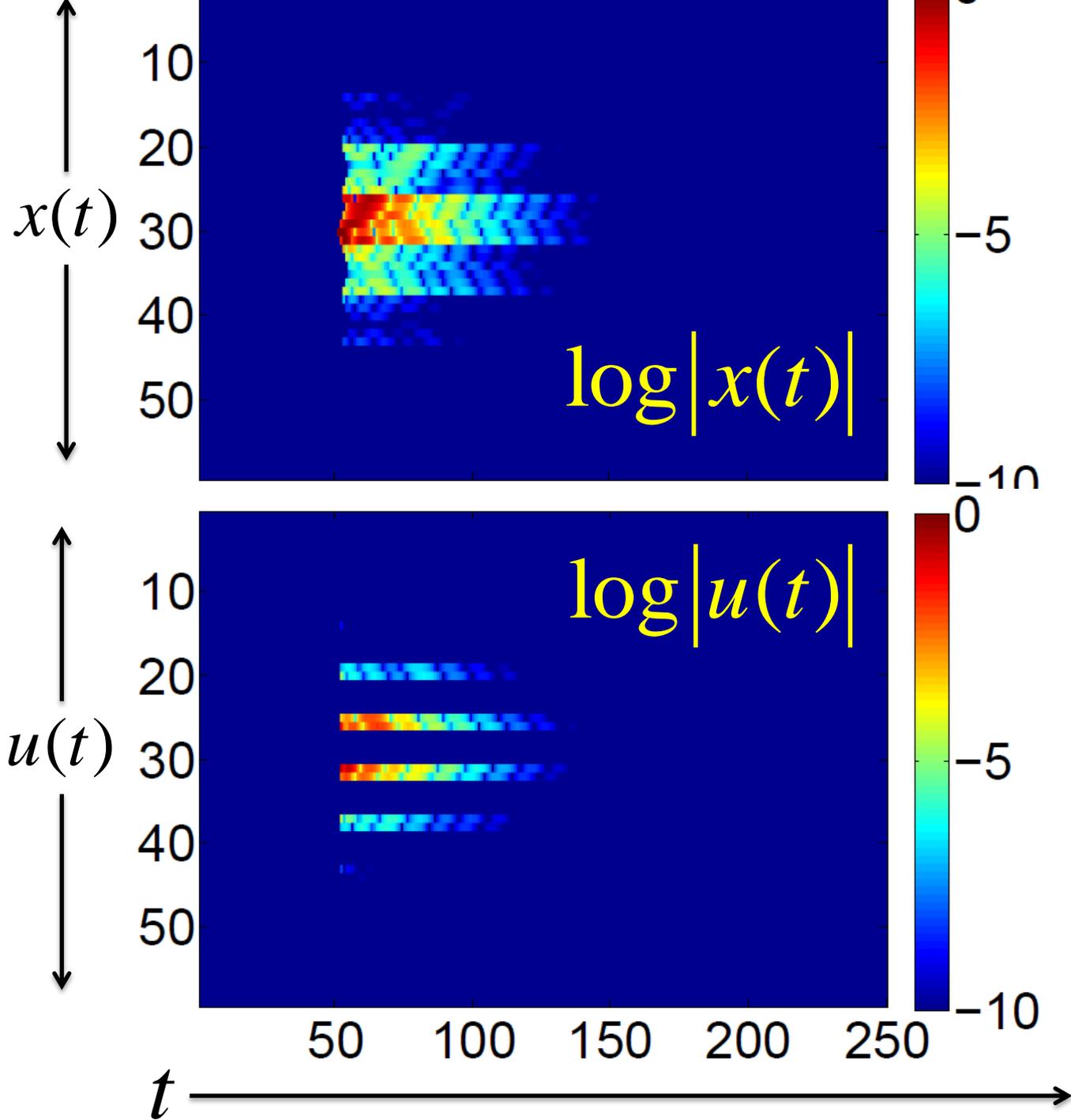




Optimal Controller

- + Norm H2 optimal
- Communication undelayed
- Design/model global/huge P
- Implementation local/huge P





**Color
code?**

Expensive?

0. Physical

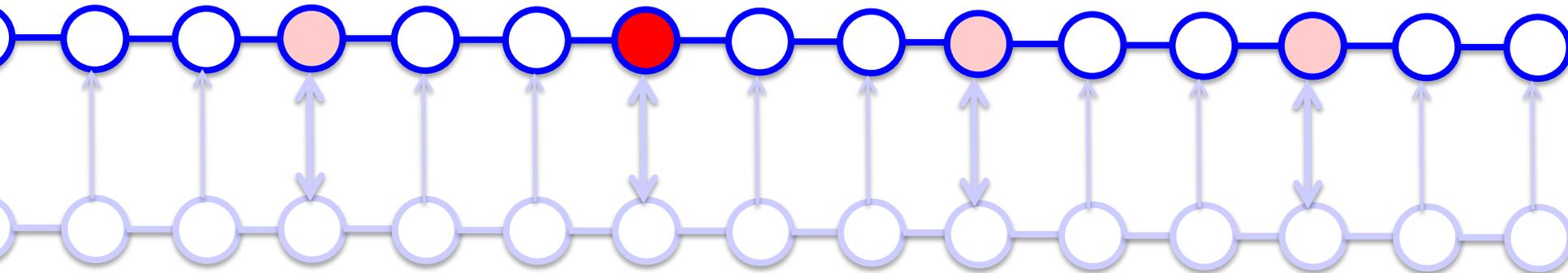
1. Actuation

2. **Comms speed**

3. Comp speed

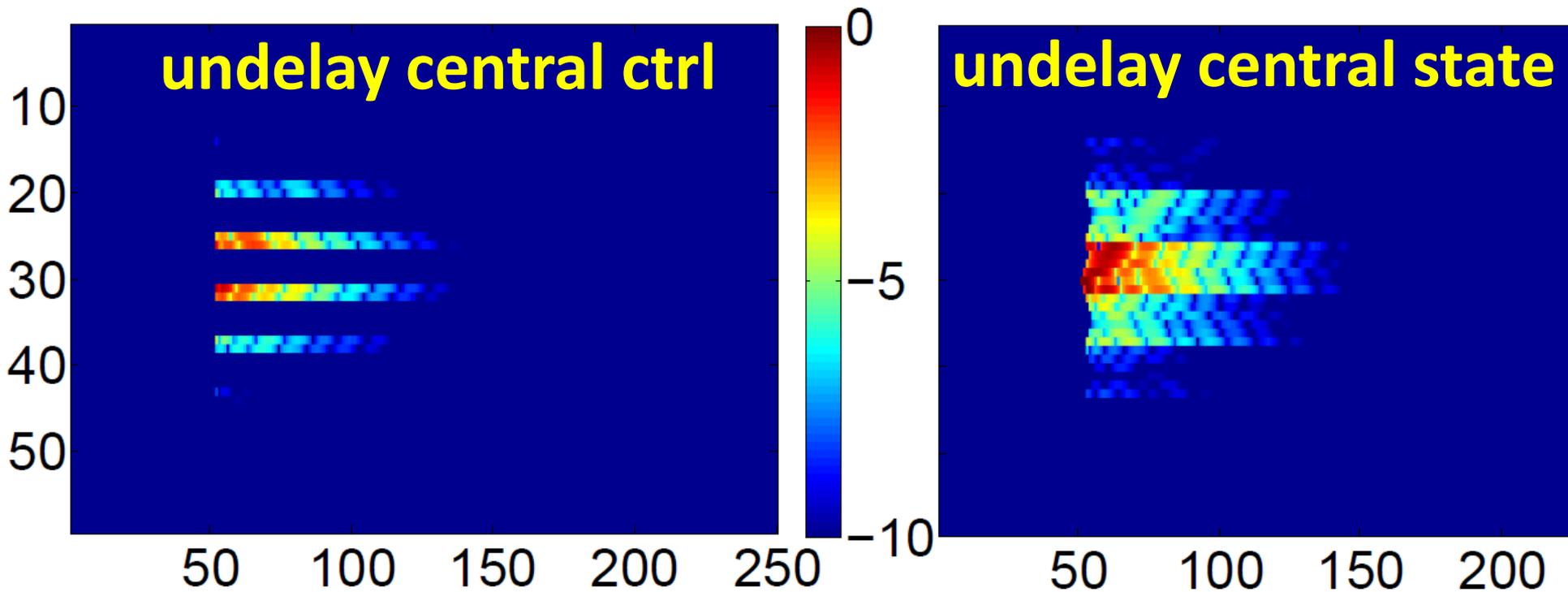
4. Sensing

...

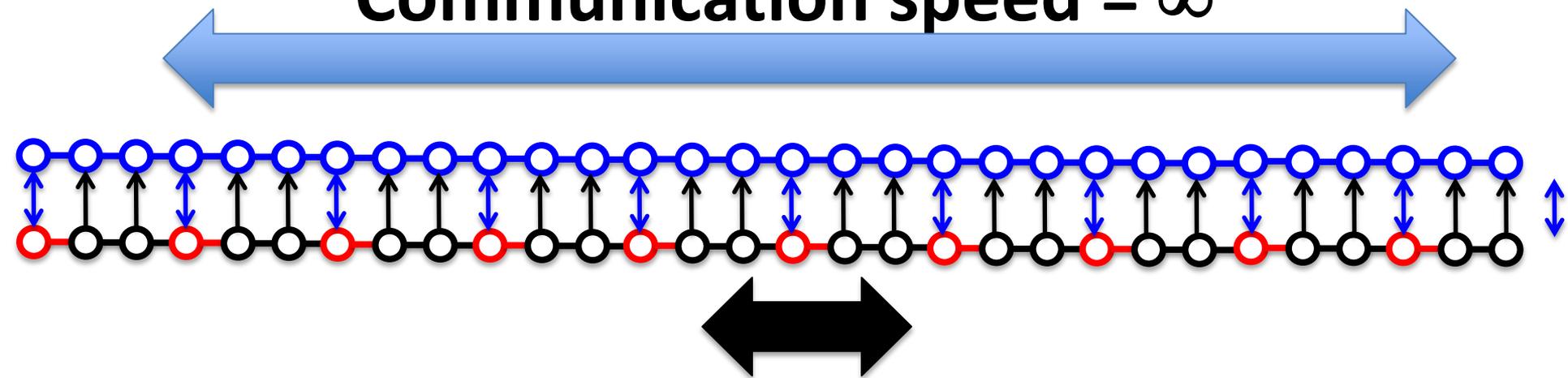


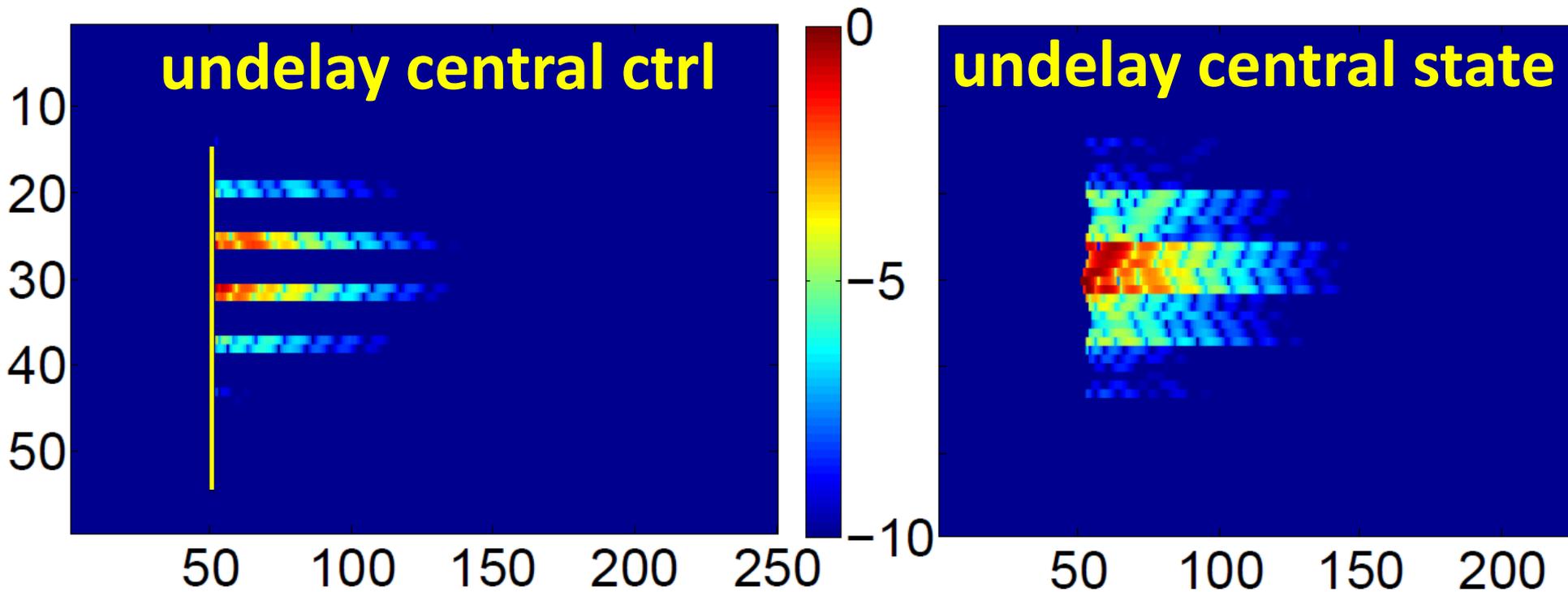
**Actuated
and sensed**

**Only
sensed**

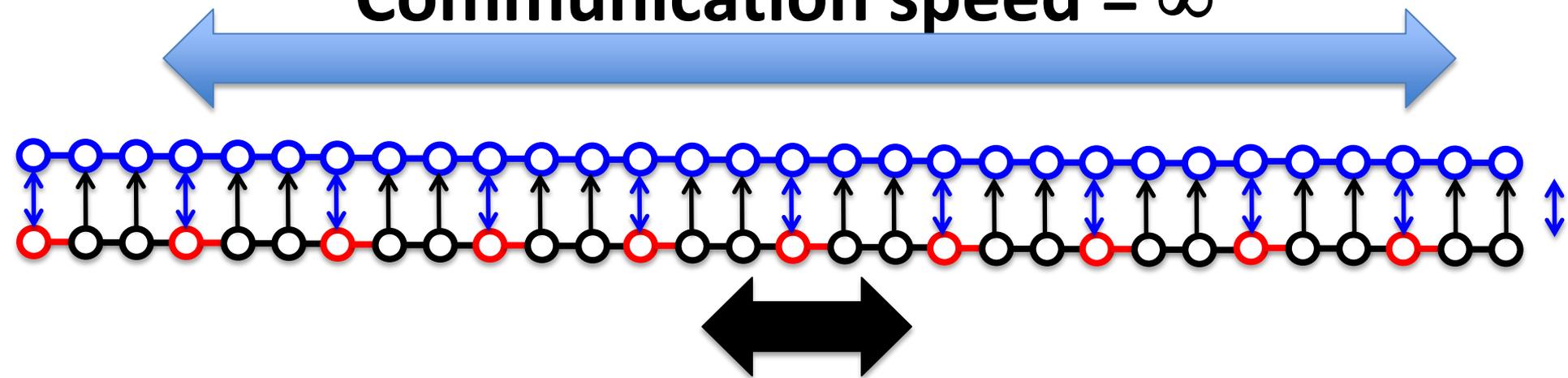


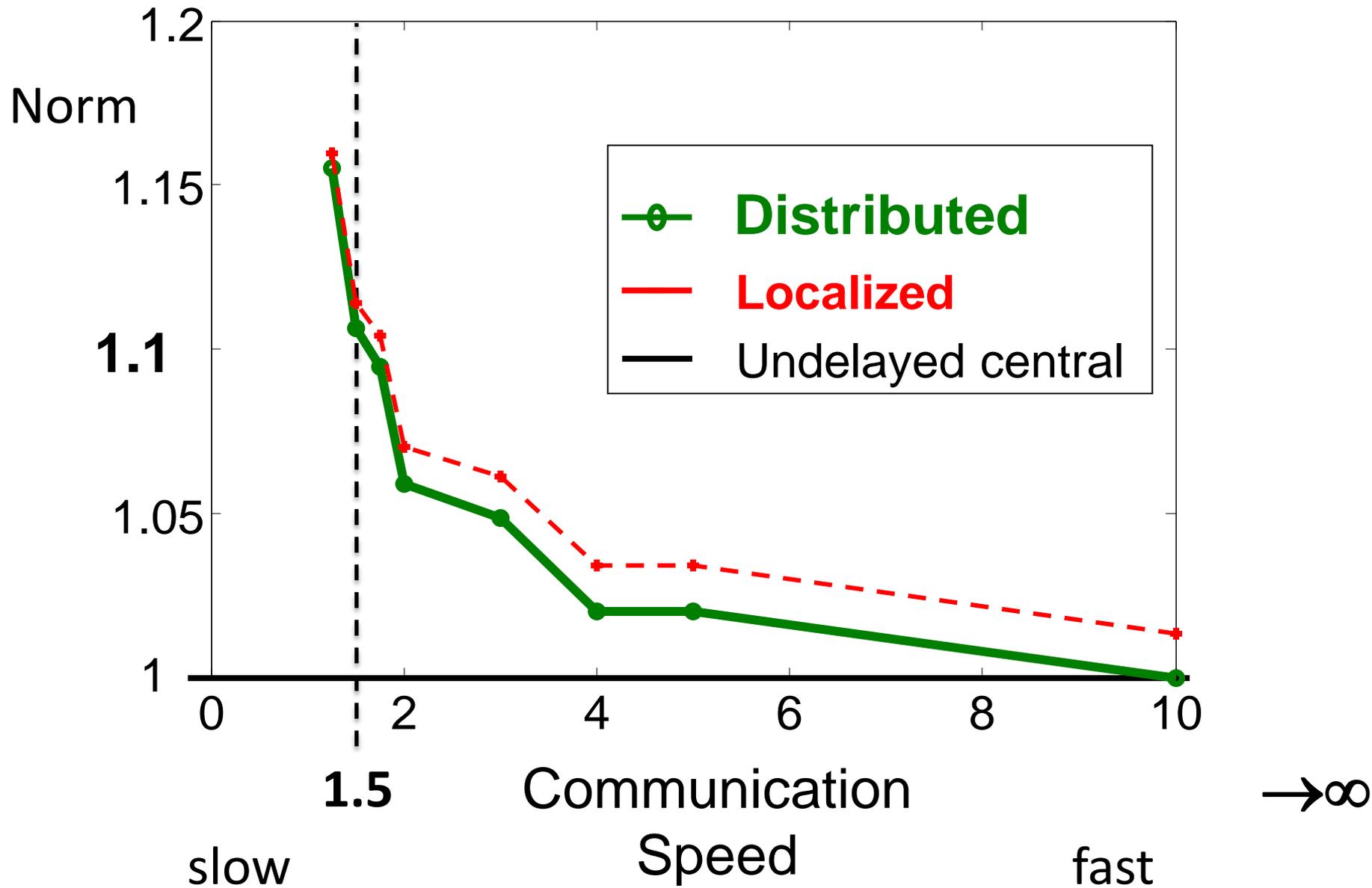
Communication speed = ∞

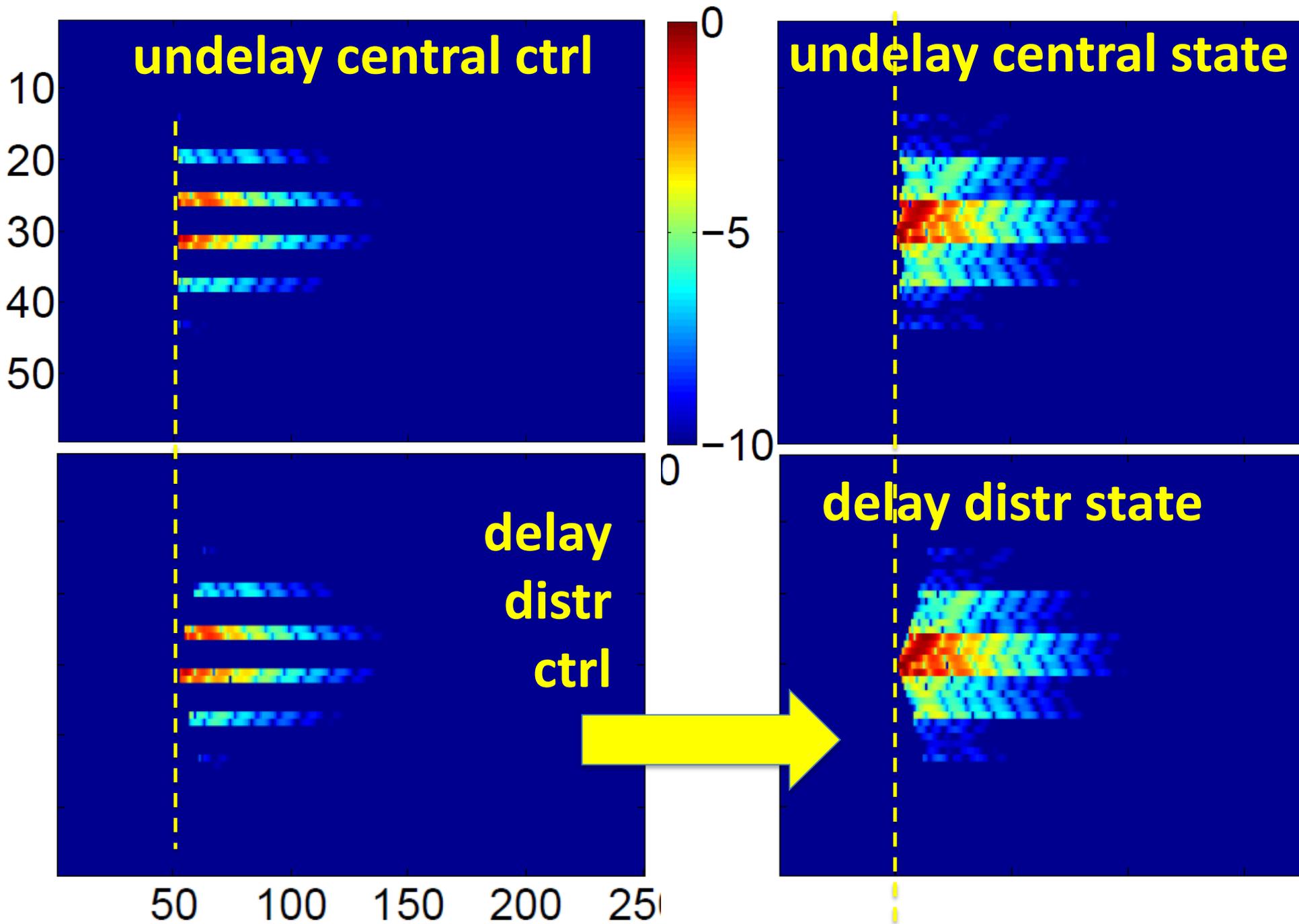




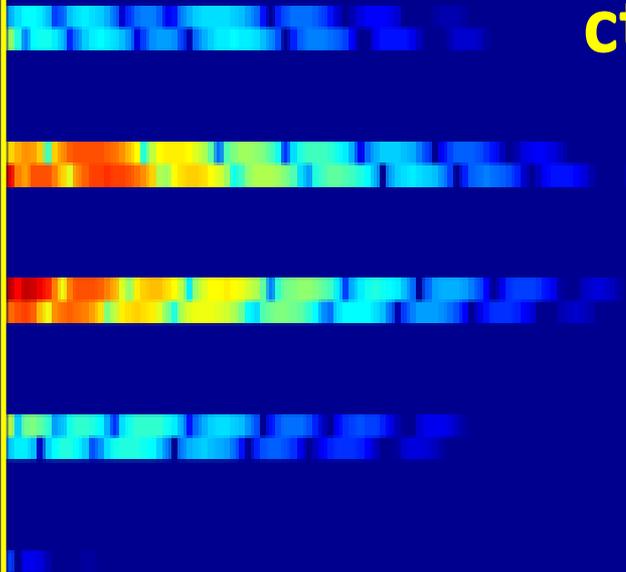
Communication speed = ∞



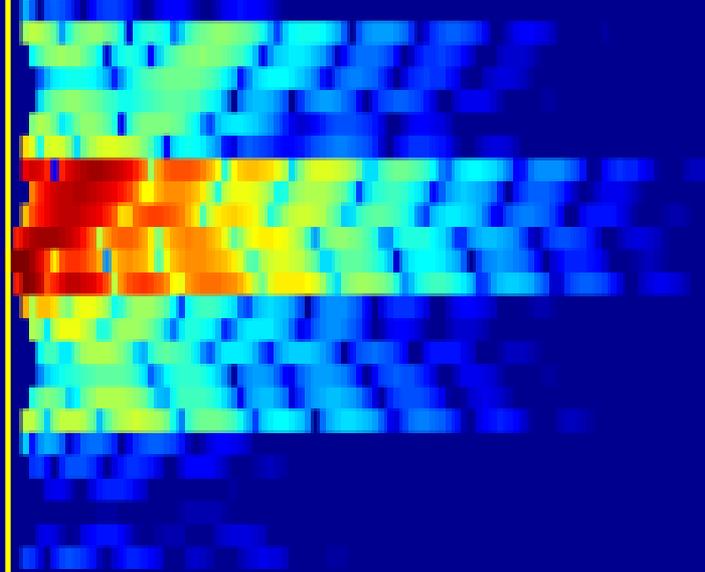




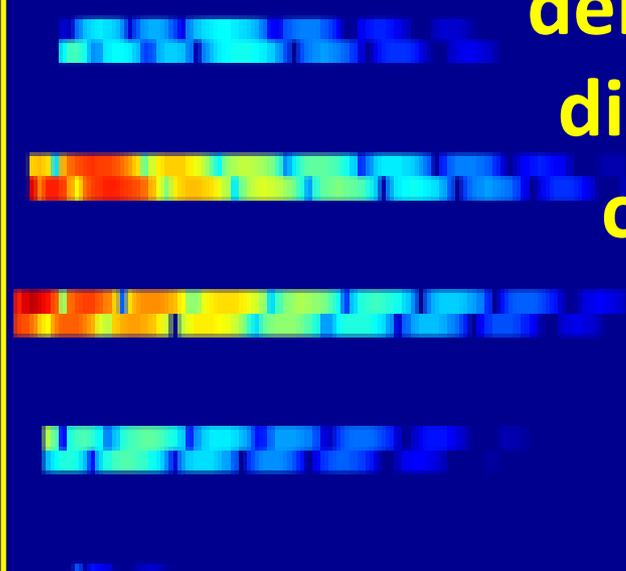
**undelay central
ctrl**



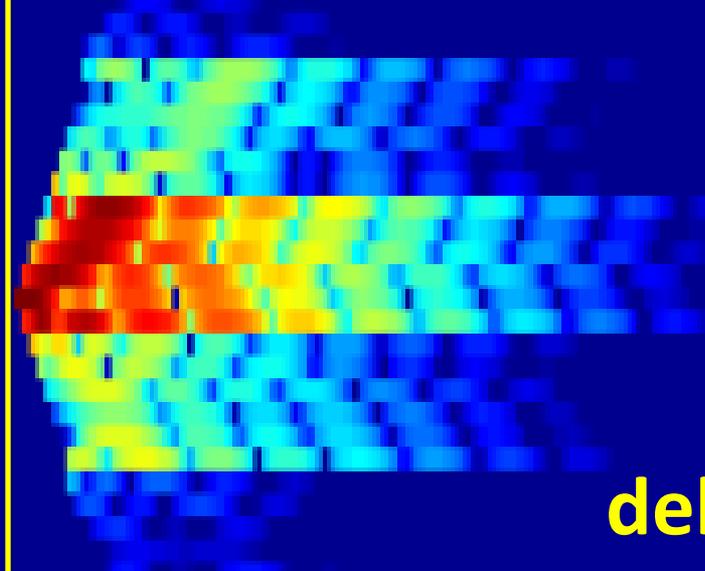
central stat



**delay
distr
ctrl**



**delay
distr state**



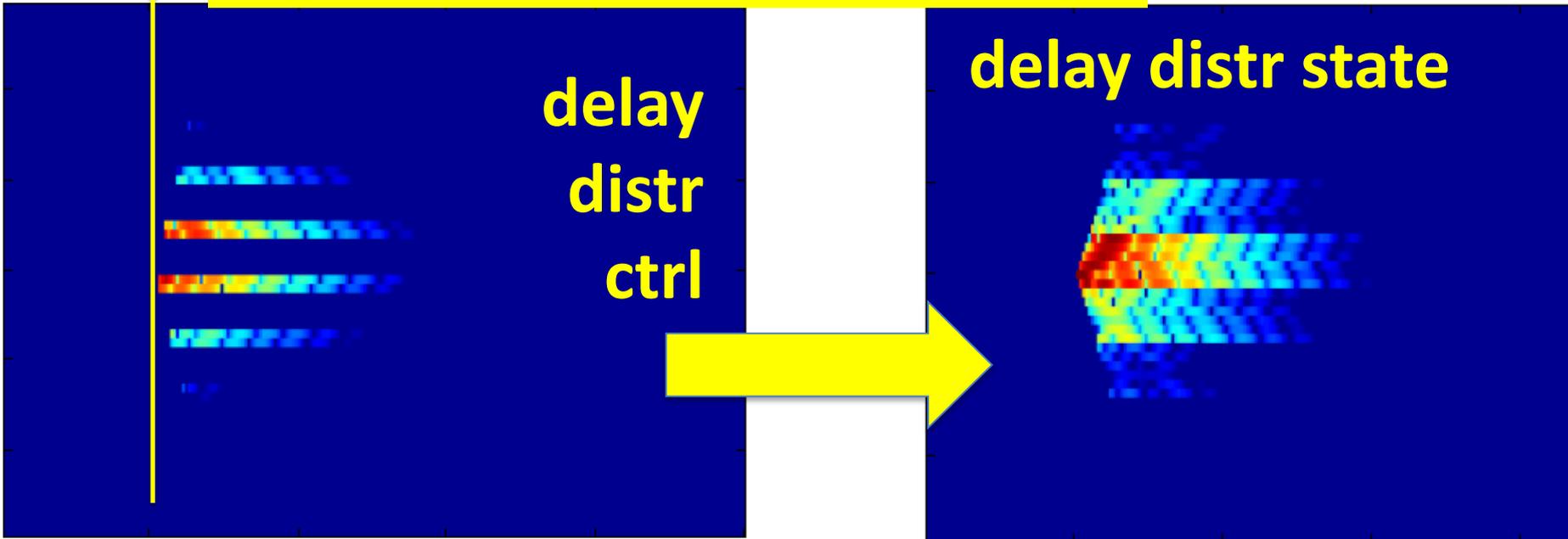
Distributed (QI) Controller

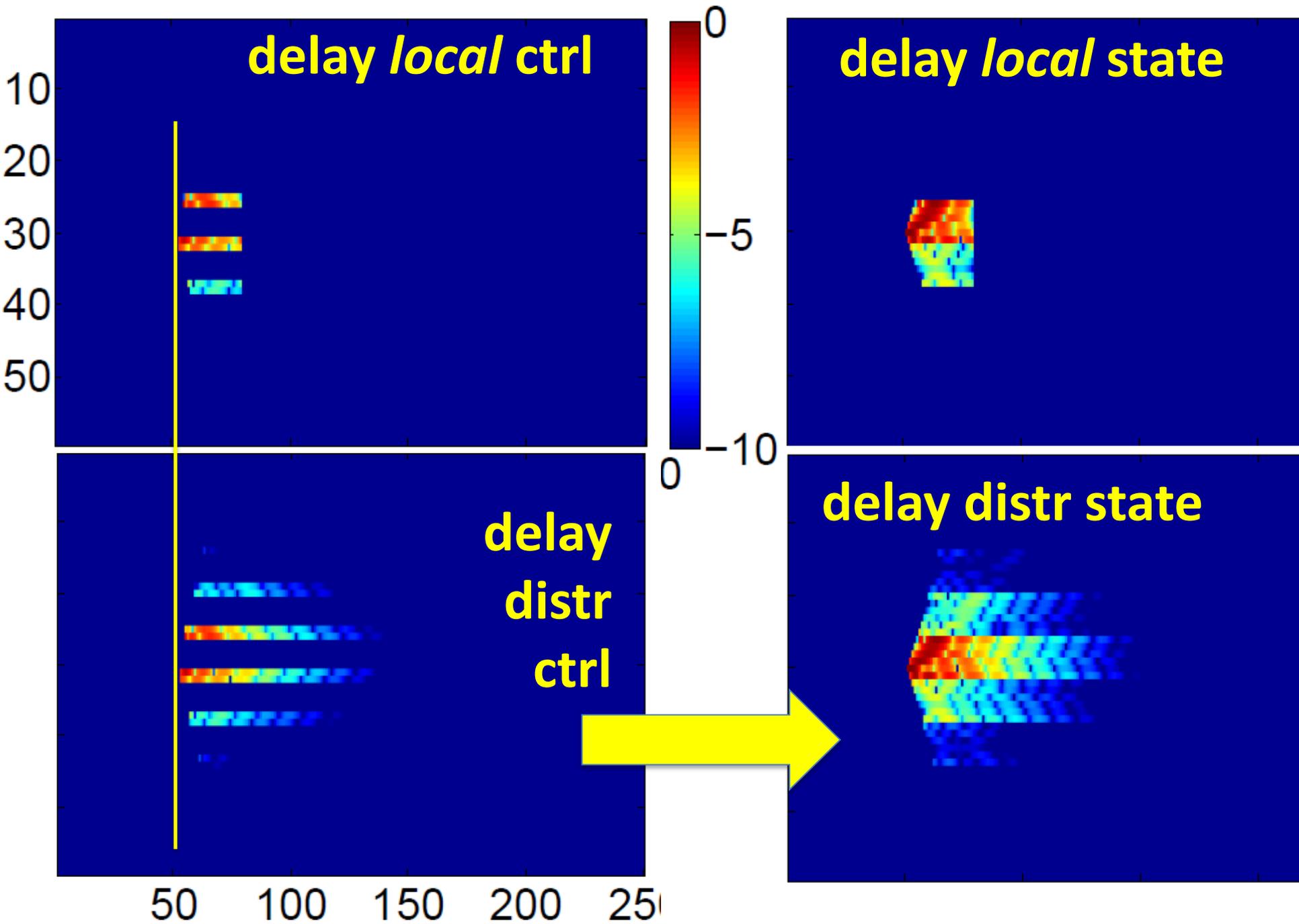
- + Norm (H2) “small”
- + Optimal for constraints
- + Communication delayed
- Design/model global/huge P
- Implementation local/huge P

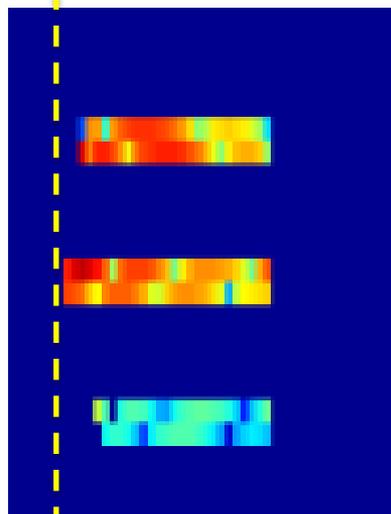
delay
distr
ctrl

delay distr state

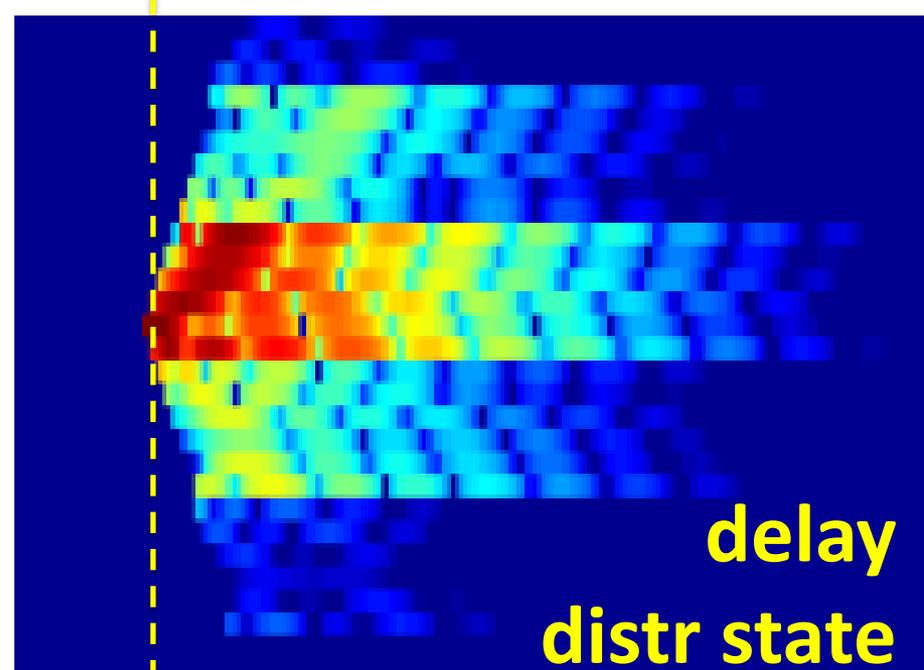
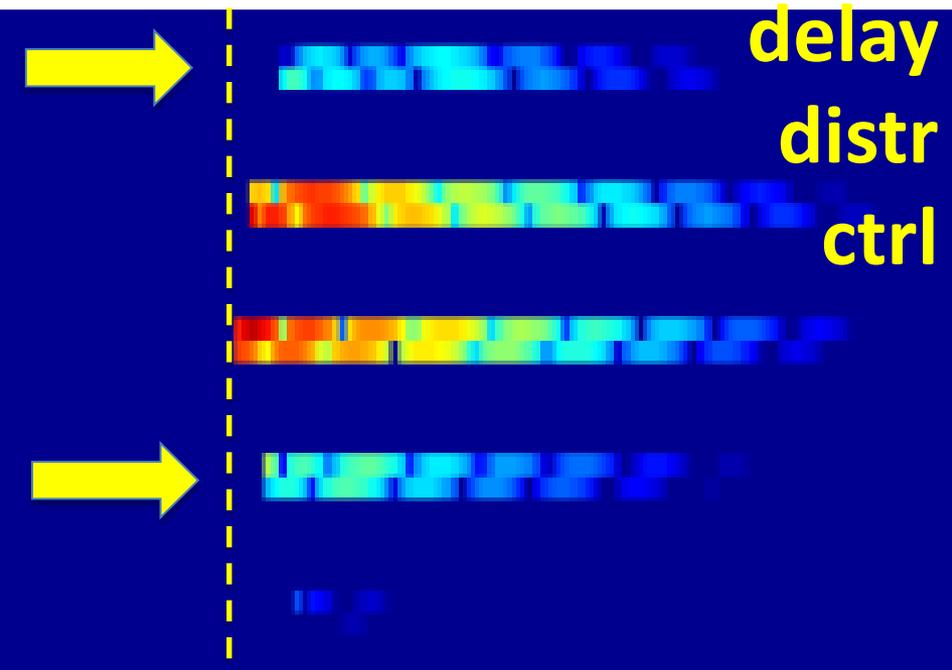
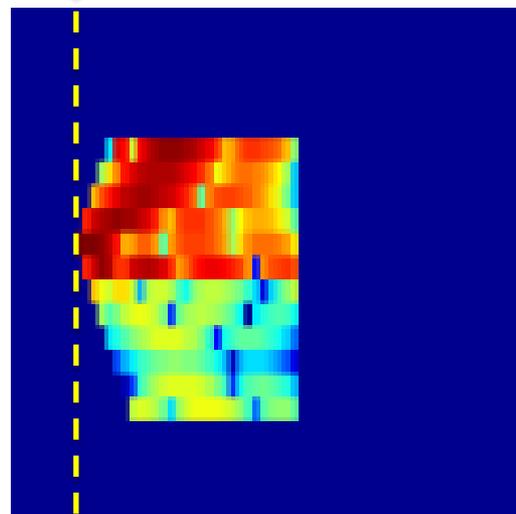
50 100 150 200 250

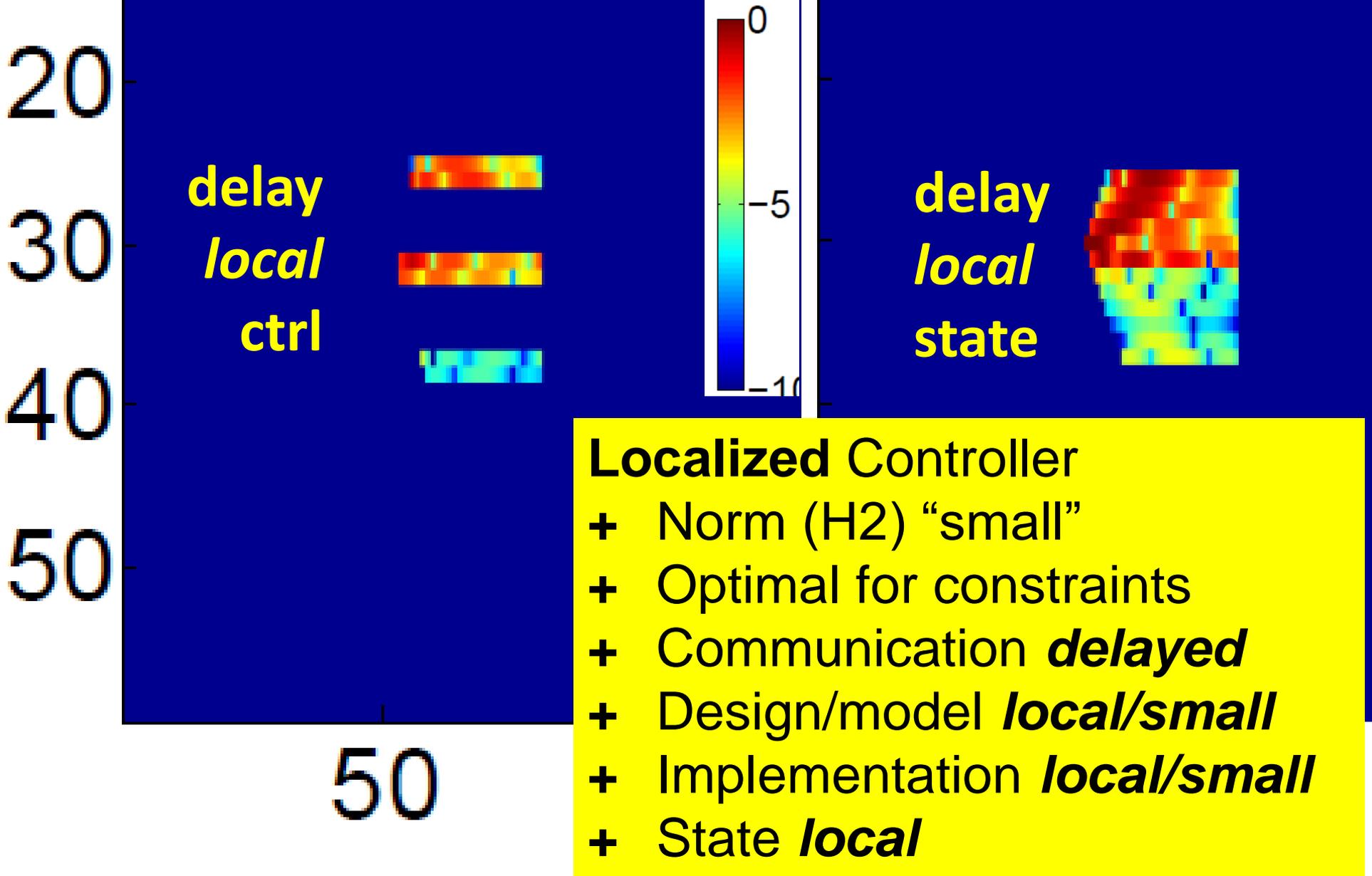






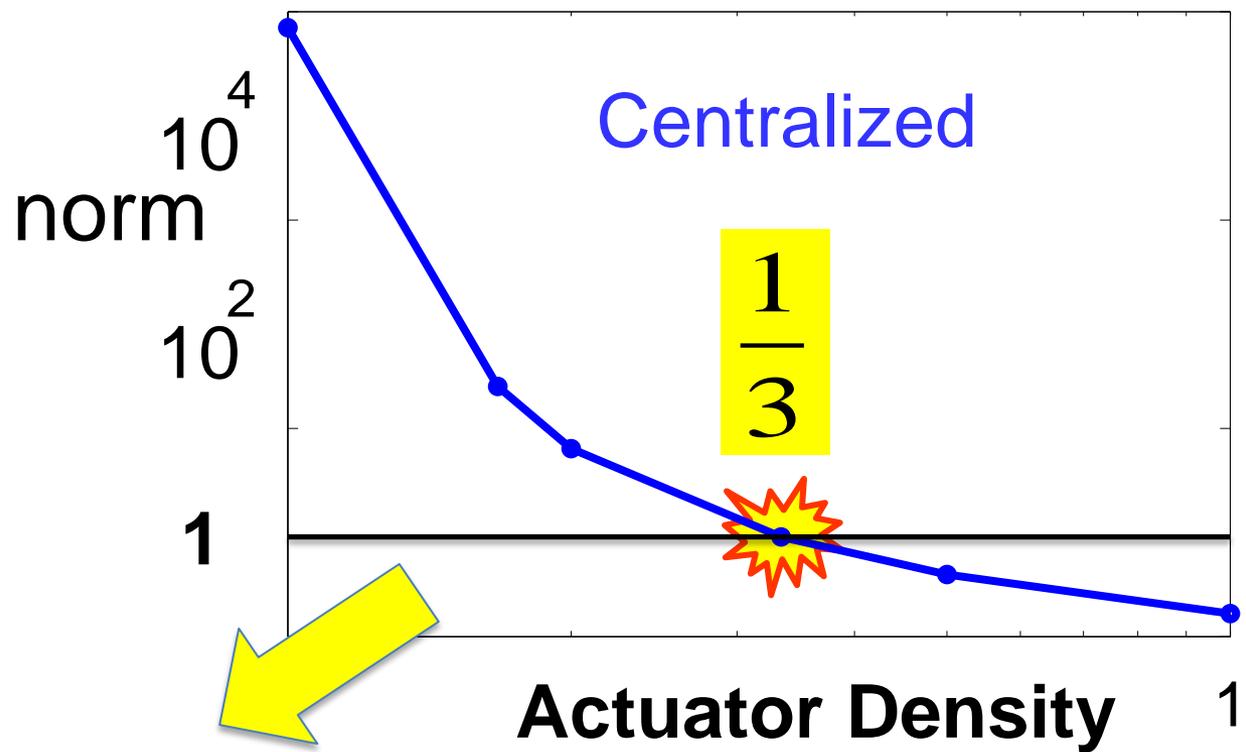
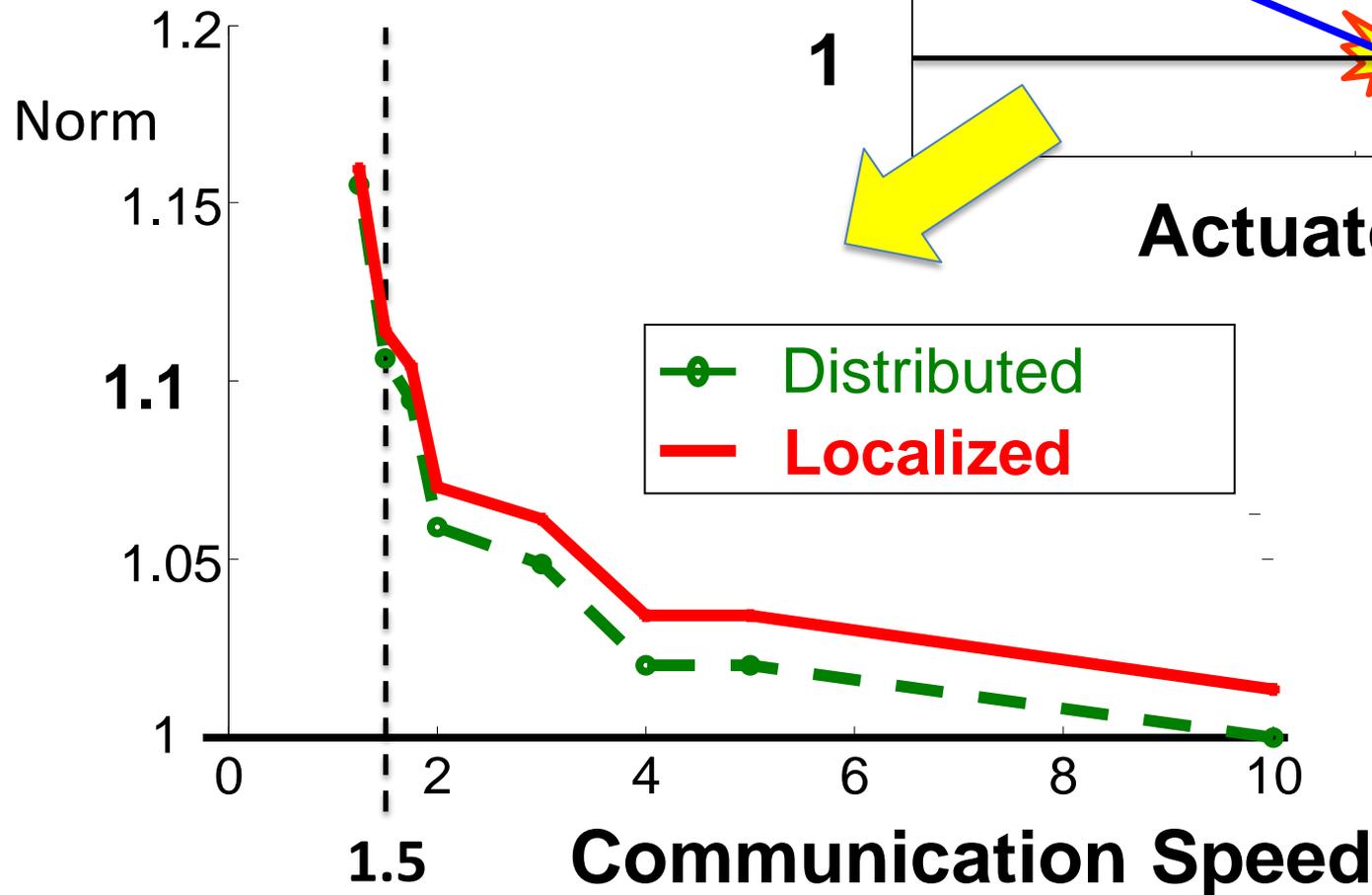
delay *local*





Everything is scalable.

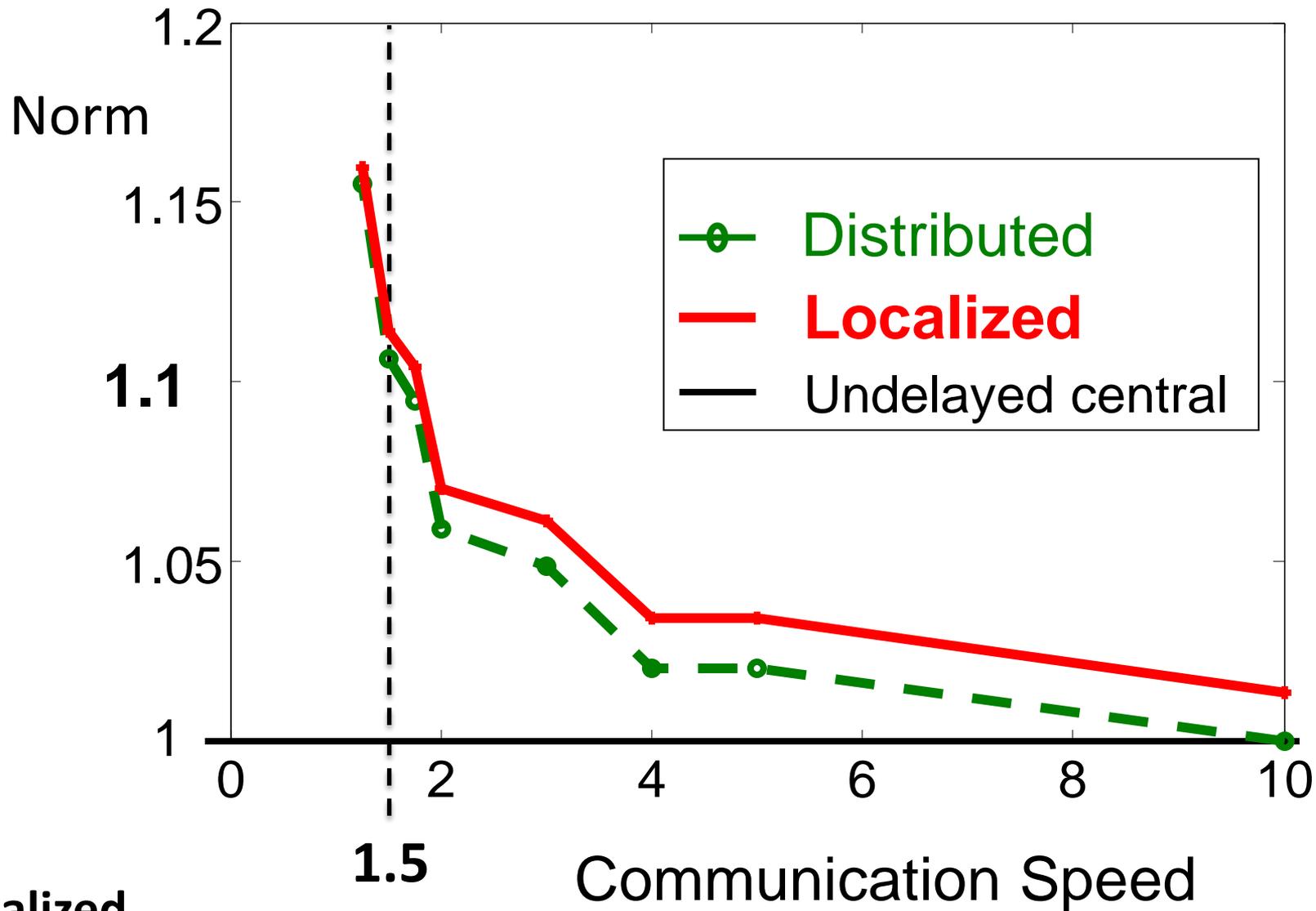
Conjecture:
Norm bad
before method
breaks



Tradeoffs

Extras

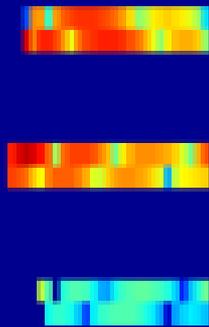
For “handout” and further study



Normalized
by undelayed
centralized

Linear equations

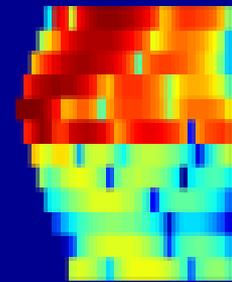
delay
local
ctrl



$$\begin{bmatrix} u[T-1] \\ \vdots \\ u[0] \end{bmatrix}$$

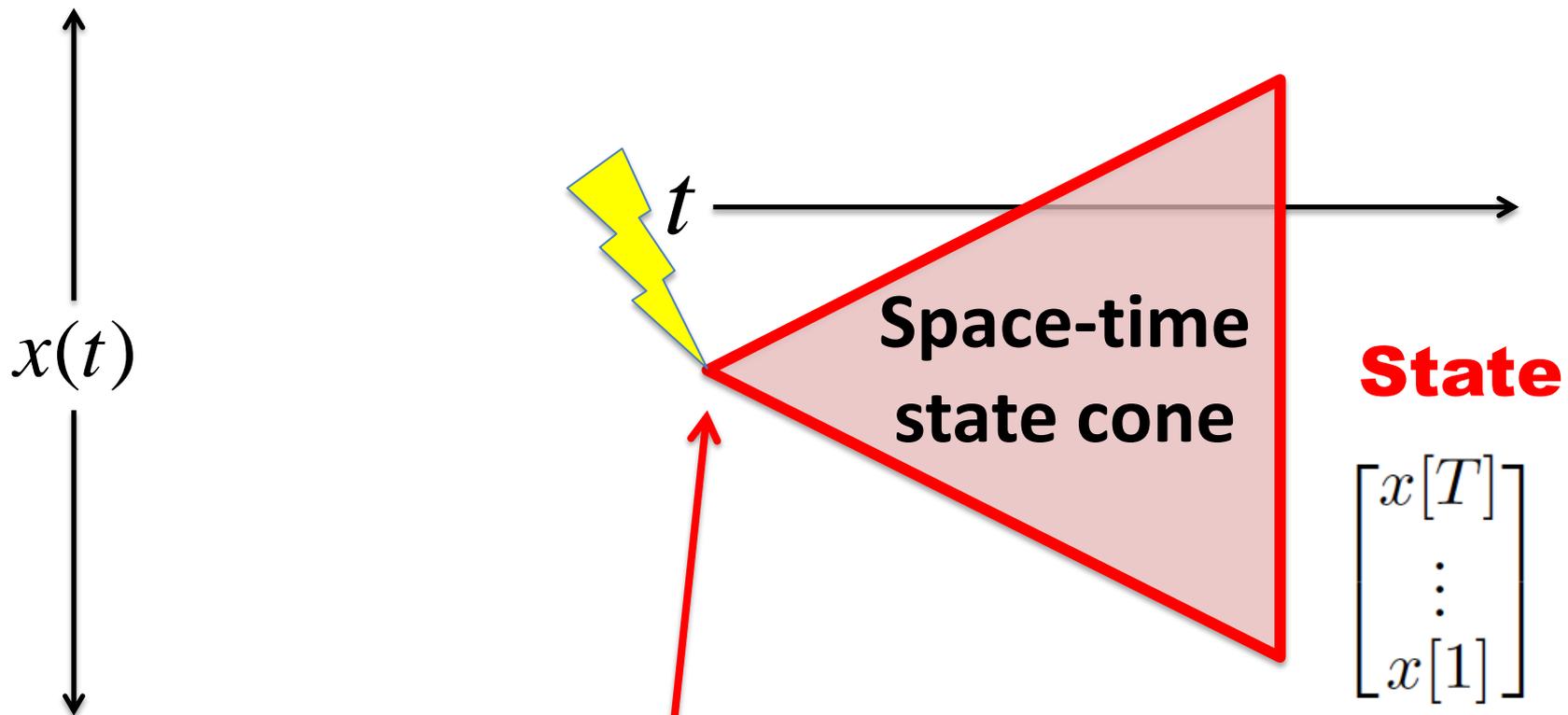
$$\begin{aligned} x[T] &= 0 \\ x &\in X \\ u &\in U \end{aligned}$$

delay
local
state



$$\begin{bmatrix} x[T] \\ \vdots \\ x[1] \end{bmatrix} =$$

$$\begin{bmatrix} x[T] \\ \vdots \\ x[1] \end{bmatrix} = \begin{bmatrix} A^T \\ \vdots \\ A \end{bmatrix} x[0] + \begin{bmatrix} B & \cdots & A^{T-1}B \\ & \ddots & \vdots \\ 0 & & B \end{bmatrix} \begin{bmatrix} u[T-1] \\ \vdots \\ u[0] \end{bmatrix}$$



$$\begin{bmatrix} x[T] \\ \vdots \\ x[1] \end{bmatrix} = \begin{bmatrix} A^T \\ \vdots \\ A \end{bmatrix} x[0] + \begin{bmatrix} B & \dots & A^{T-1}B \\ & \ddots & \vdots \\ 0 & & B \end{bmatrix} \begin{bmatrix} u[T-1] \\ \vdots \\ u[0] \end{bmatrix}$$

$x(t)$

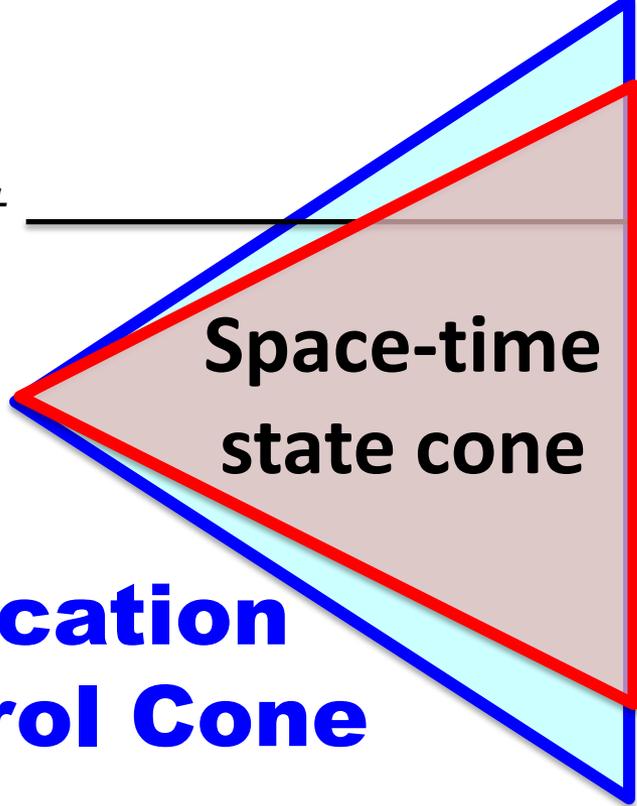
t

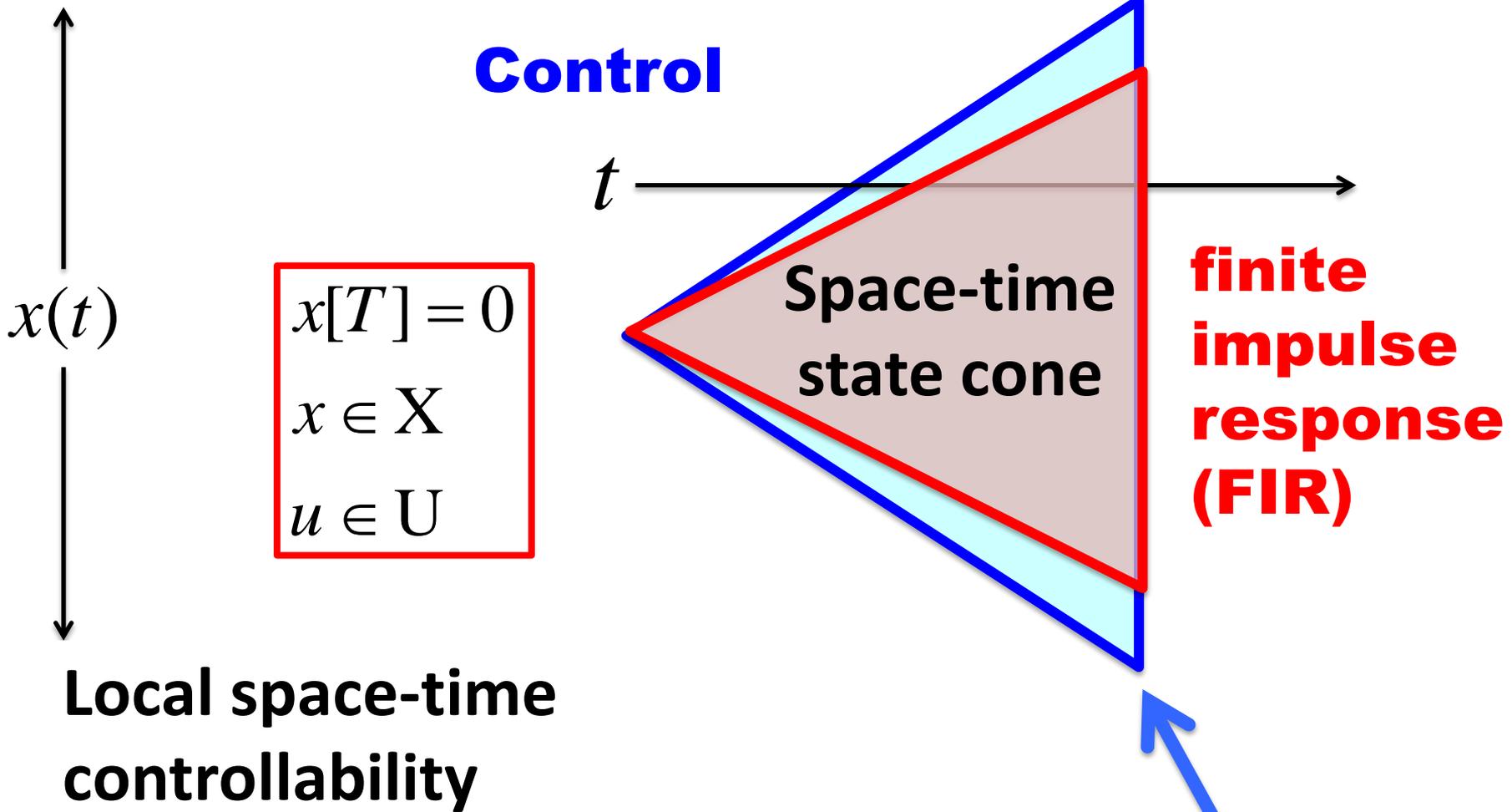
**Communication
and Control Cone**

Space-time
state cone

State

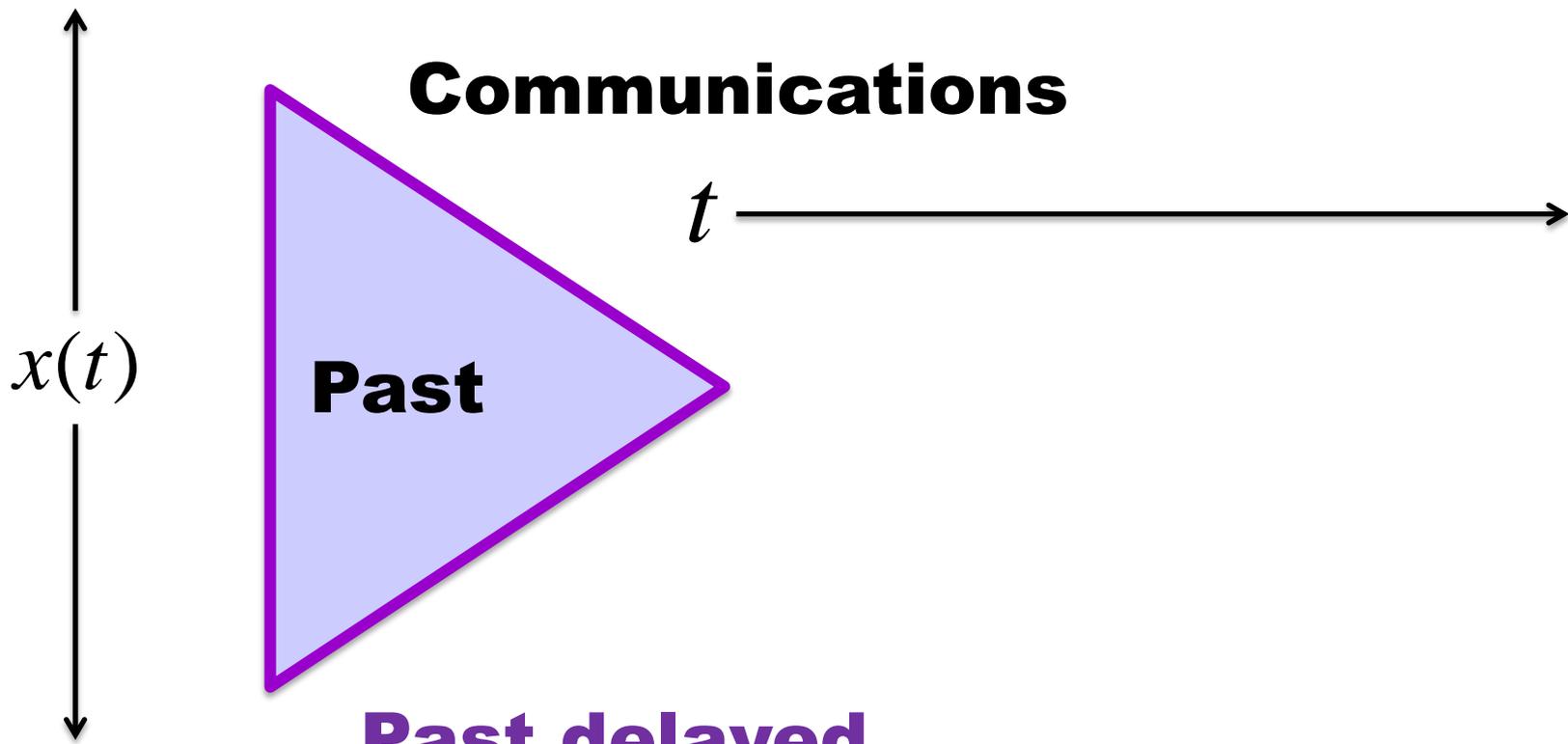
**finite
impulse
response
(FIR)**



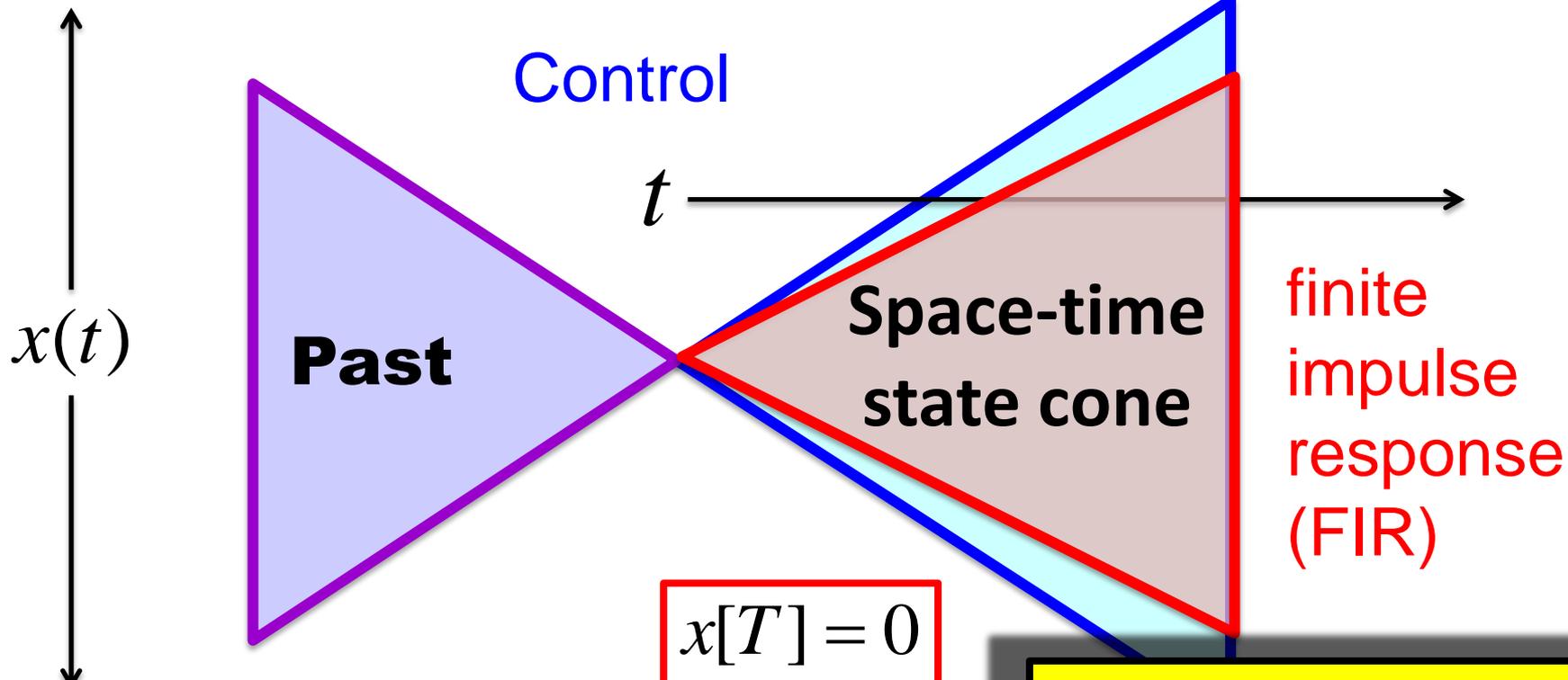


$$\begin{bmatrix} x[T] \\ \vdots \\ x[1] \end{bmatrix} = \begin{bmatrix} A^T \\ \vdots \\ A \end{bmatrix} x[0] + \begin{bmatrix} B & \cdots & A^{T-1}B \\ & \ddots & \vdots \\ 0 & & B \end{bmatrix} \begin{bmatrix} u[T-1] \\ \vdots \\ u[0] \end{bmatrix}$$

Communications



**Past delayed
state needed
to compute
control**

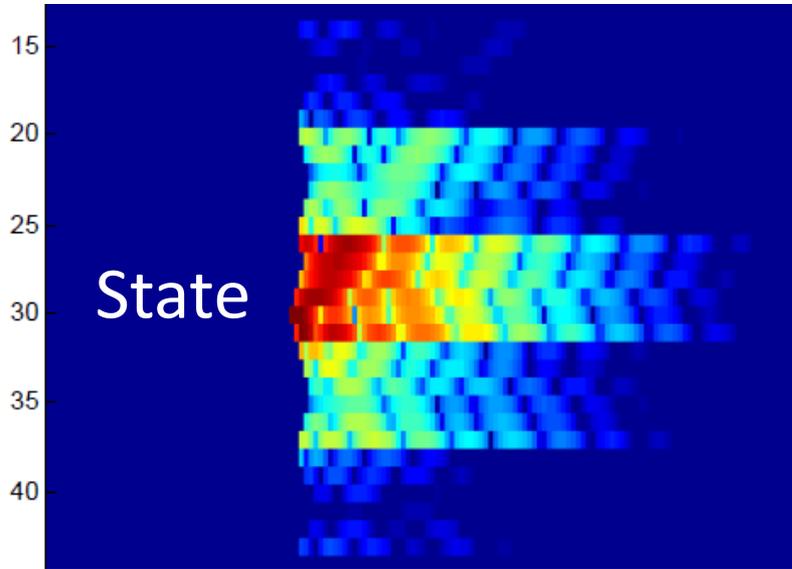


Local space-time controllability

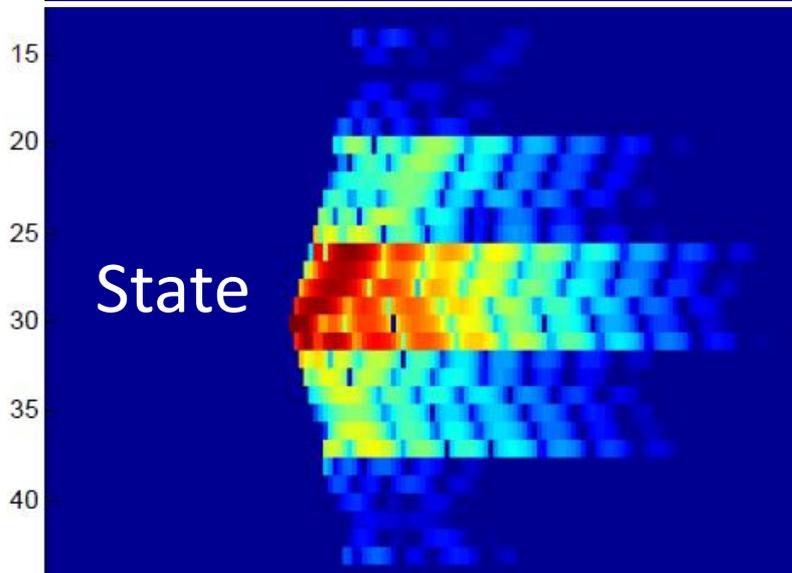
$$\begin{aligned}
 &x[T] = 0 \\
 &x \in X \\
 &u \in U
 \end{aligned}$$

This can *linearly* constrain *any* optimization

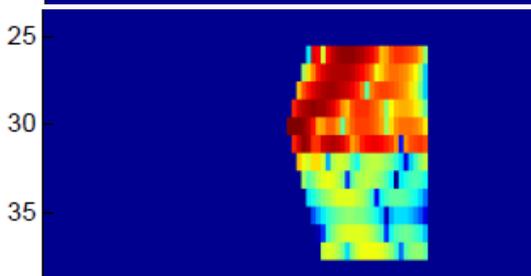
$$\begin{bmatrix} x[T] \\ \vdots \\ x[1] \end{bmatrix} = \begin{bmatrix} A^T \\ \vdots \\ A \end{bmatrix} x[0] + \begin{bmatrix} B & \cdots & A^{T-1}B \\ & \ddots & \vdots \\ 0 & & B \end{bmatrix} \begin{bmatrix} u[T-1] \\ \vdots \\ u[0] \end{bmatrix}$$



**Optimal
undelayed
centralized
state (old)**

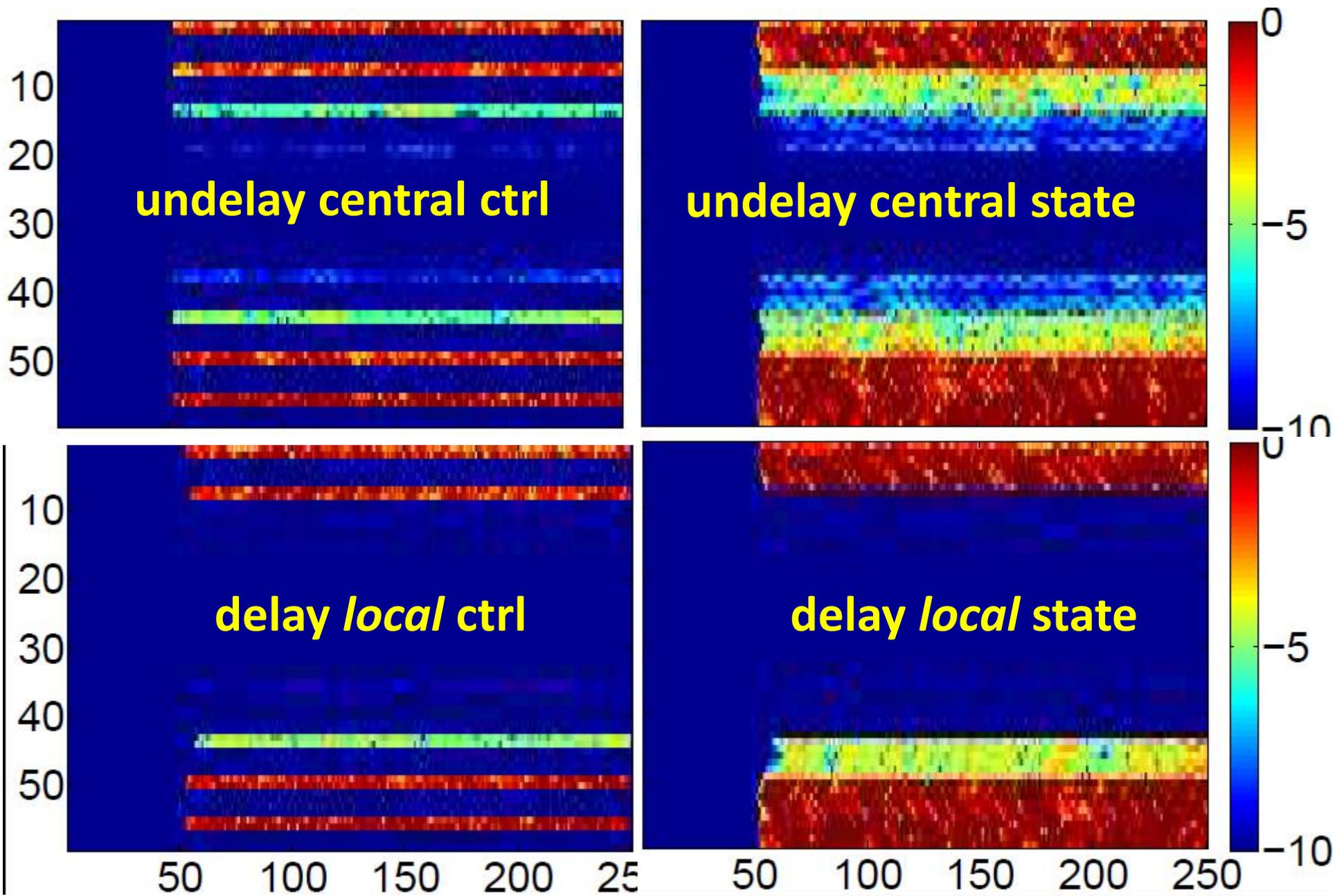


**Optimal delayed
distributed (newish)
(but not scalable)**



**Optimal delayed *localized*
(very new, scalable)**

AWGN in C2, L26, C29



Control

Localized Controller

- + Norm (H2) ***small***
- + Optimal for constraints
- + Communication is ***delayed***
- + Design/model ***local/small***
- + Implementation ***local/small***
- + State ***local***

finite
impulse
response
(FIR)

$x(t)$

**Local space-time
controllability**

$$\begin{aligned} x &\in X \\ u &\in U \end{aligned}$$

$$\begin{bmatrix} x[T] \\ \vdots \\ x[1] \end{bmatrix} = \begin{bmatrix} A^T \\ \vdots \\ A \end{bmatrix} x[0] + \begin{bmatrix} B \\ \vdots \\ 0 \end{bmatrix} \begin{bmatrix} \cdot \\ \cdot \\ \cdot \end{bmatrix}$$

This can ***linearly***
constrain any
optimization

Localized Controller

- + Norm (H2) small
- + Optimal for constraints
- + Design/model is local
- + Implementation is local
- + State stays local

- Bandwidth is ∞

? **Output feedback?**

? **Approximately local?**

? **Layering?**

? **Nonlinear, MPC, etc?**

? **Comms codesign?**

See also Javad's new relaxations

Mostly good news, but incomplete

Extensions

- Scalable optimal control
 - Localizable control: Y.-S. Wang, N. Matni, S. You and J. C. Doyle ACC '14
 - Localized LQR control: Y.-S. Wang, N. Matni, and J. C. Doyle CDC'14
 - Output feedback progress
- Dealing with varying-delays (jitter)
 - Two player LQR with varying delays: N. Matni and J. C. Doyle CDC' 13, N. Matni, A. Lamperski and J. C. Doyle IFAC '14

More Nikolai Matni

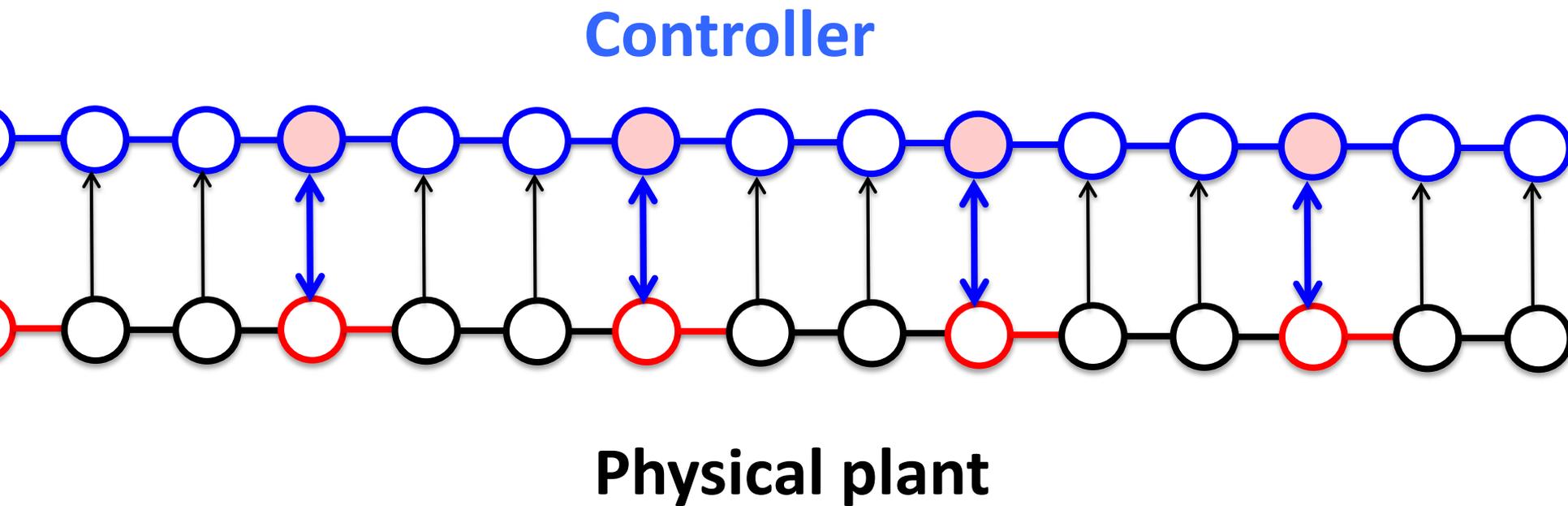
- Distributed/scalable system identification
 - Low-rank + Low-order decompositions: N. Matni and A. Rantzer, CDC' 14
- Structured Robustness
 - Distributed Controllers Satisfying an H_∞ norm bound: N. Matni CDC '14
- Regularization for Design
 - Topology/interconnection design: N. Matni, CDC '13 (best student paper), TCNS '14
 - More broadly (including actuator/sensor placement): N. Matni and V. Chandrasekaran, CDC '14

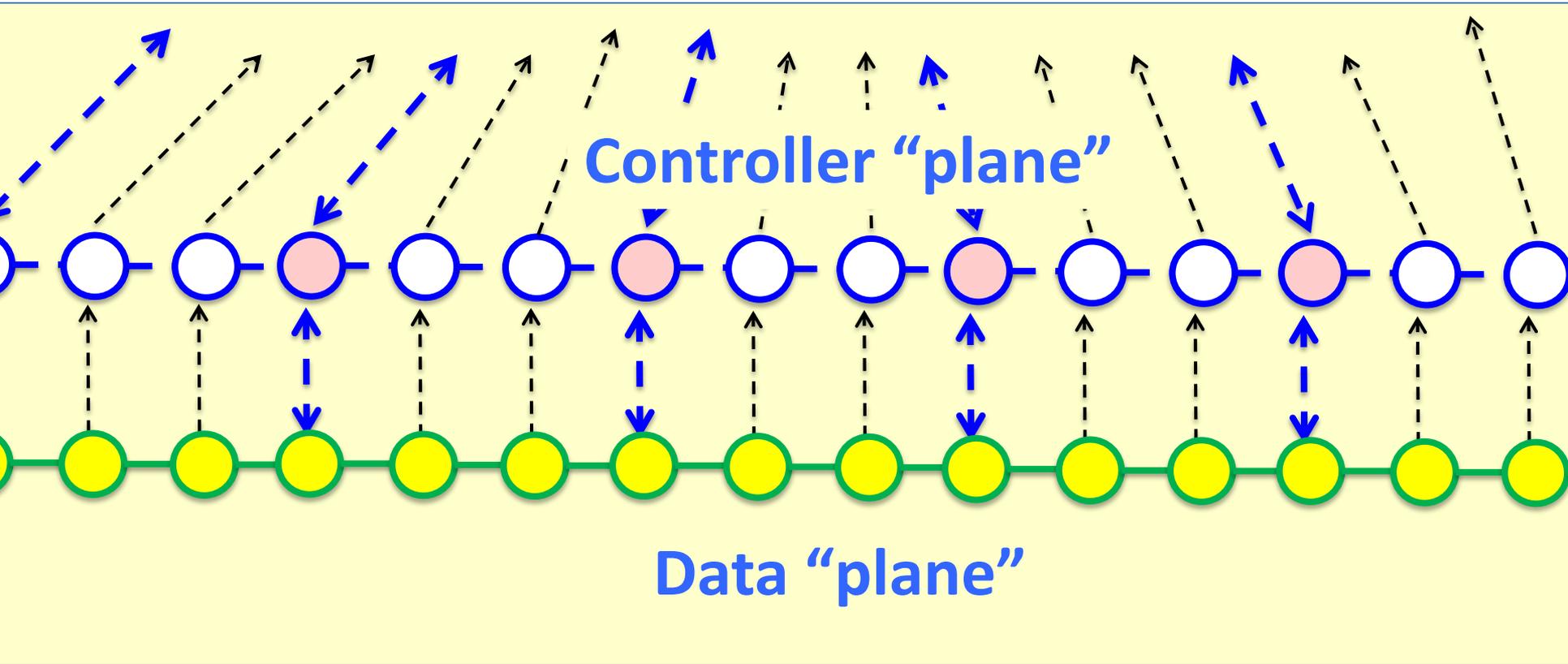
More Extensions/Apps

- Apps: neuro, smartgrid, CPS, cells
- IMC/RHC, etc (all of centralized control theory)
- Cyber theory: Delay jitter (uncertainty)
- Cyber: Comms co-design (CDC student prize paper)
- Physical: Robustness (unmodeled dynamics, noise)
- Cyber-phys: System ID, ML, adaptive
- SDN (Software defined nets, OpenDaylight)

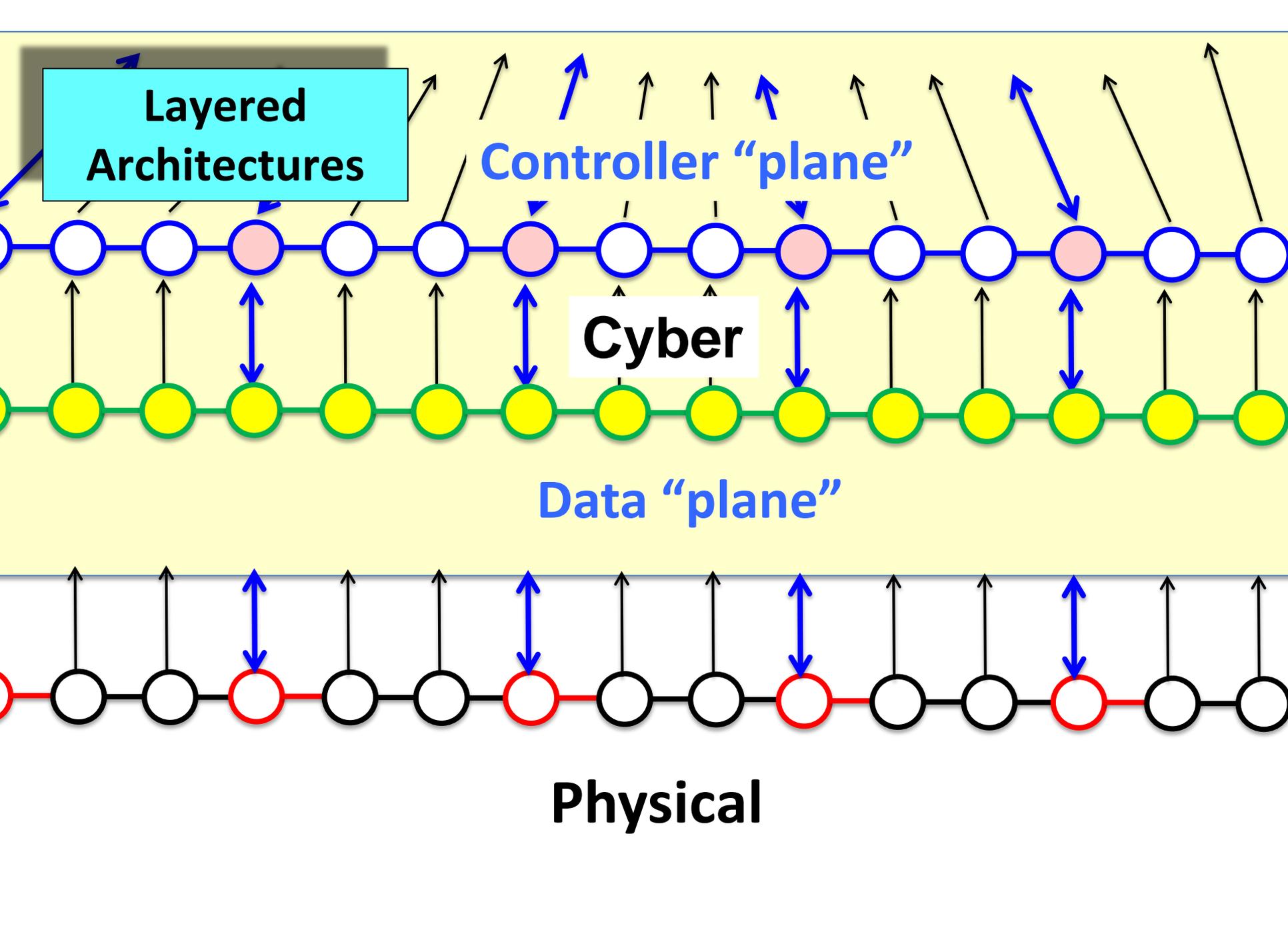
- Revisit “layering as optimization”?
- Poset causality (streamlining)?
- Quantization and network coding?

Revisit layering as optimization decomposition
Chiang, Low, Calderbank, Doyle, 2007

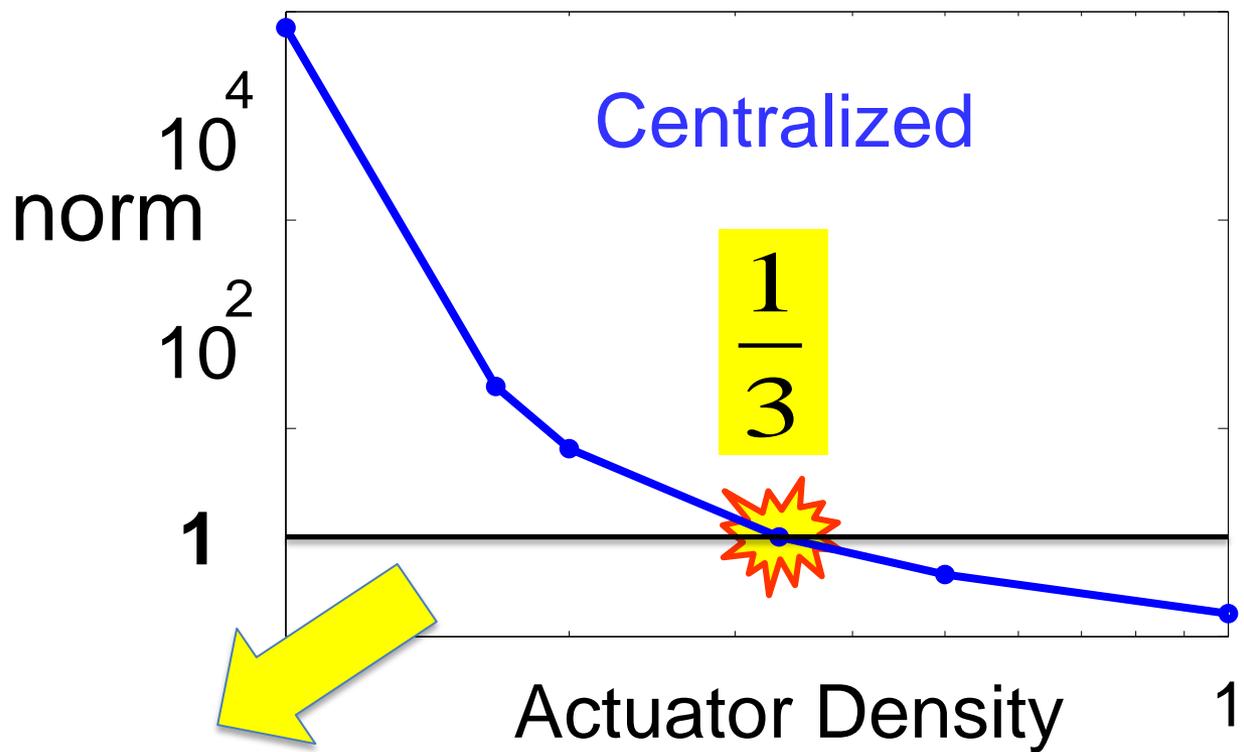
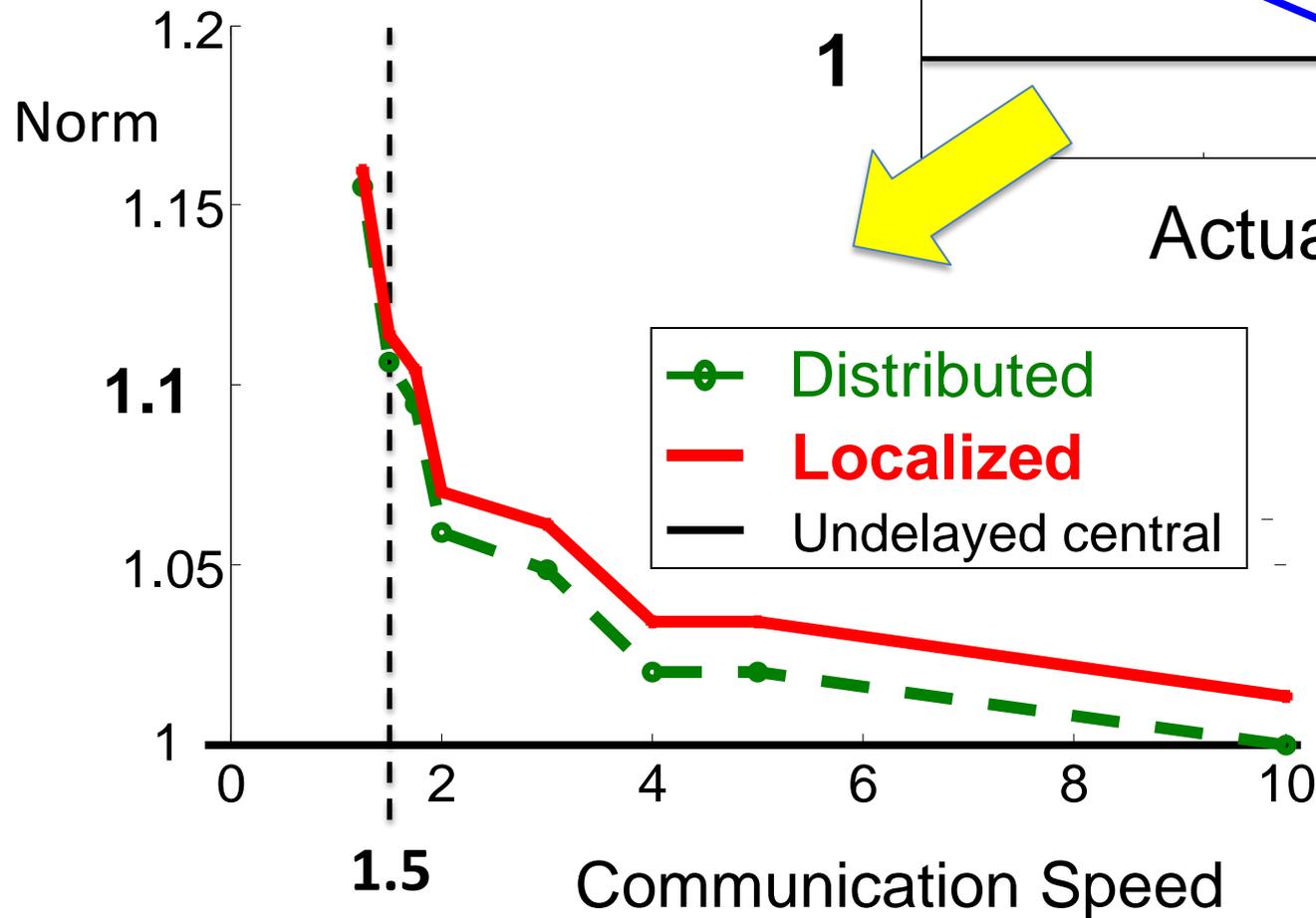




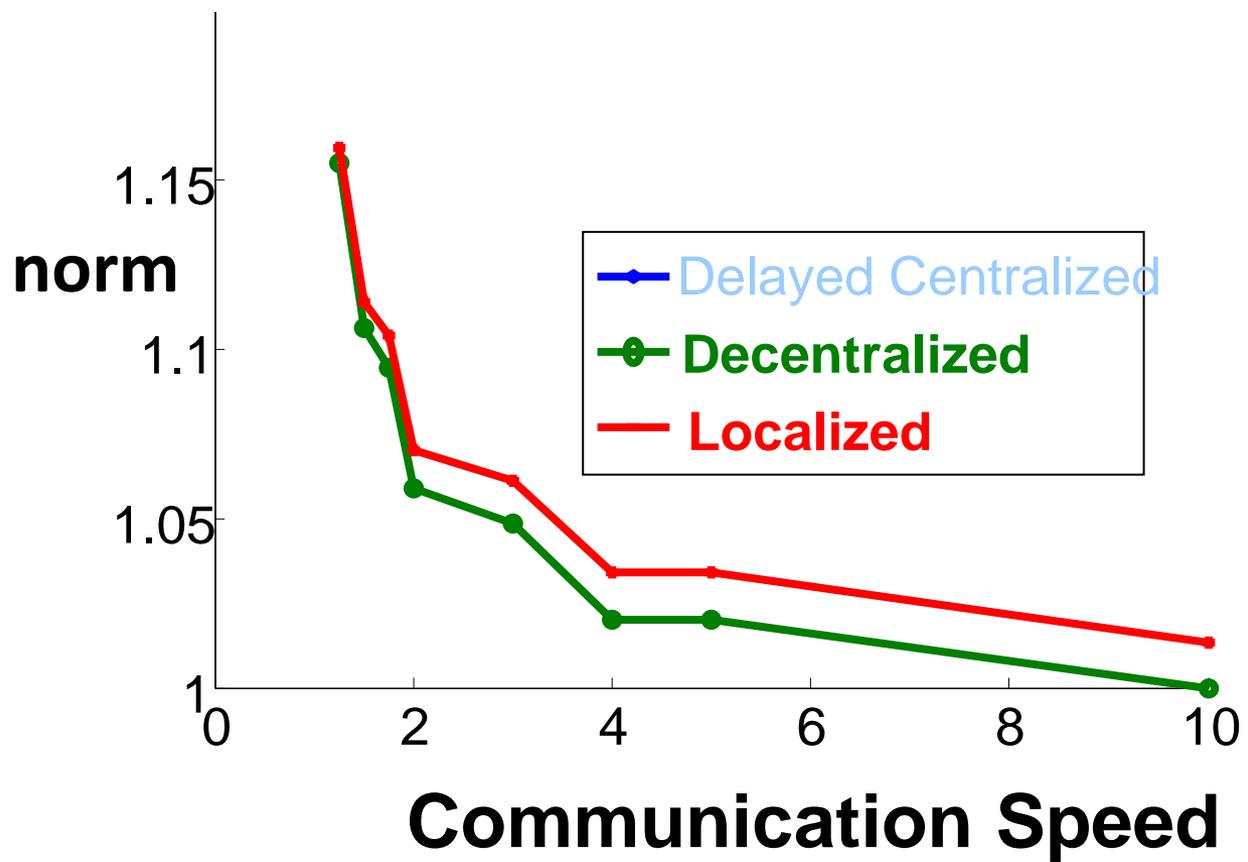
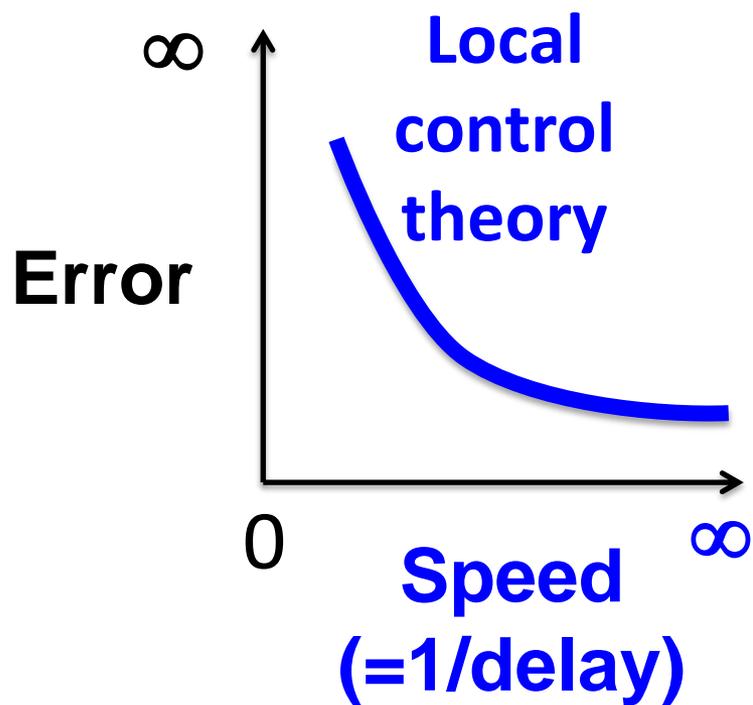
SDN/ODP

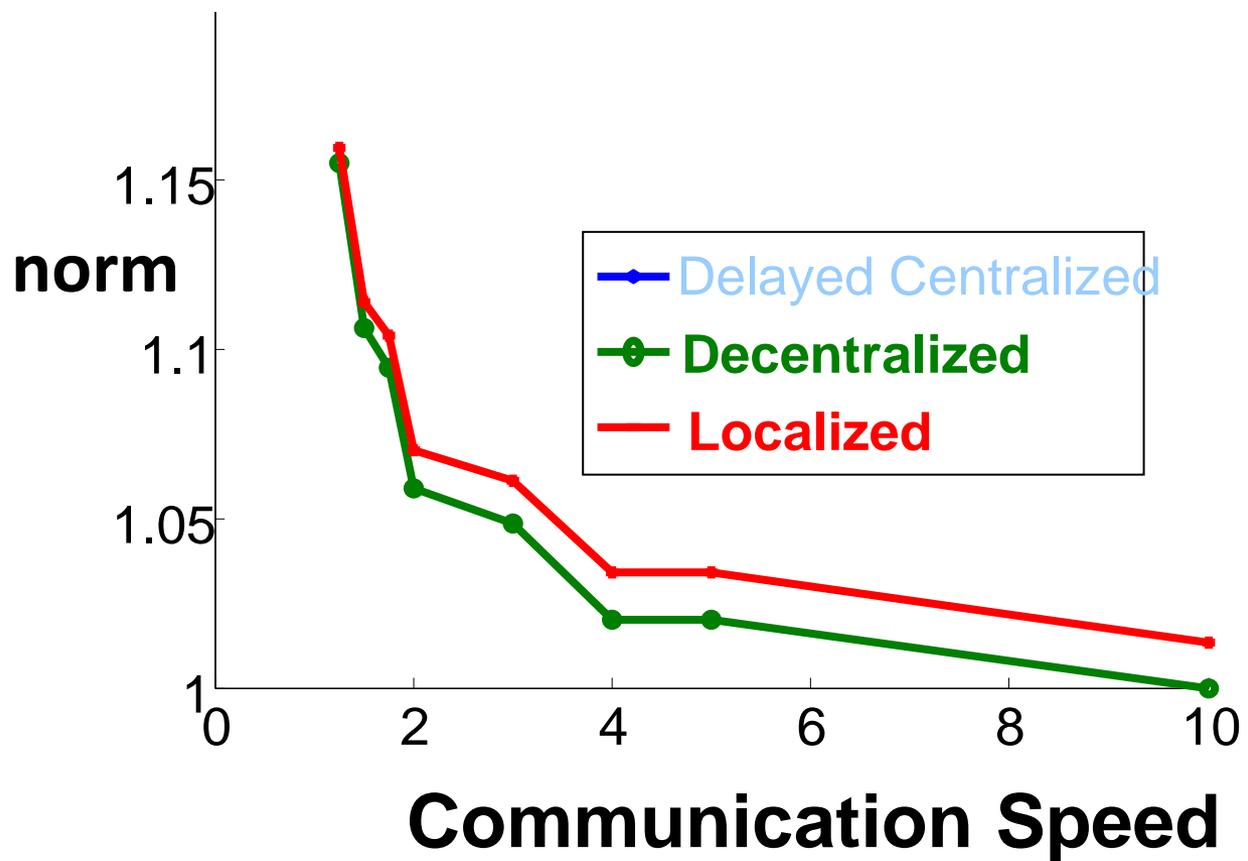
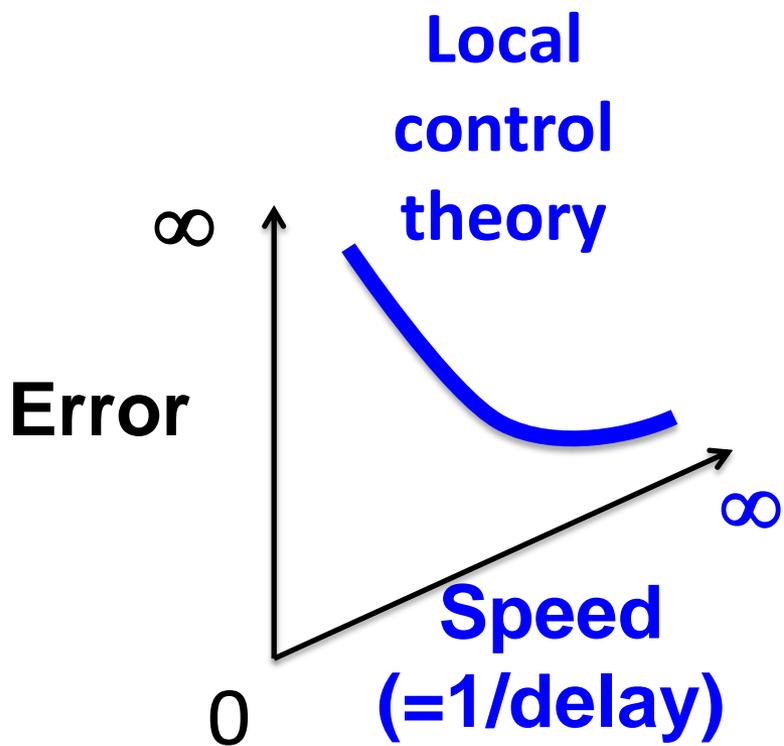


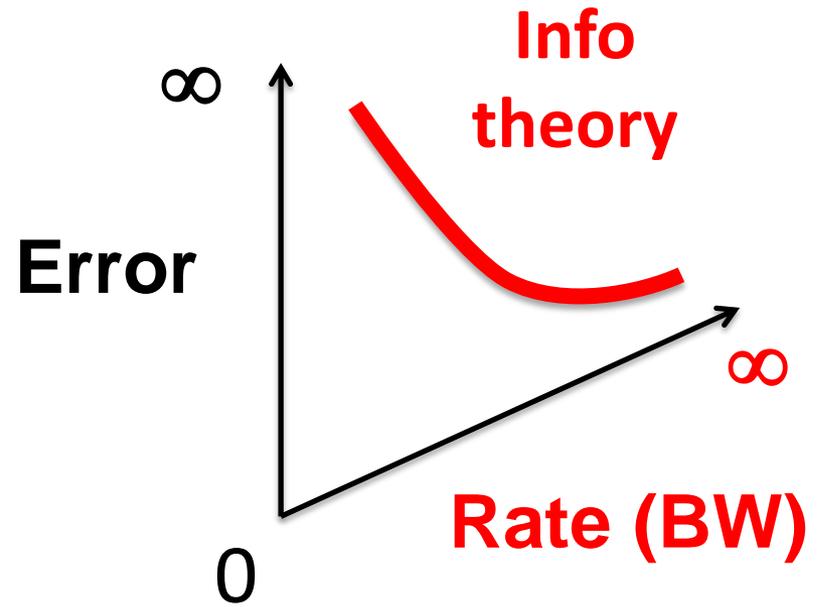
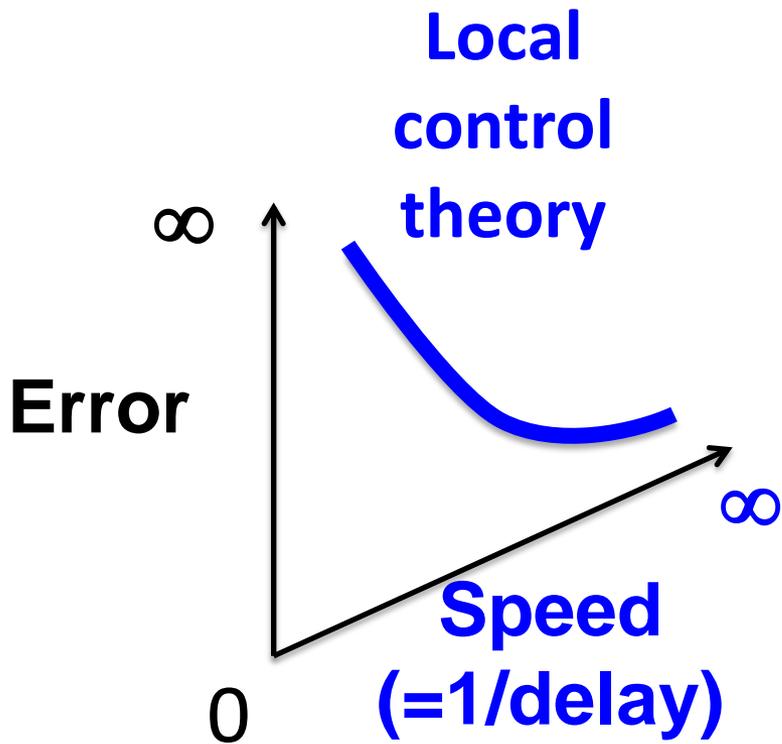
Conjecture:
Norm bad
before method
breaks



Tradeoffs

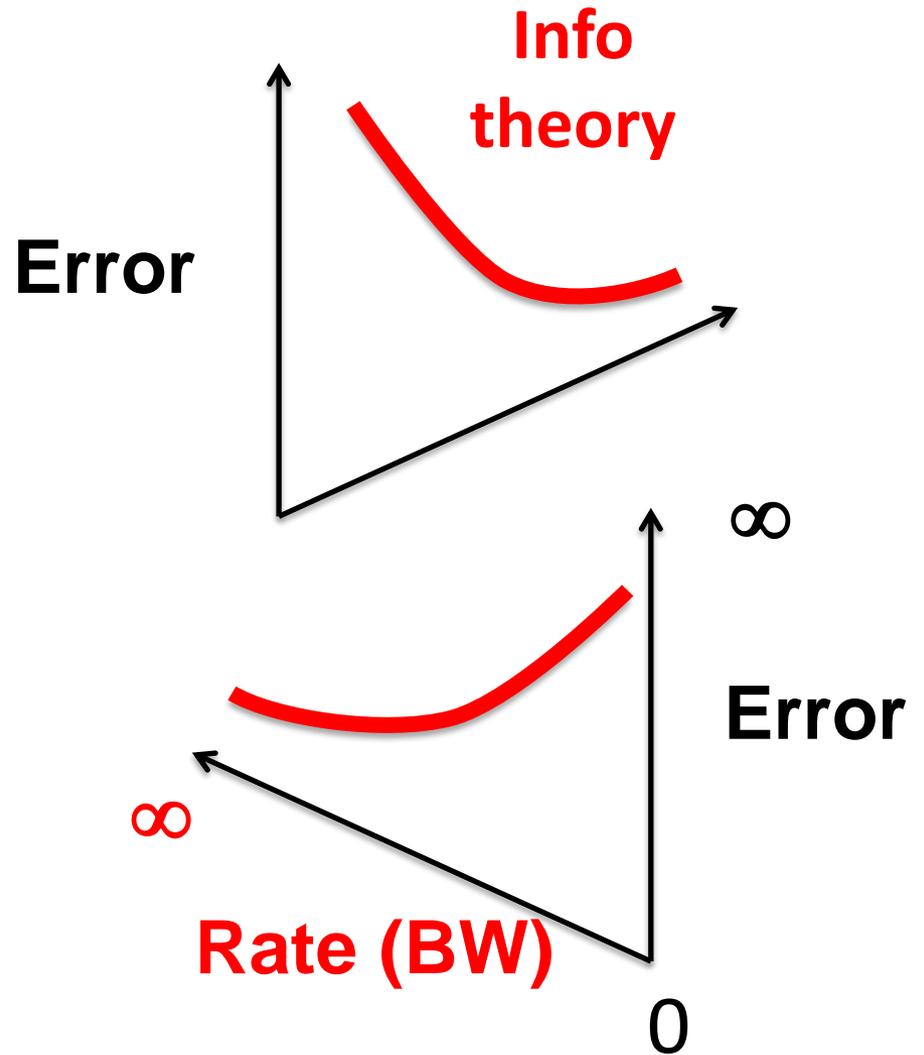
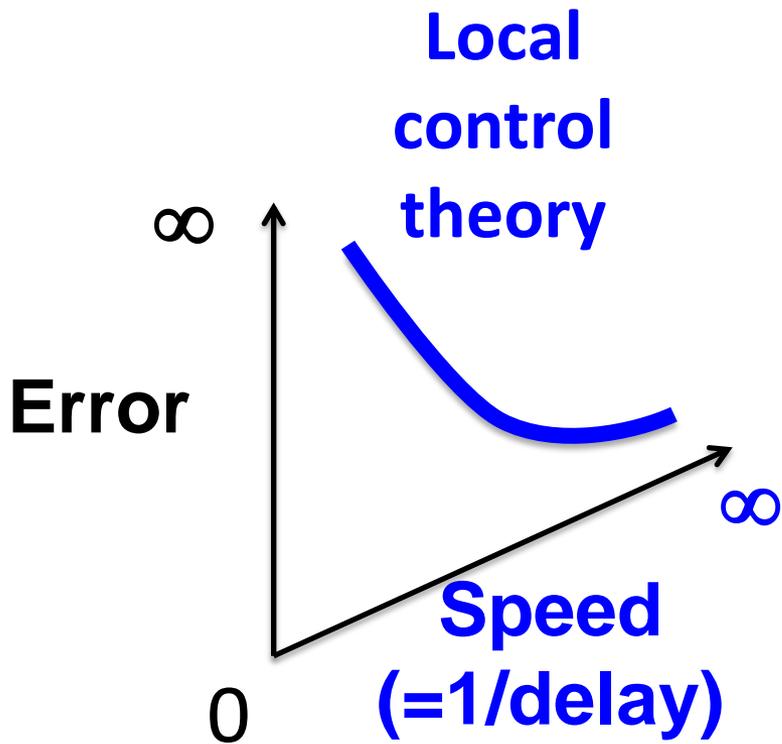




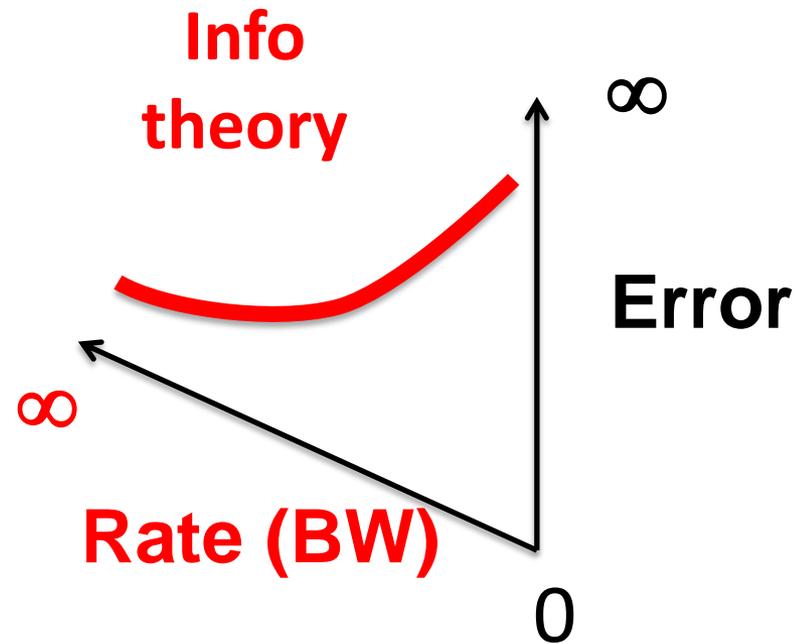
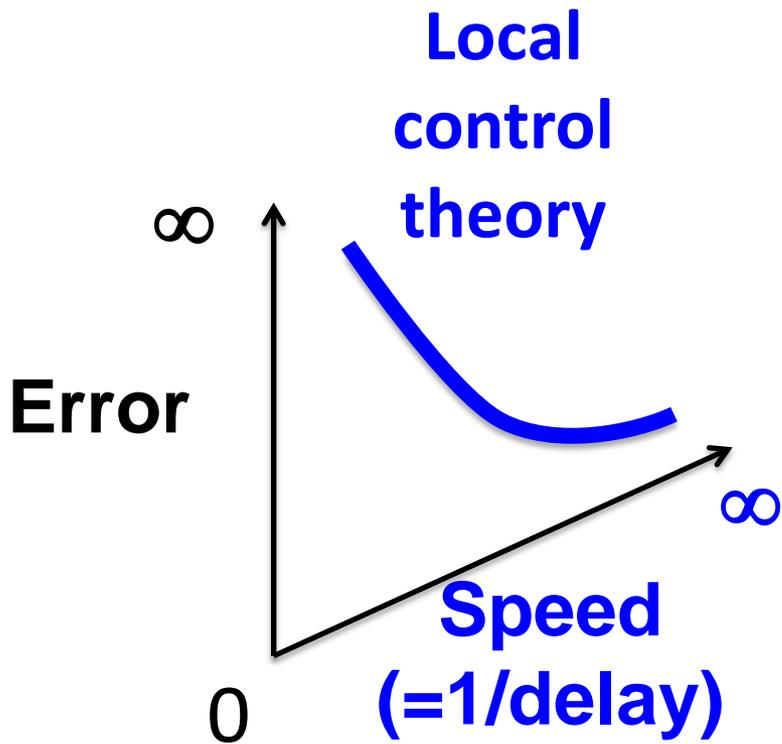


Due to quantization, loss, noise

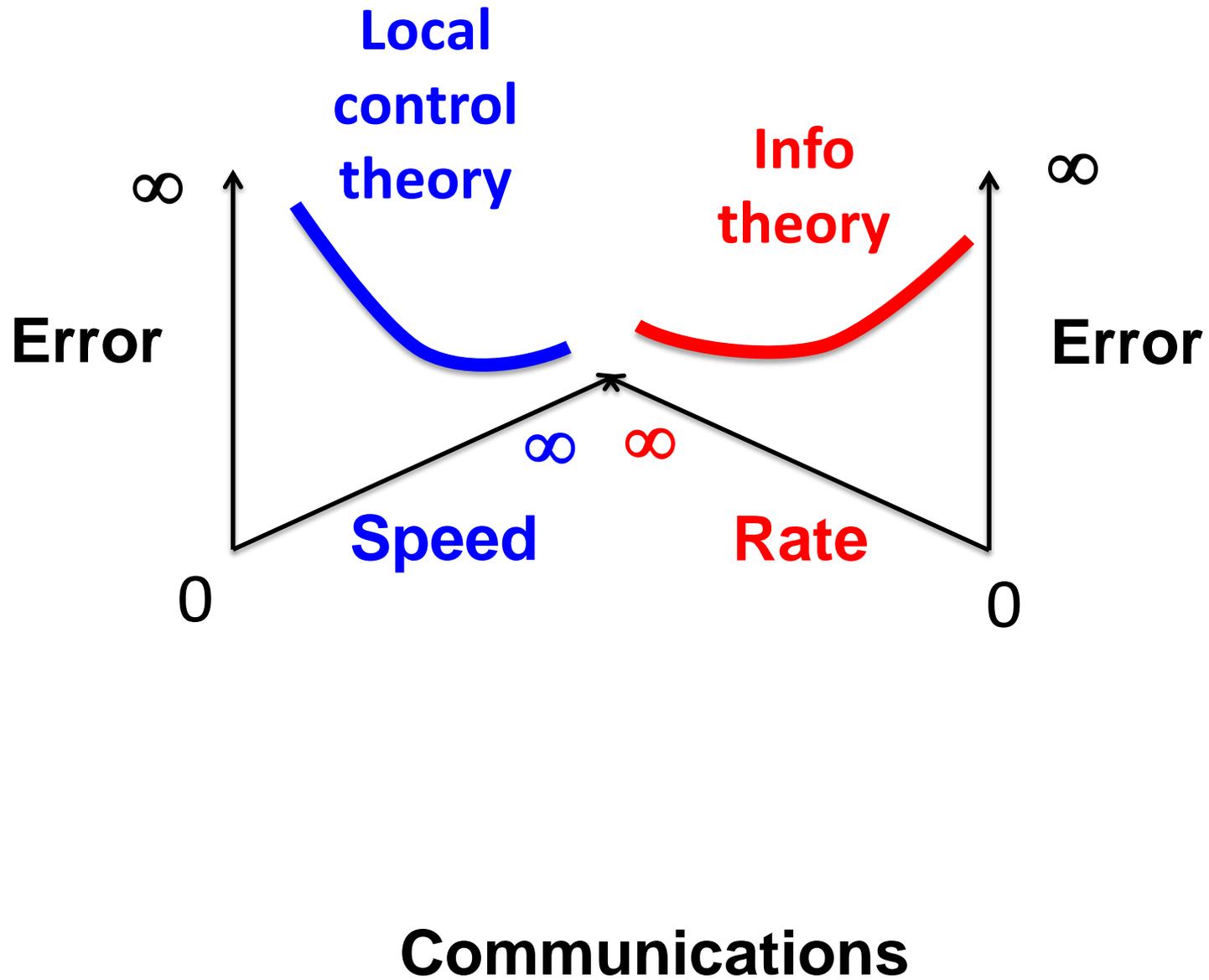
Communications

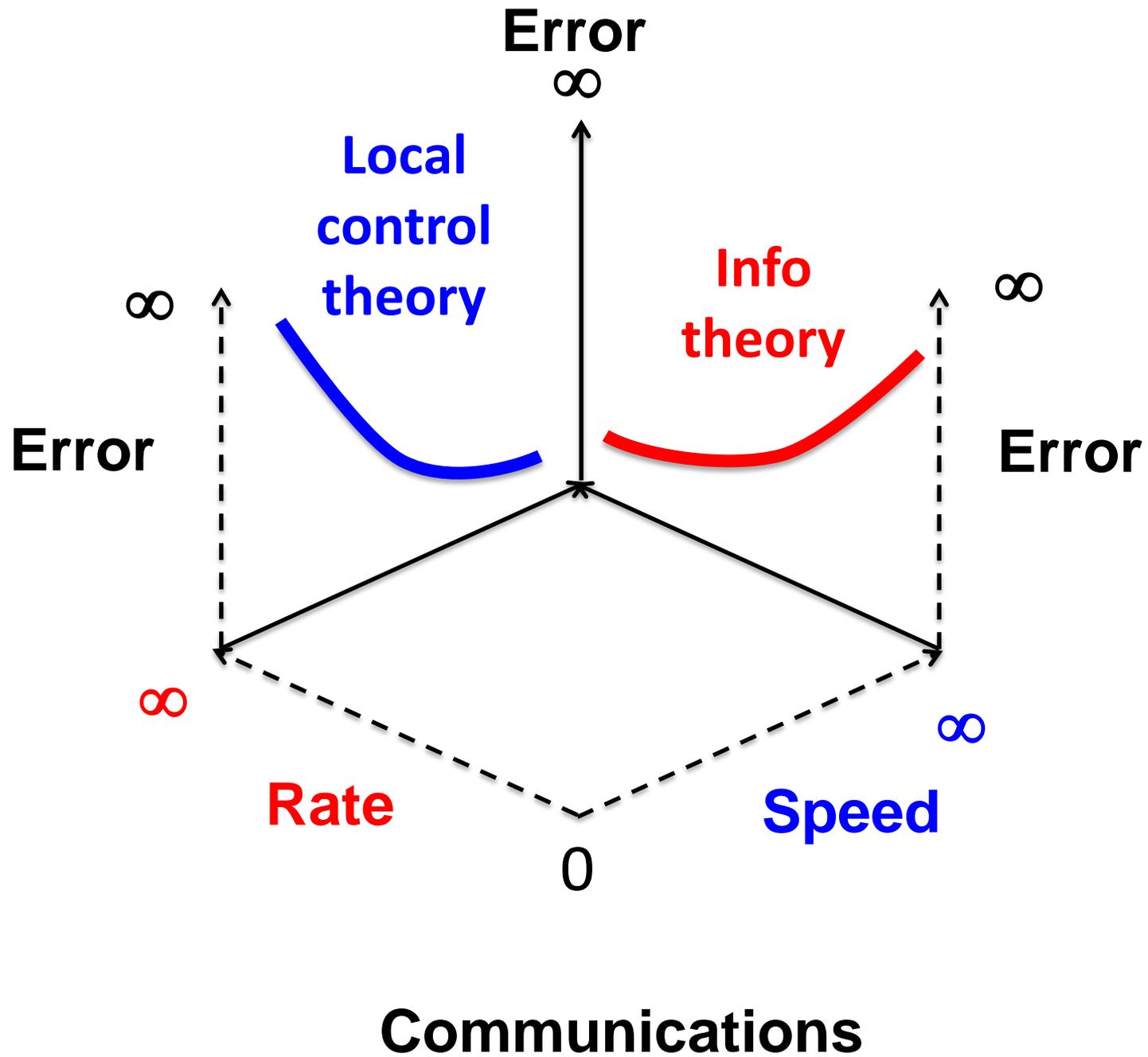


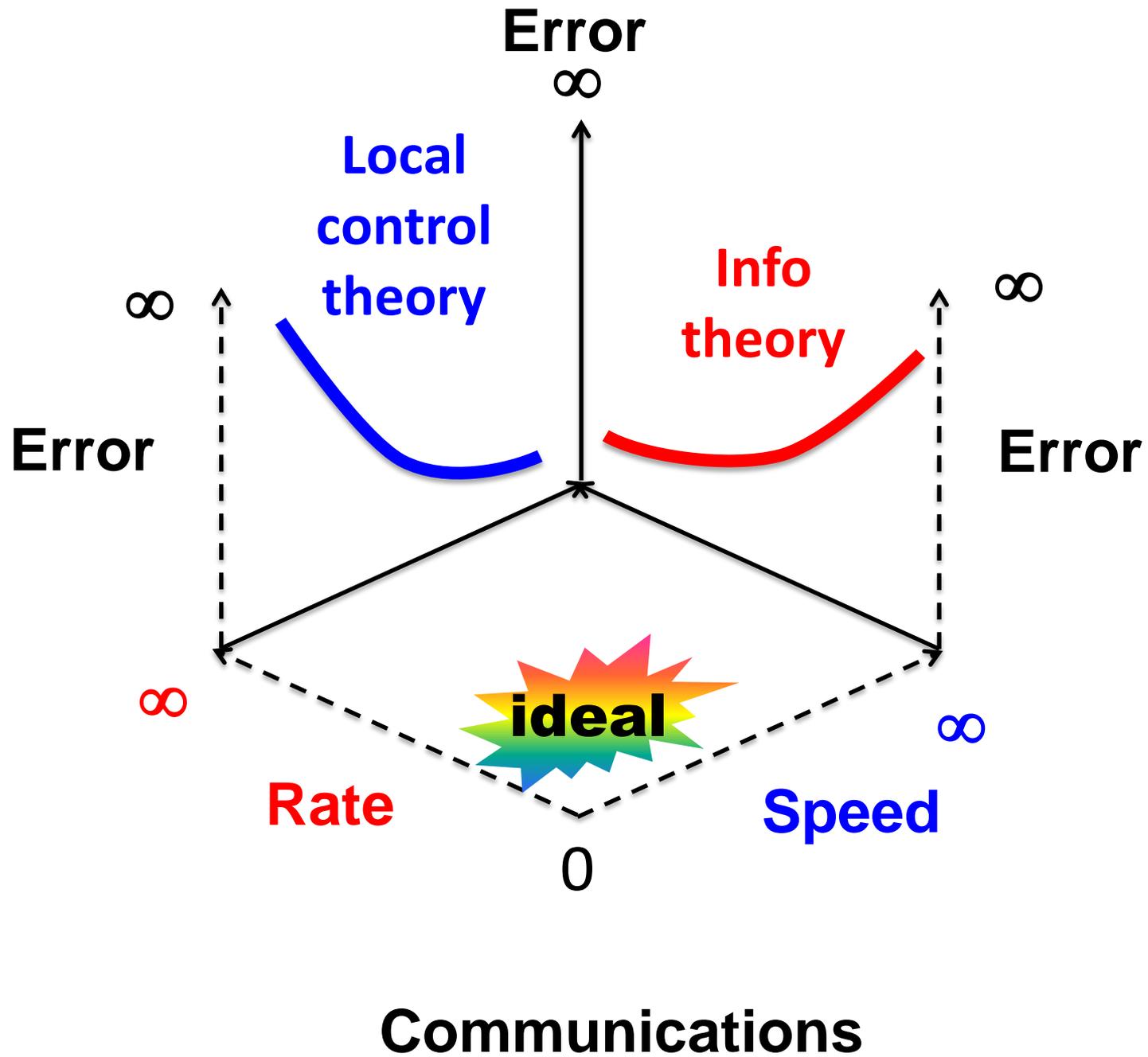
Communications

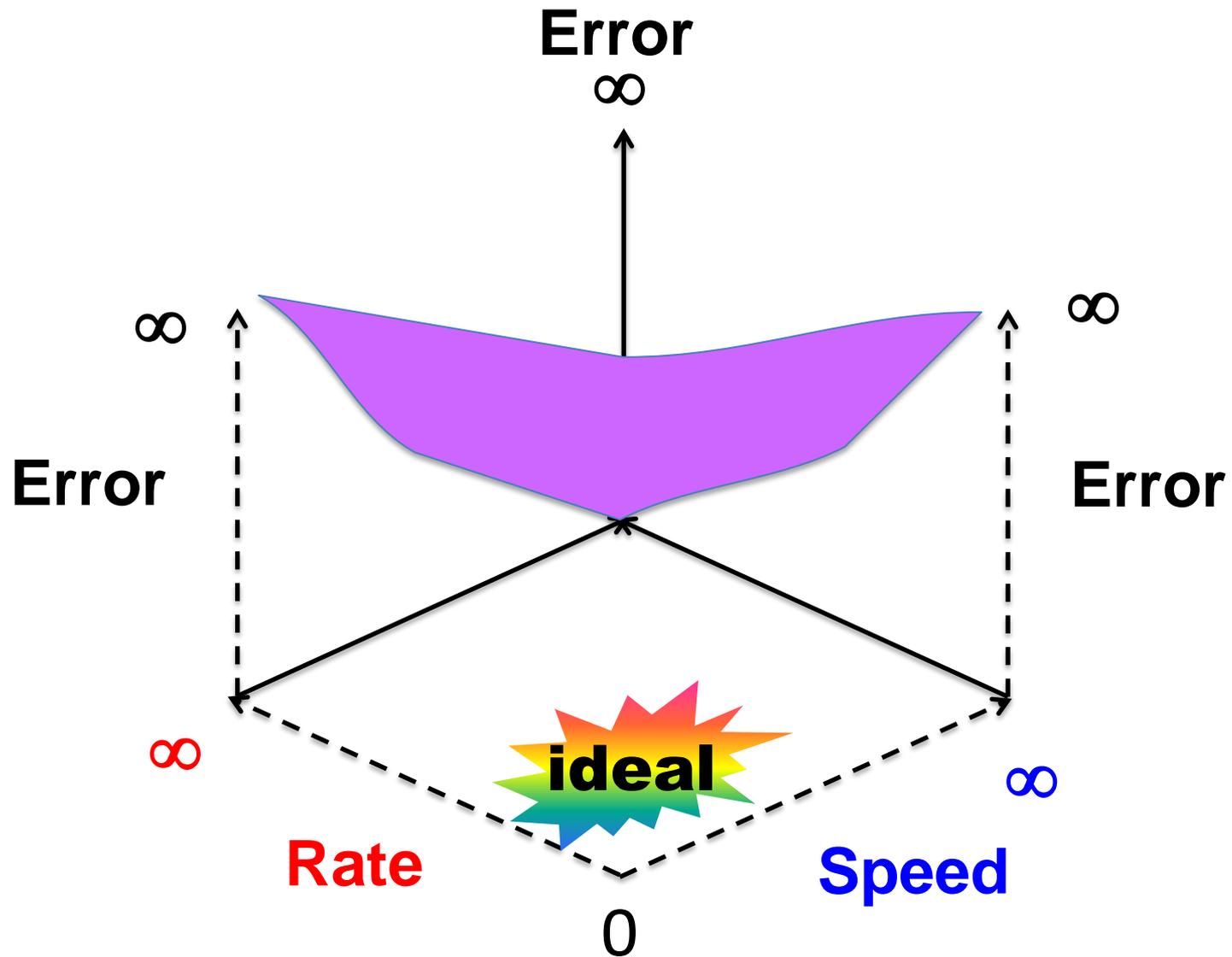


Communications

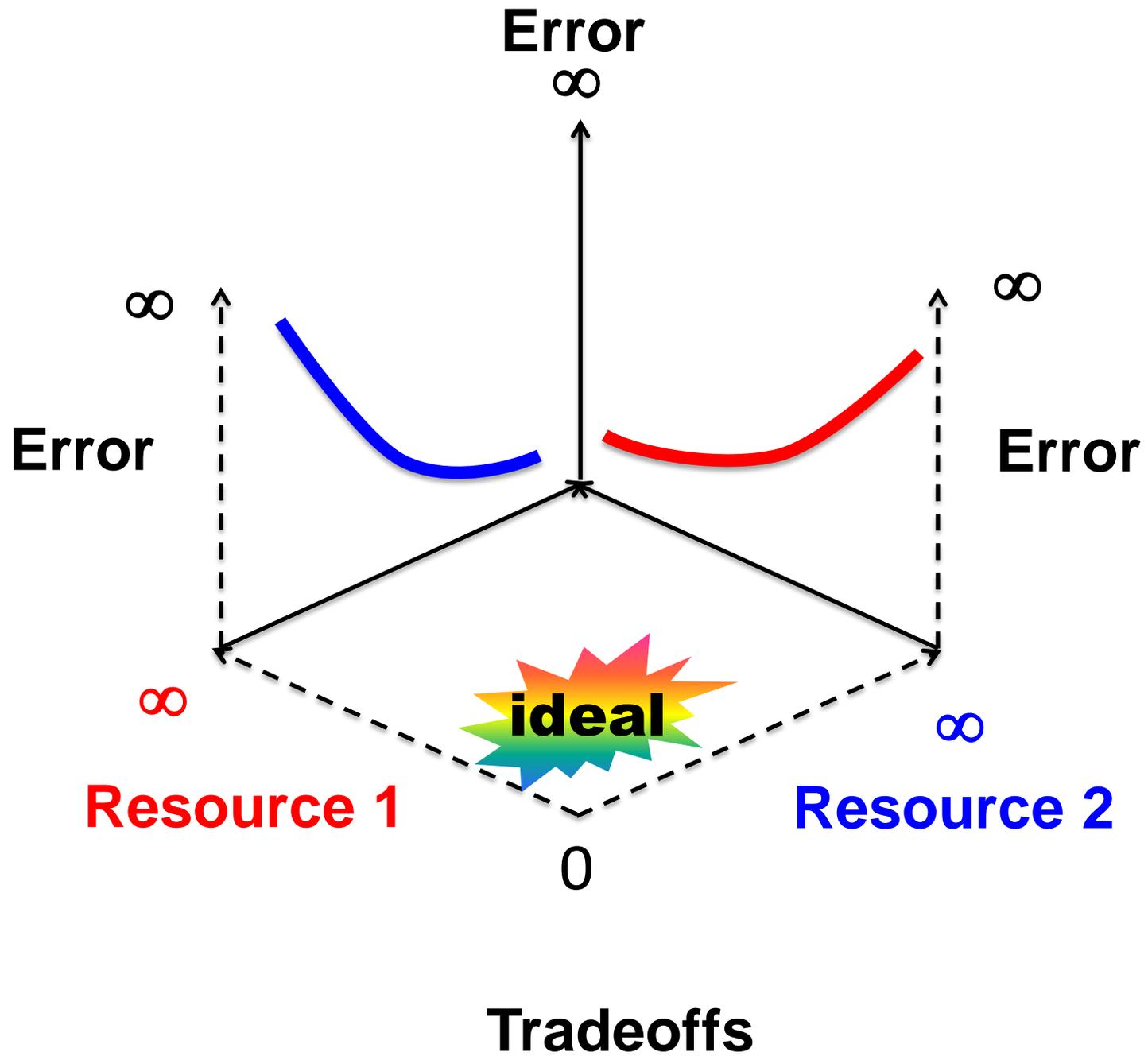




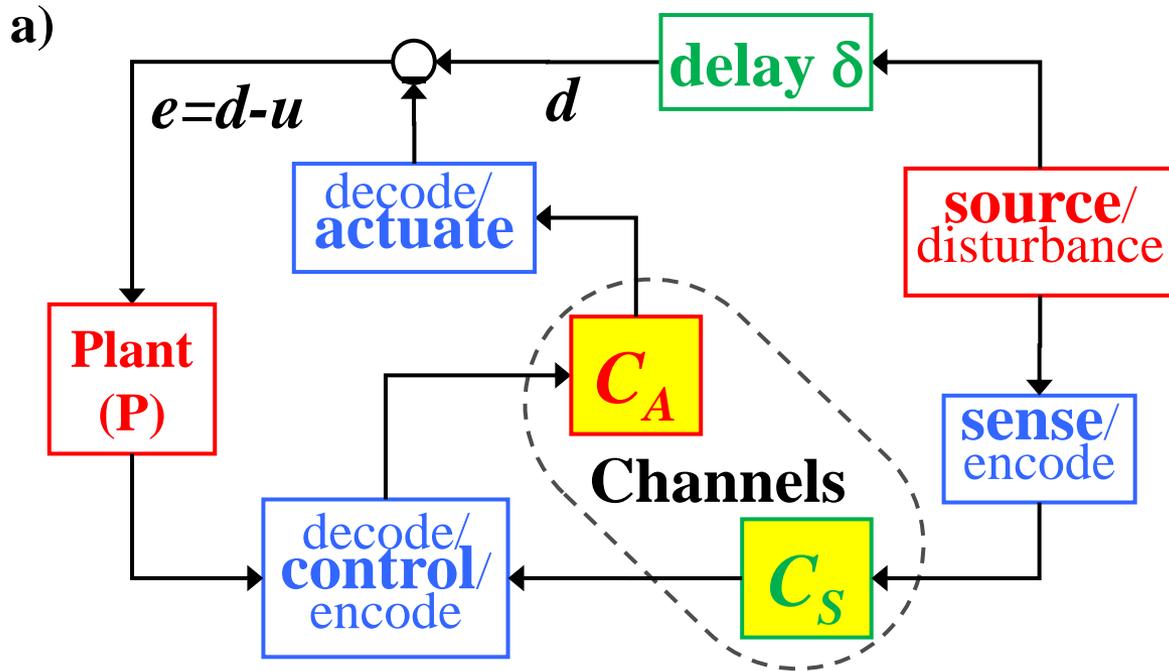




Local control



Control over limited channels (Martins et al)



$$\int f(\omega) d\omega @ \frac{1}{\pi} \int_0^{\infty} f(\omega) d\omega$$

b) $P(p) = \infty \quad p \geq 0$

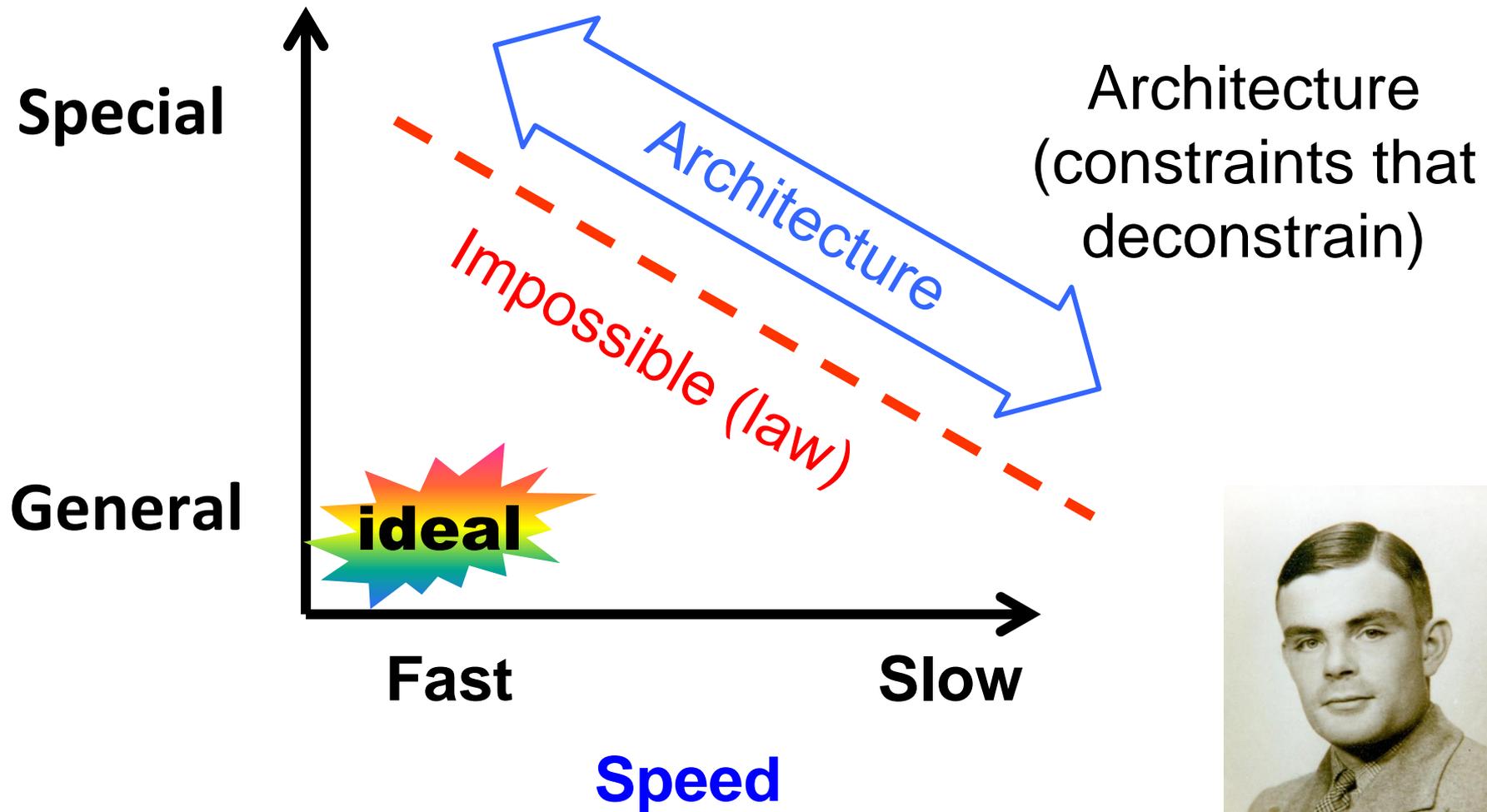
c) $S(j\omega) @ \frac{E(j\omega)}{D(j\omega)}$

d) $\int \log |S| d\omega \geq p - C_S$

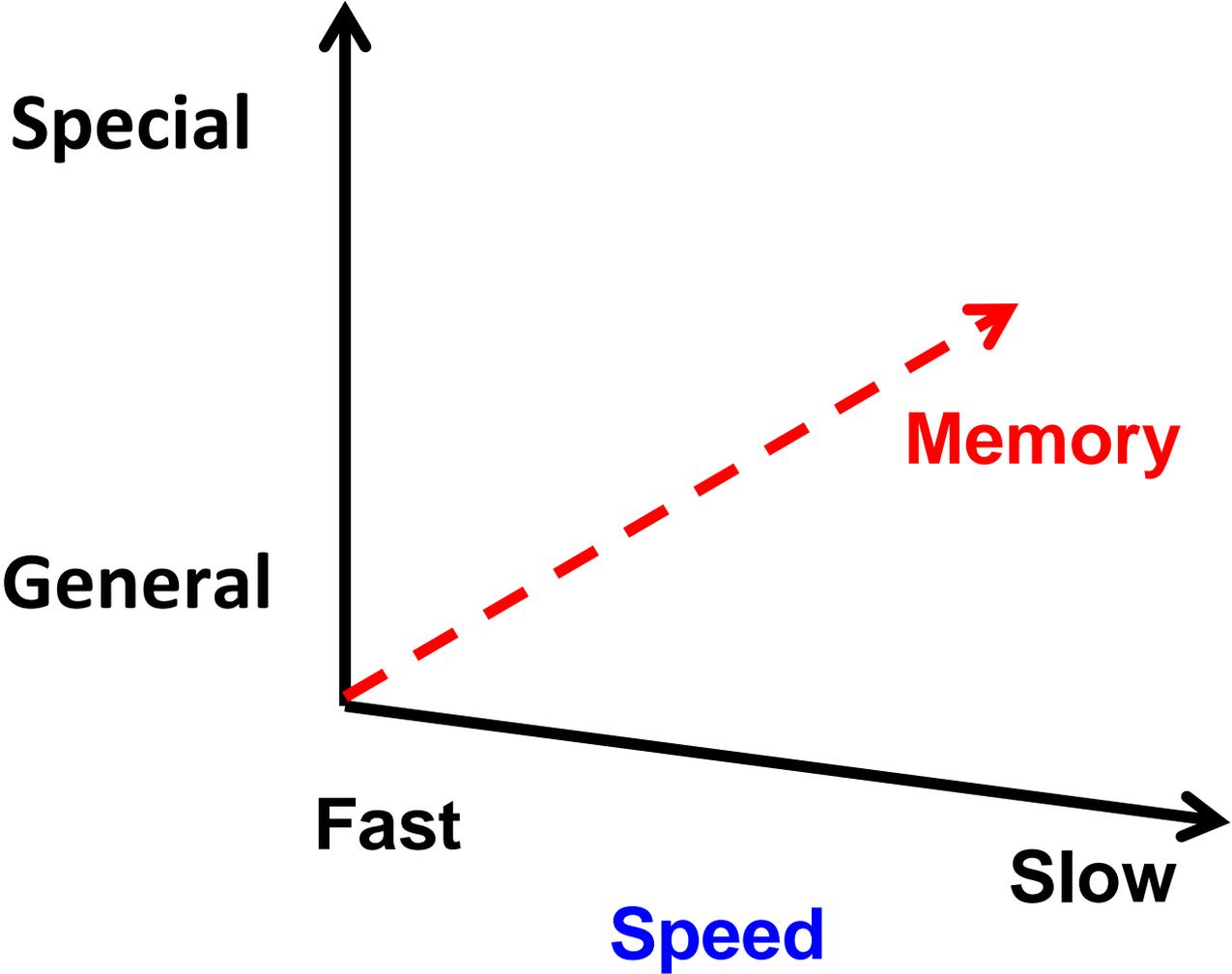
e) $\int \min(0, \log |S|) d\omega \geq p - C_A$

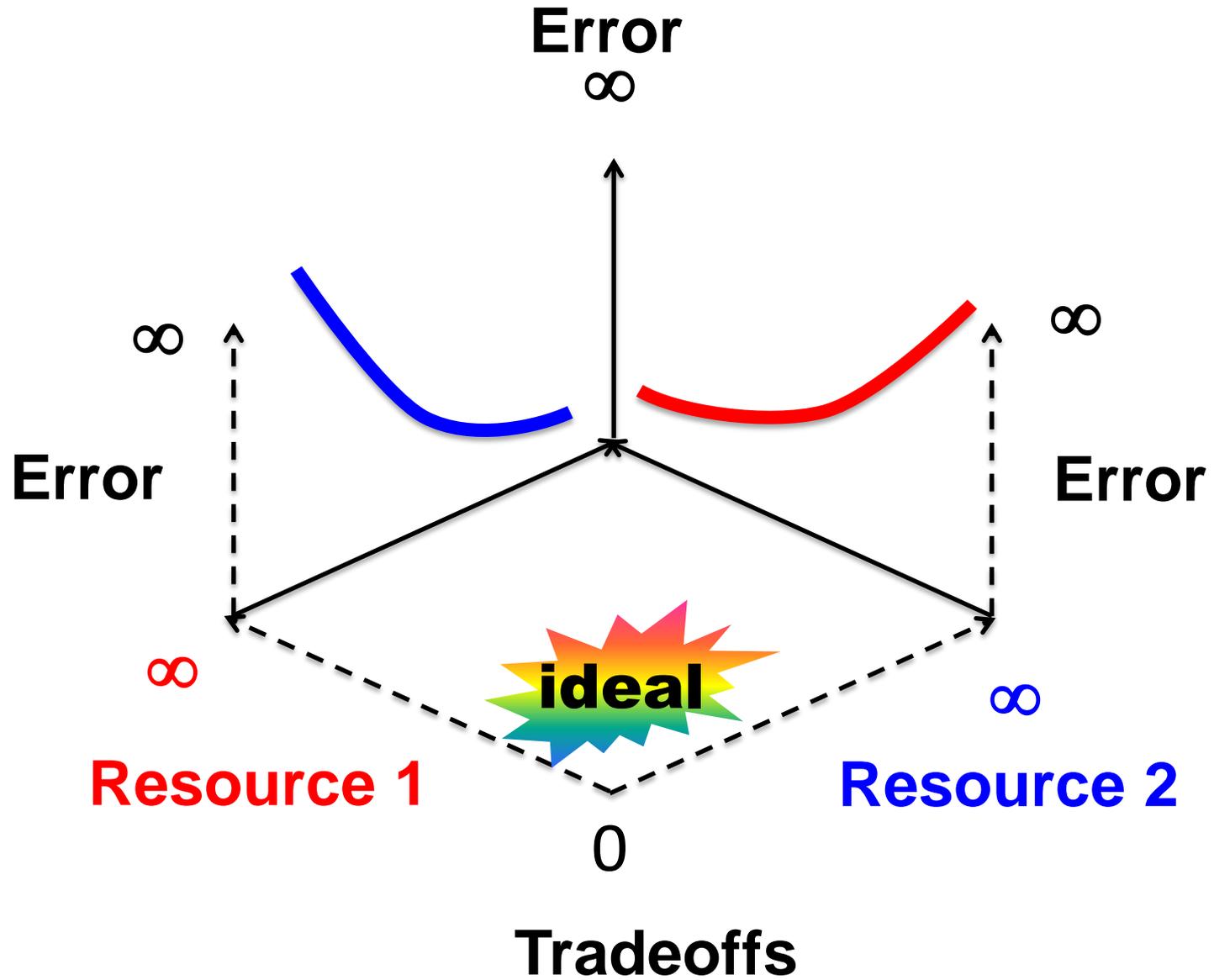
f) $P(z) = 0 \Rightarrow \int \ln |S(j\omega)| \frac{z}{z^2 + \omega^2} d\omega \geq \frac{1}{2} \ln \left| \frac{z+p}{z-p} \right| \left(\geq \frac{p}{z} \text{ if } p < z \right)$

Universal laws and architectures (Turing)

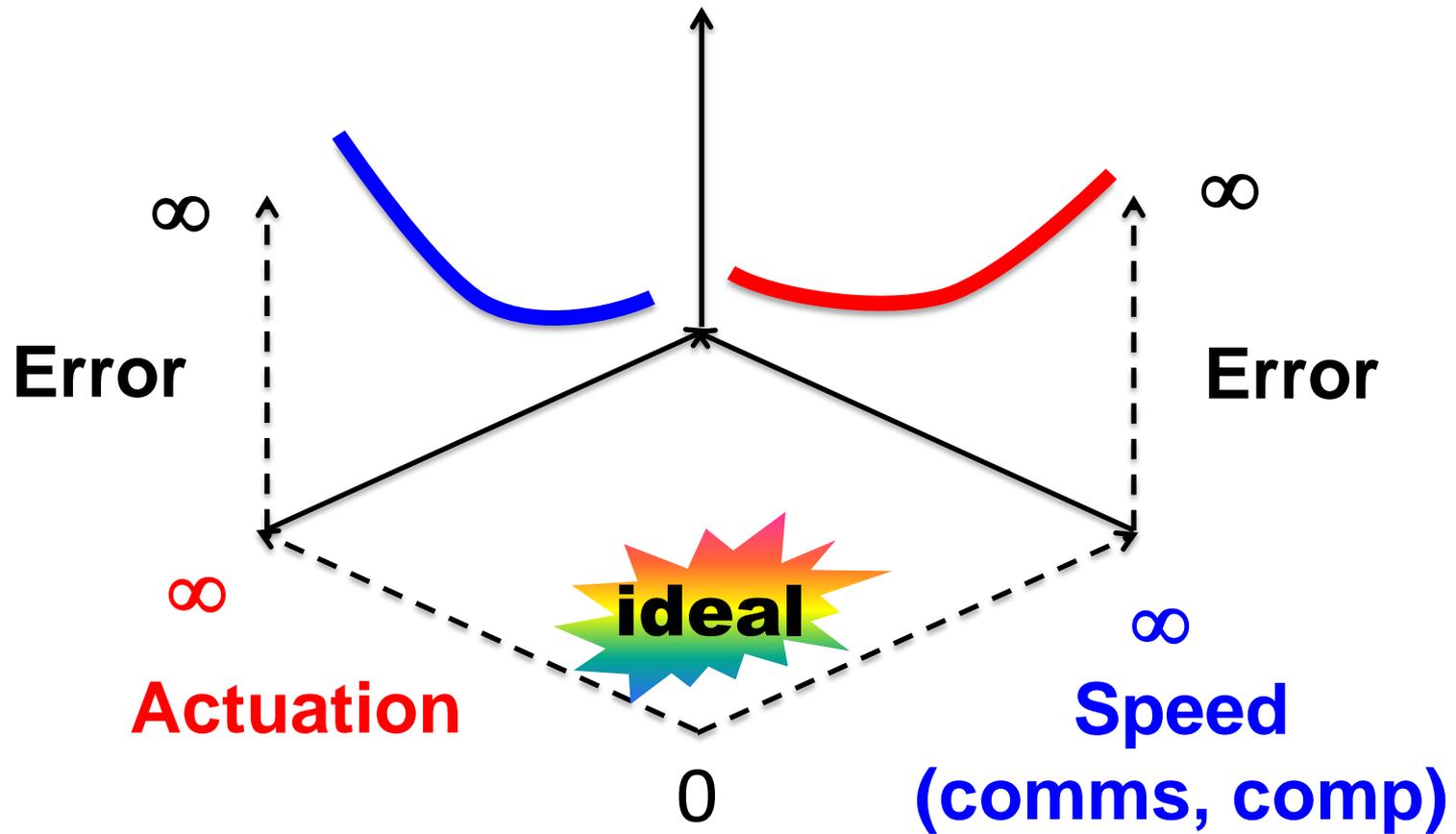


Memory is cheap, reusable, powerful.
Time is not.



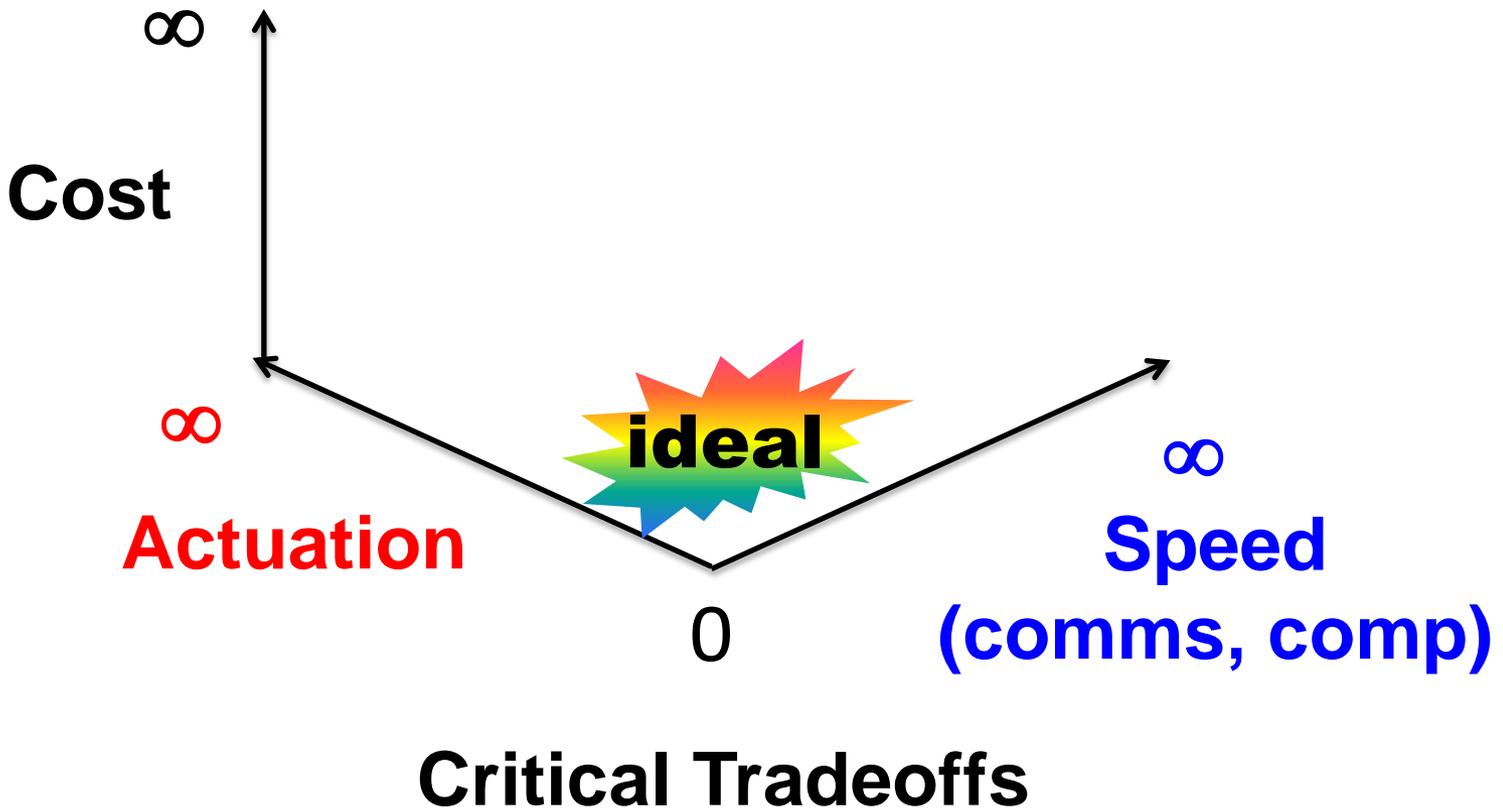


- **Cheap: memory, bandwidth, sensors**
- **Not : time (1/speed), actuators**
- Brains/bodies, cells, CyberPhySys, ...



Critical Tradeoffs

All costs are ultimately “physical.”

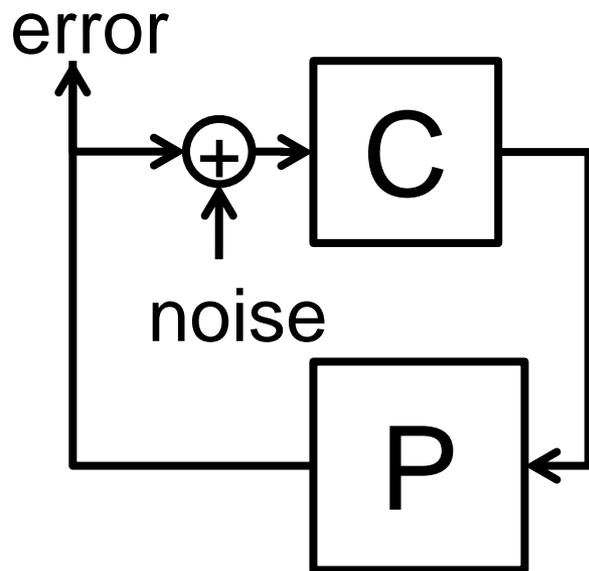


Understand this more deeply?

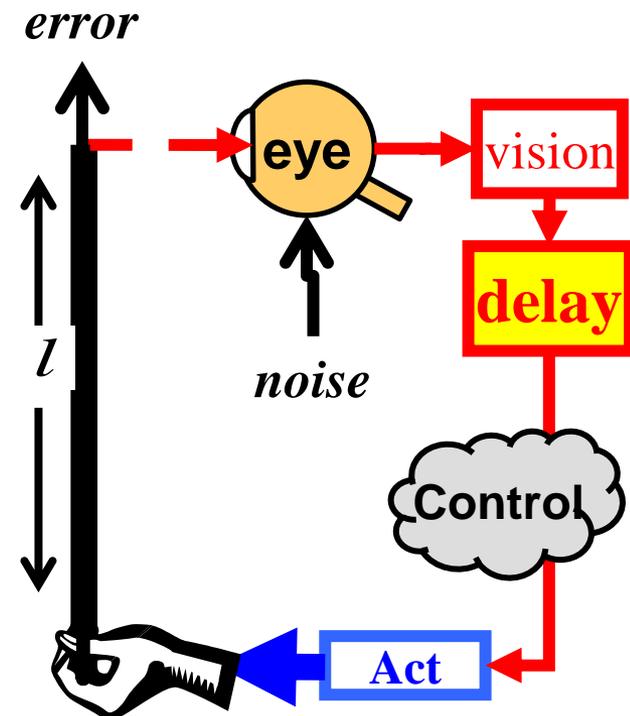
$$\left. \exp\left(\int \ln |T| \right) \right\} \geq \exp(p\tau) \left| \frac{z+p}{z-p} \right|$$
$$\|T\|_{\infty}$$

Mechanics+
Gravity +
Light +
Control theory

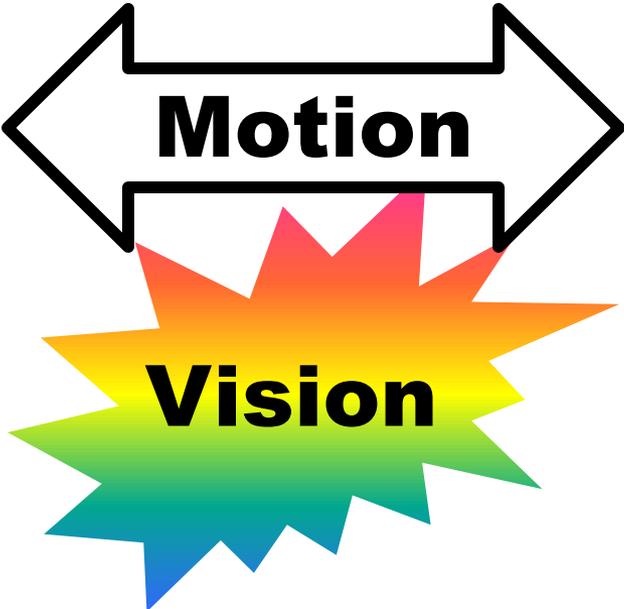
$$|T(j\omega)| = \left| \frac{E}{N} \right|$$



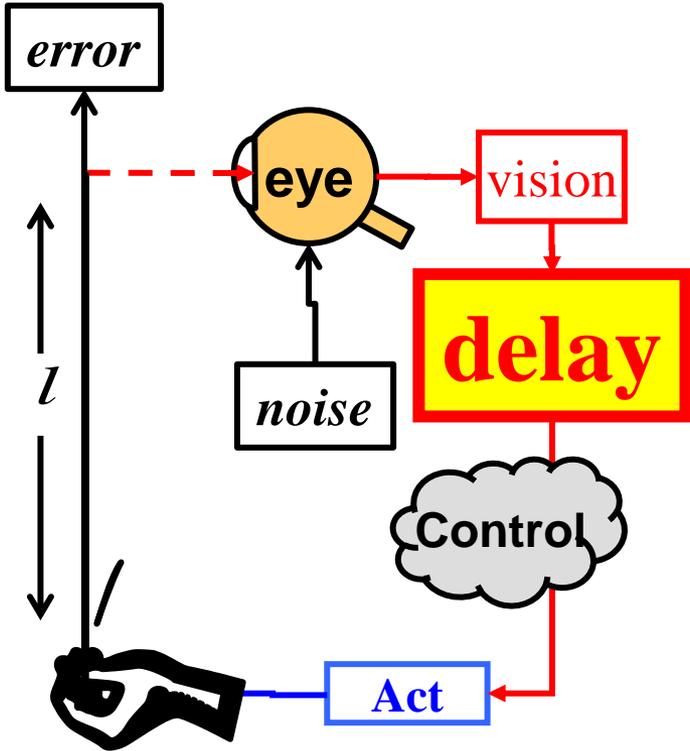
+ Neuroscience

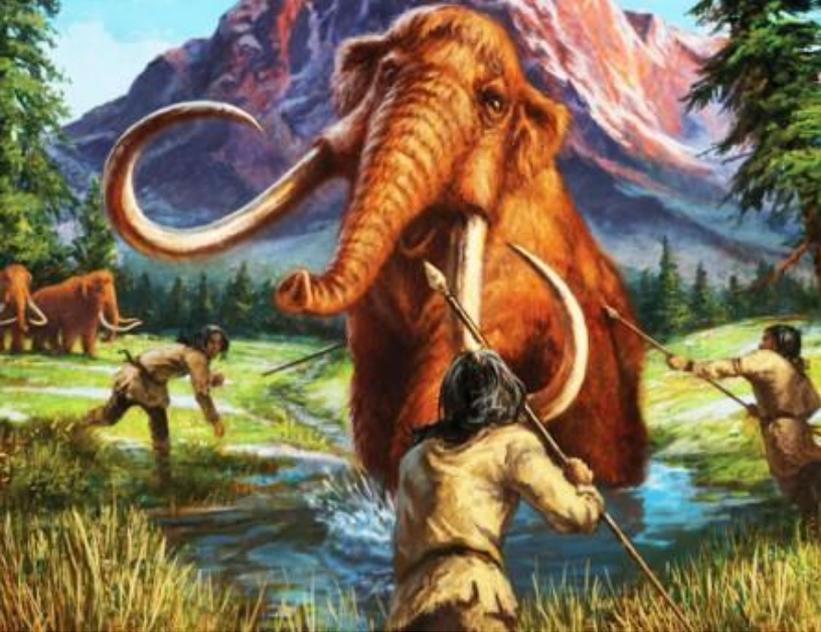


Understand this more deeply?



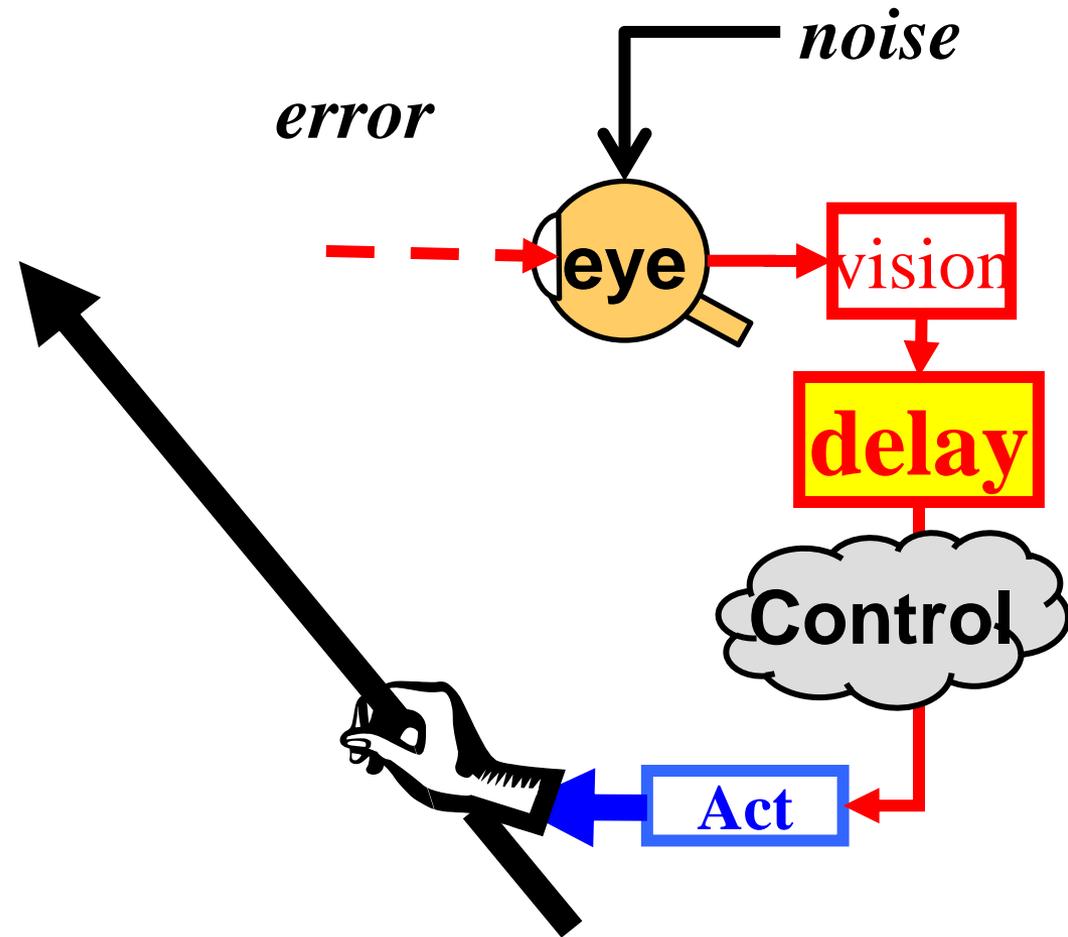
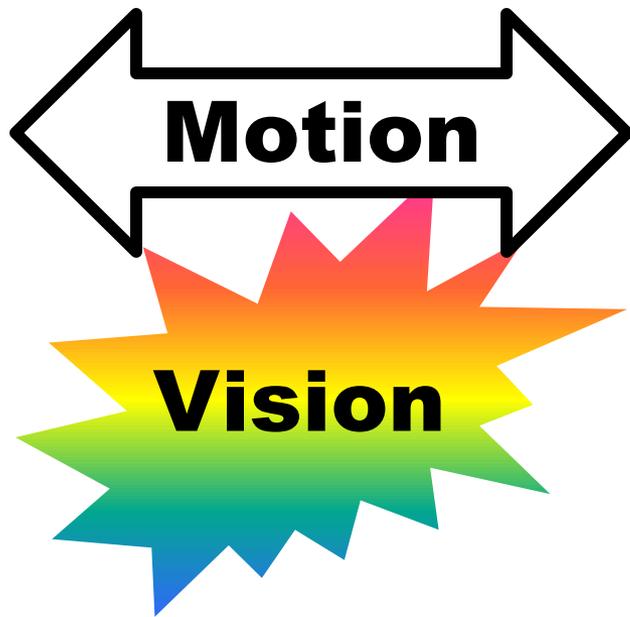
+ Neuroscience

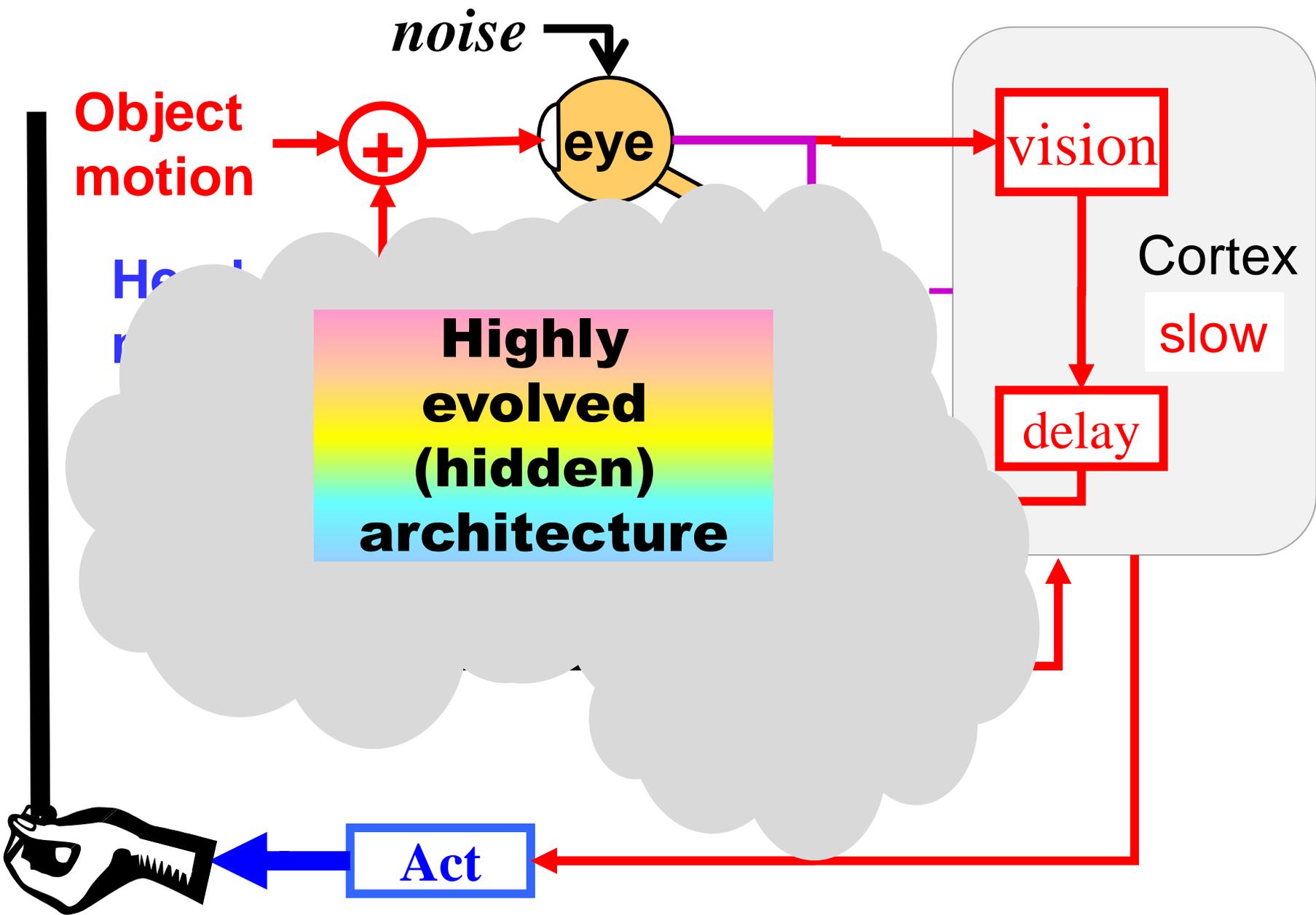




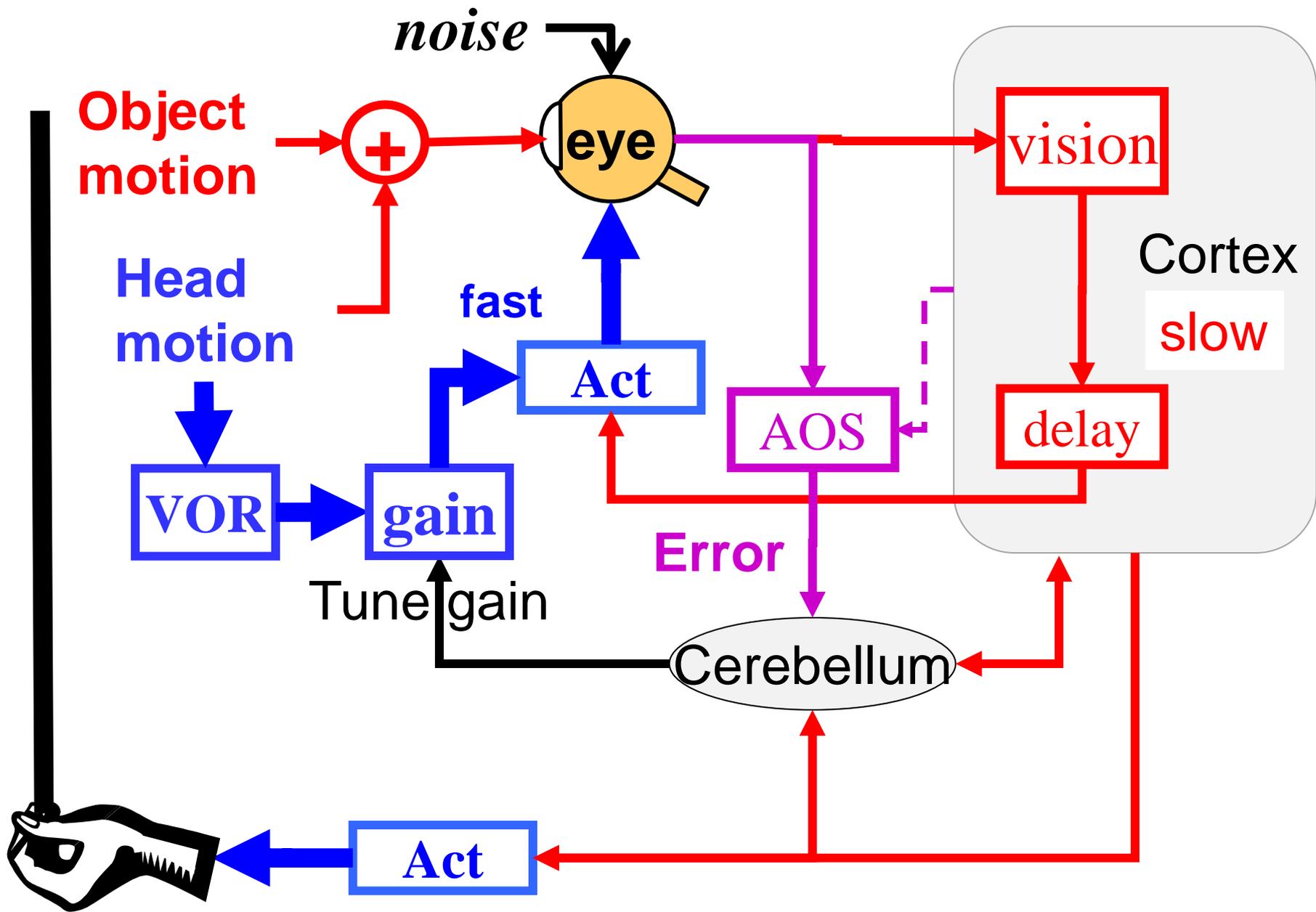
Robust vision w/motion

- Object motion
- Self motion





AOS = Accessory Optical system



AOS = Accessory Optical system

**Layering
Feedback**

**Explain this
amazing
system.**

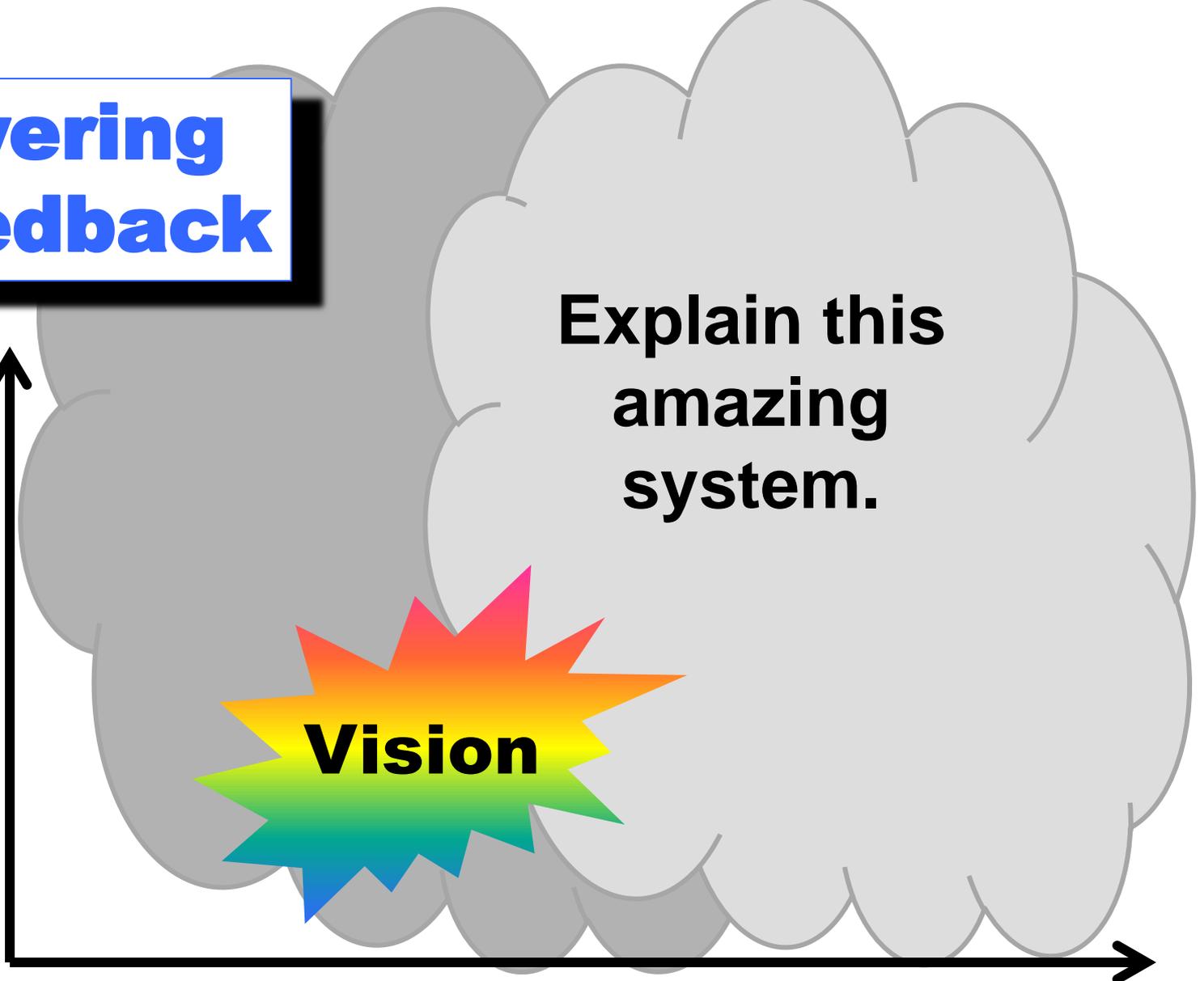
Vision

Slow

Fast

Flexible

Inflexible



Robust vision with

- Hand motion
- Head motion



Experiment

- Motion/vision control without blurring
- Which is easier and faster?

Why?

- Mechanism
- Tradeoff

Slow

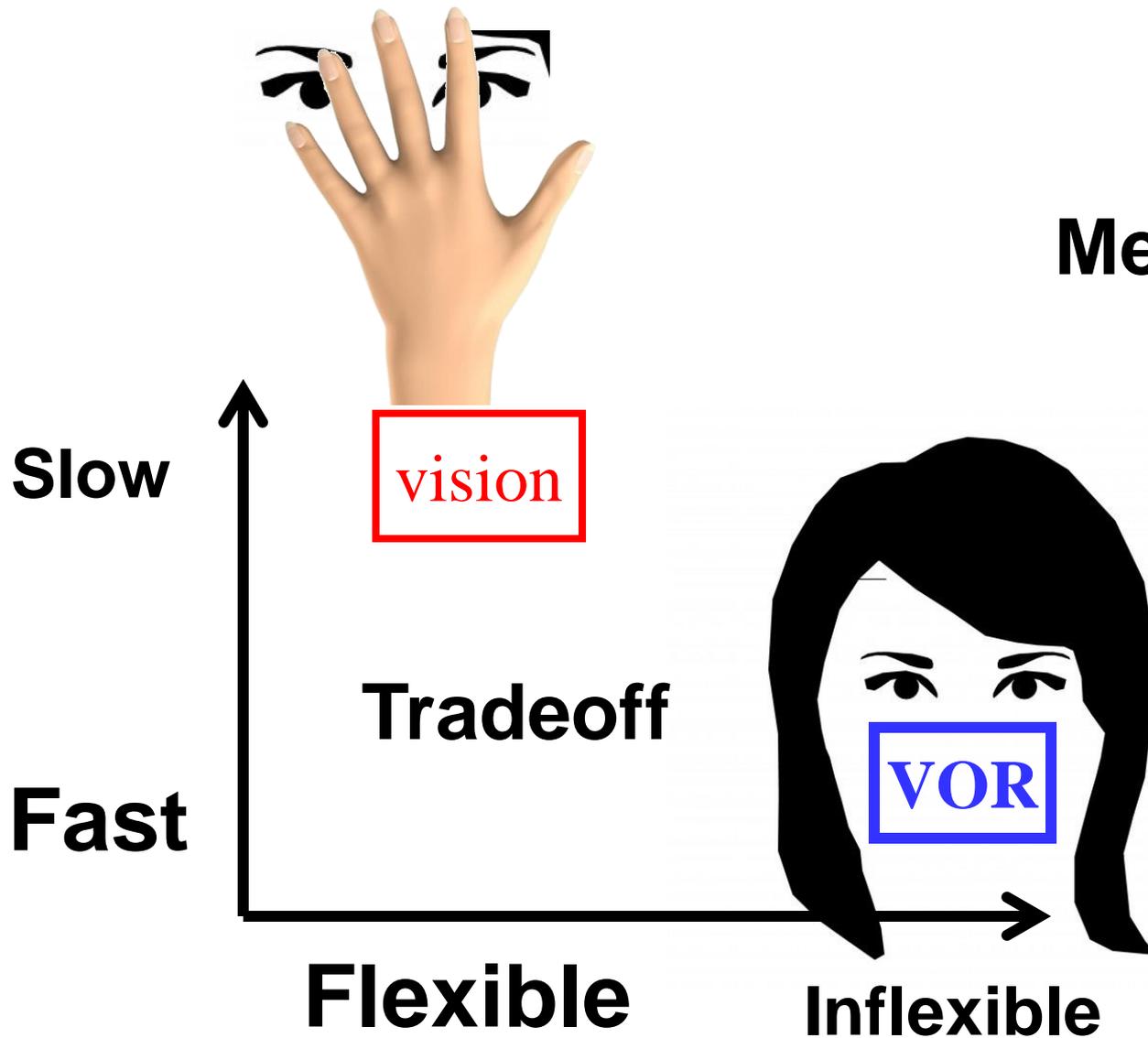


vision

Fast

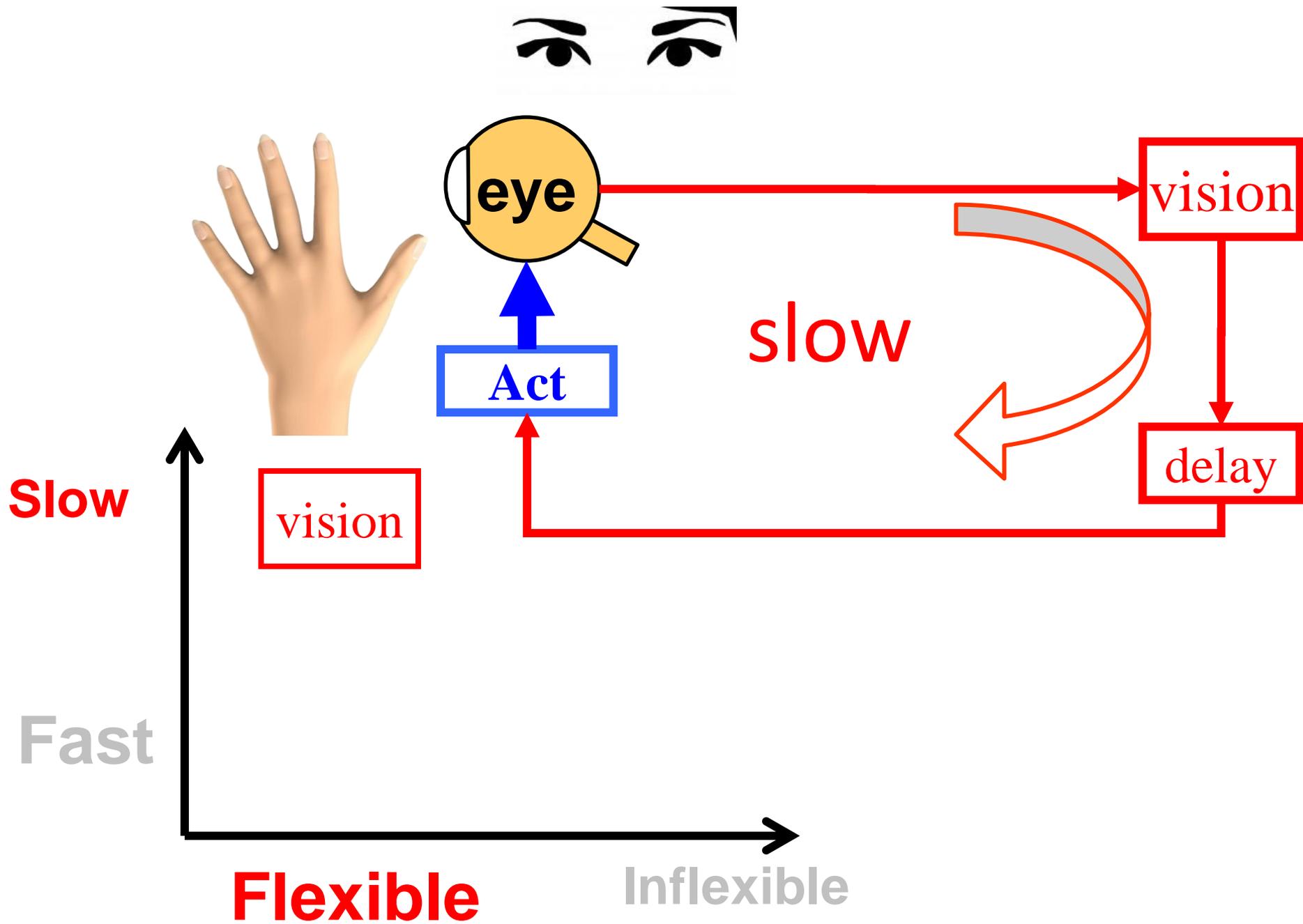


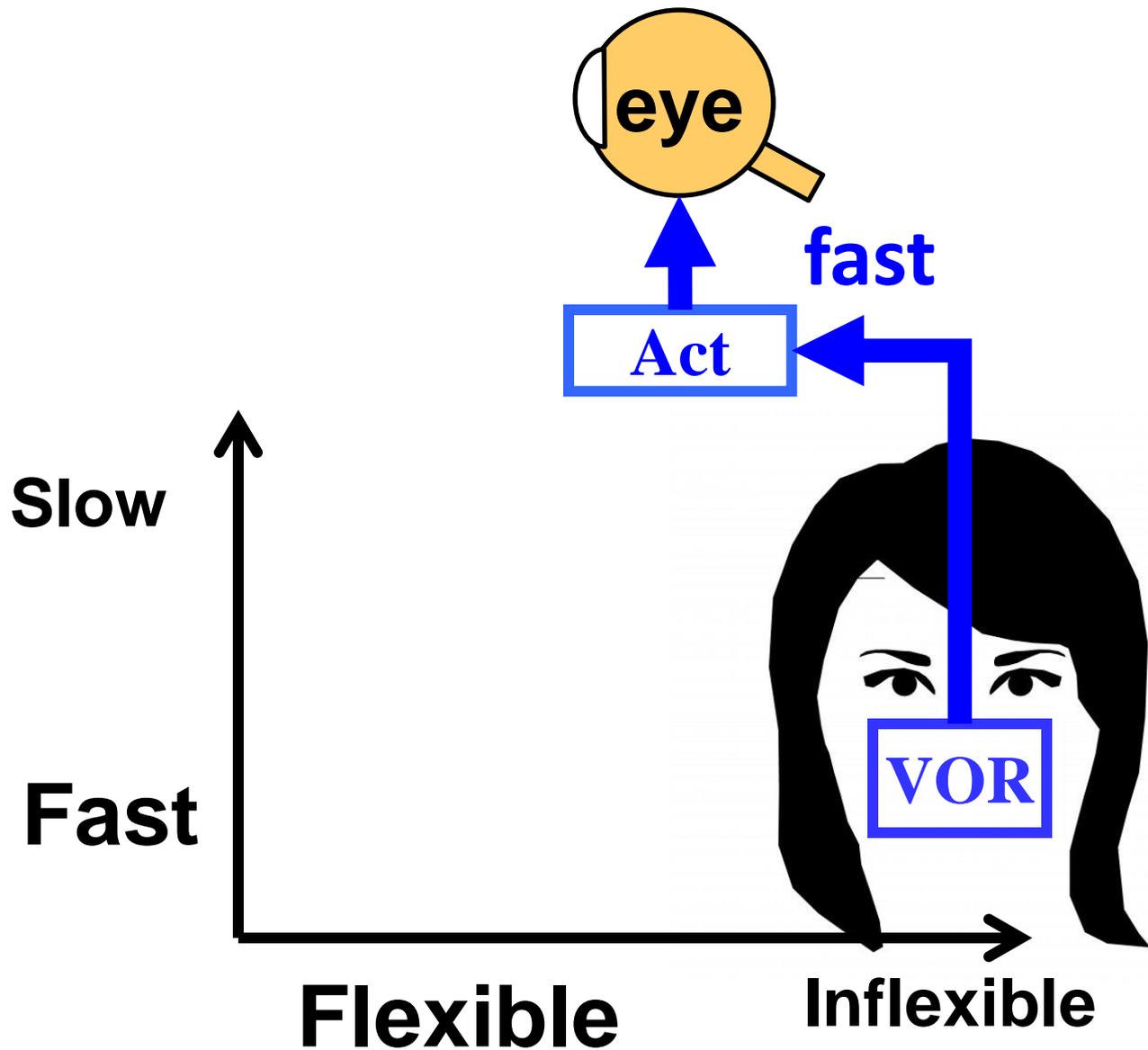
VOR



Mechanism

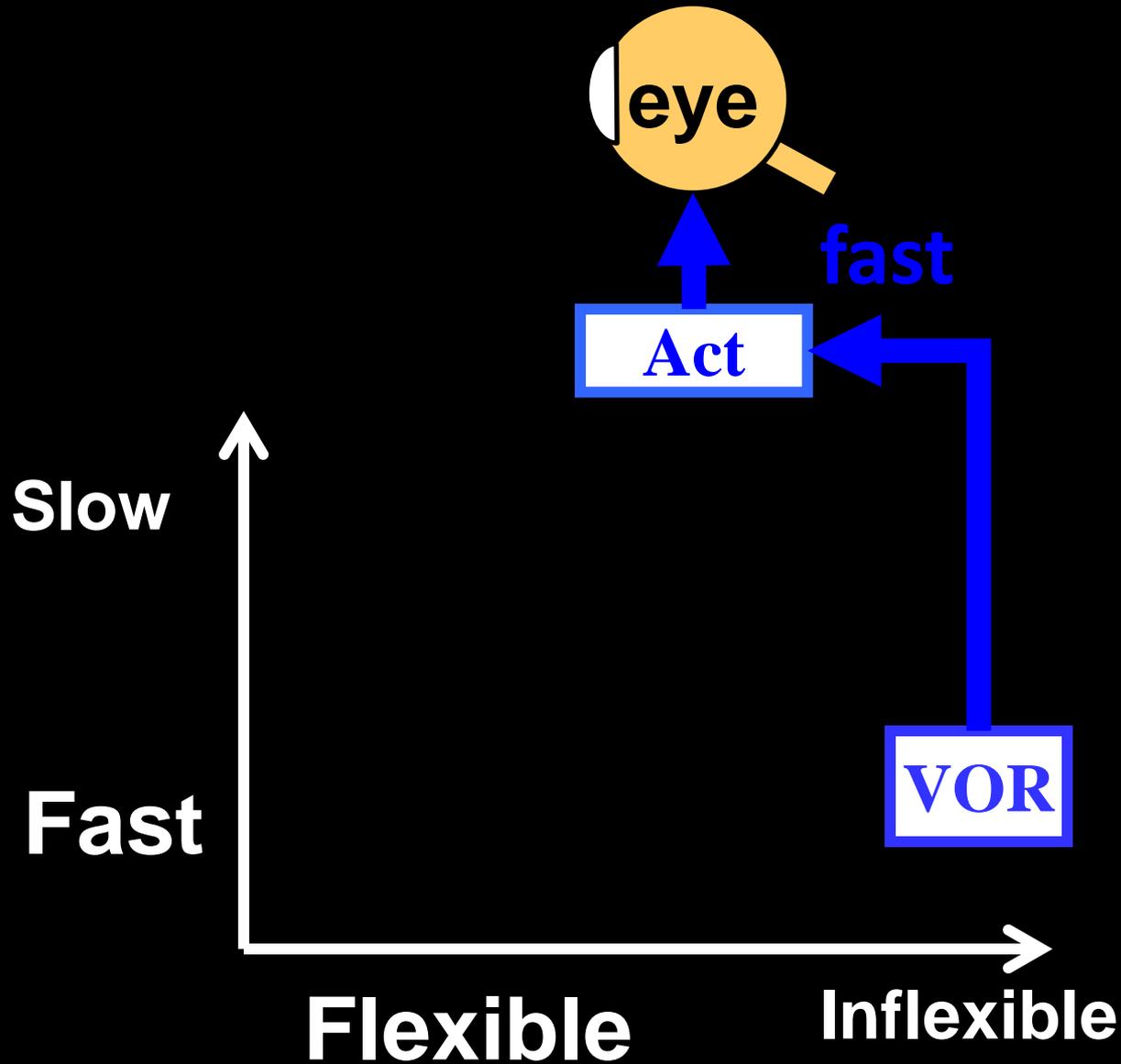
Vestibular
Ocular
Reflex
(VOR)

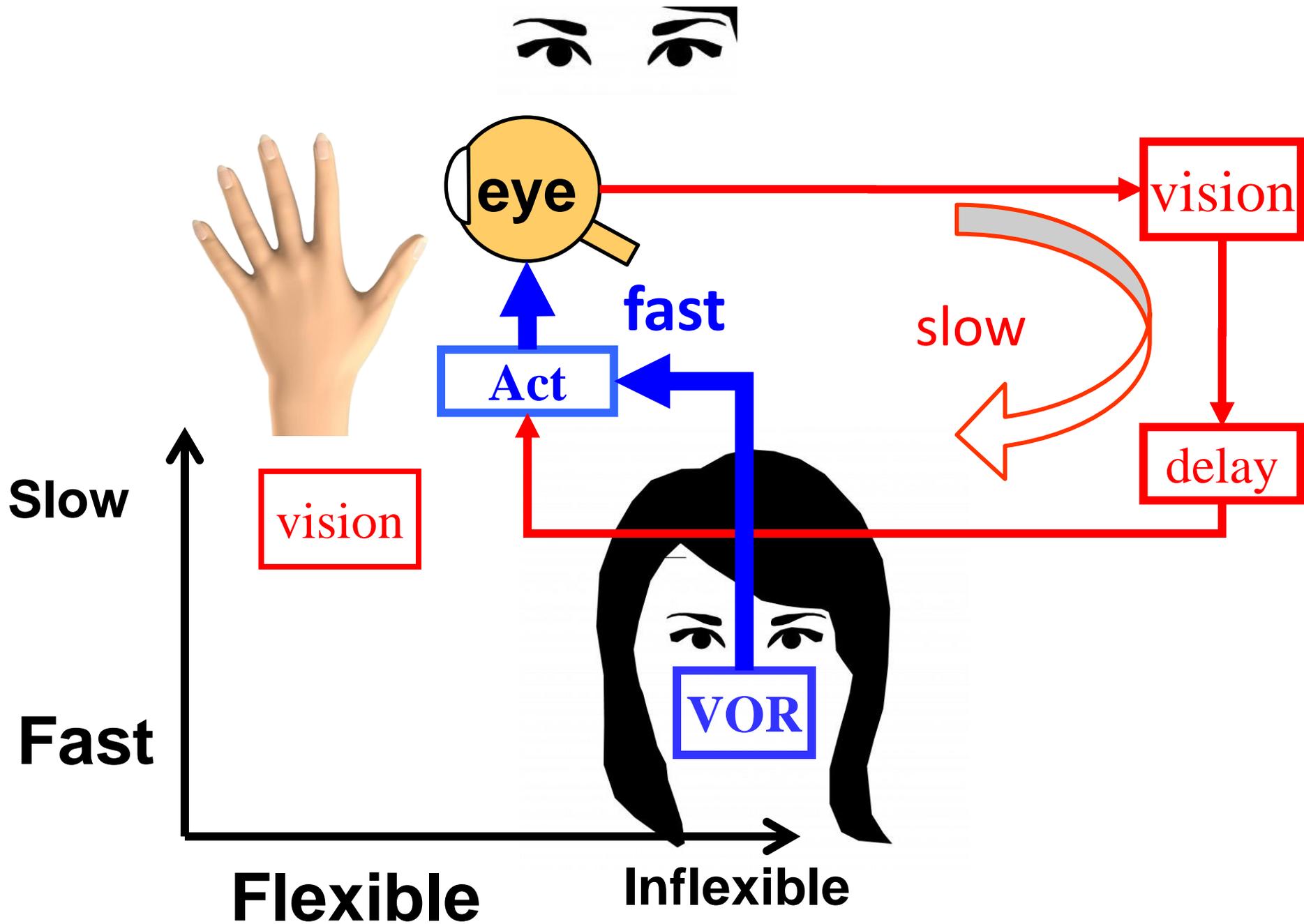




**Vestibular
Ocular
Reflex
(VOR)**

It works in the dark or with your eyes closed, but you can't tell.





**Layering
Feedback**

Slow

Fast

**Highly
evolved
(hidden)
architecture**

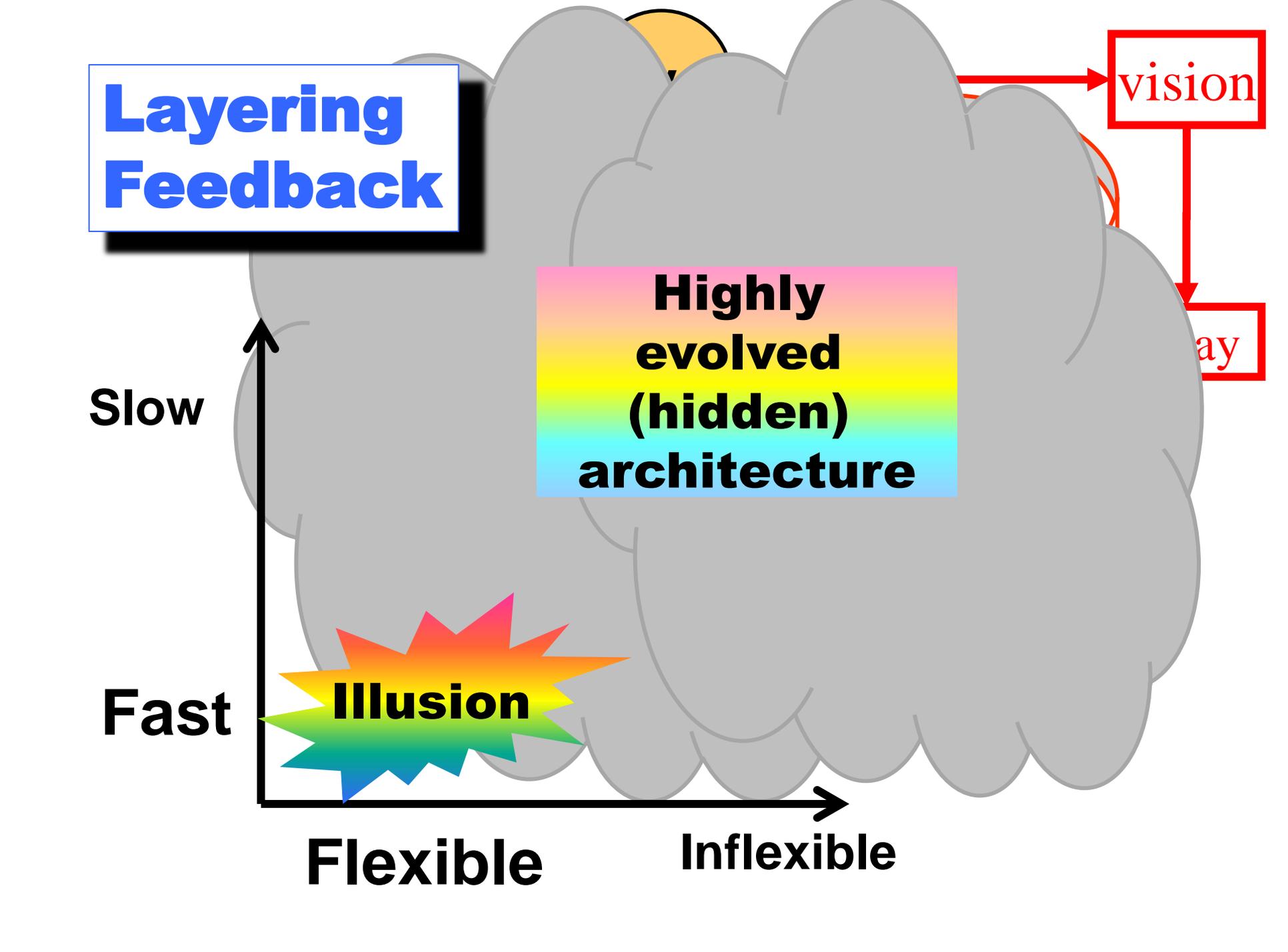
Illusion

Flexible

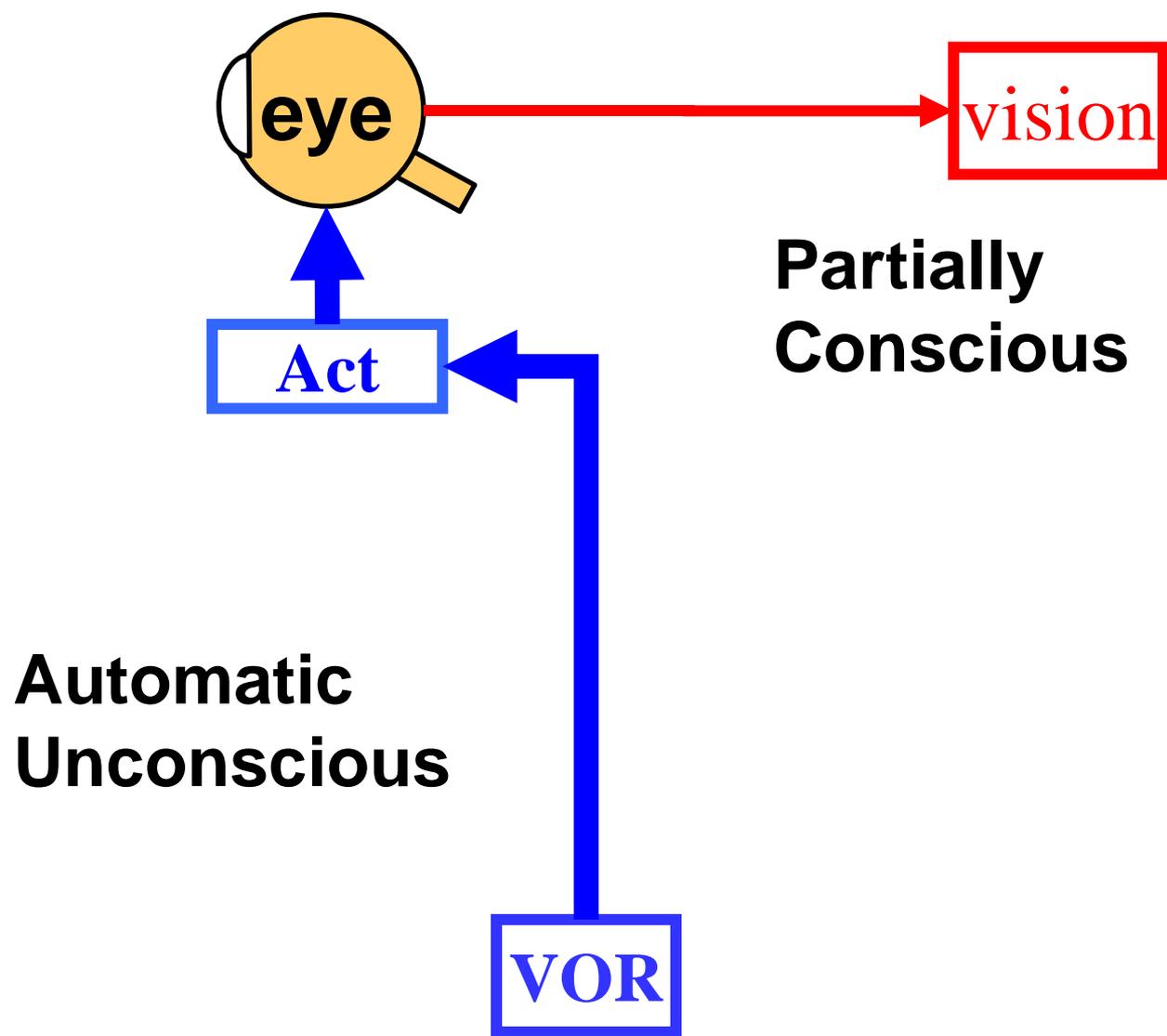
Inflexible

vision

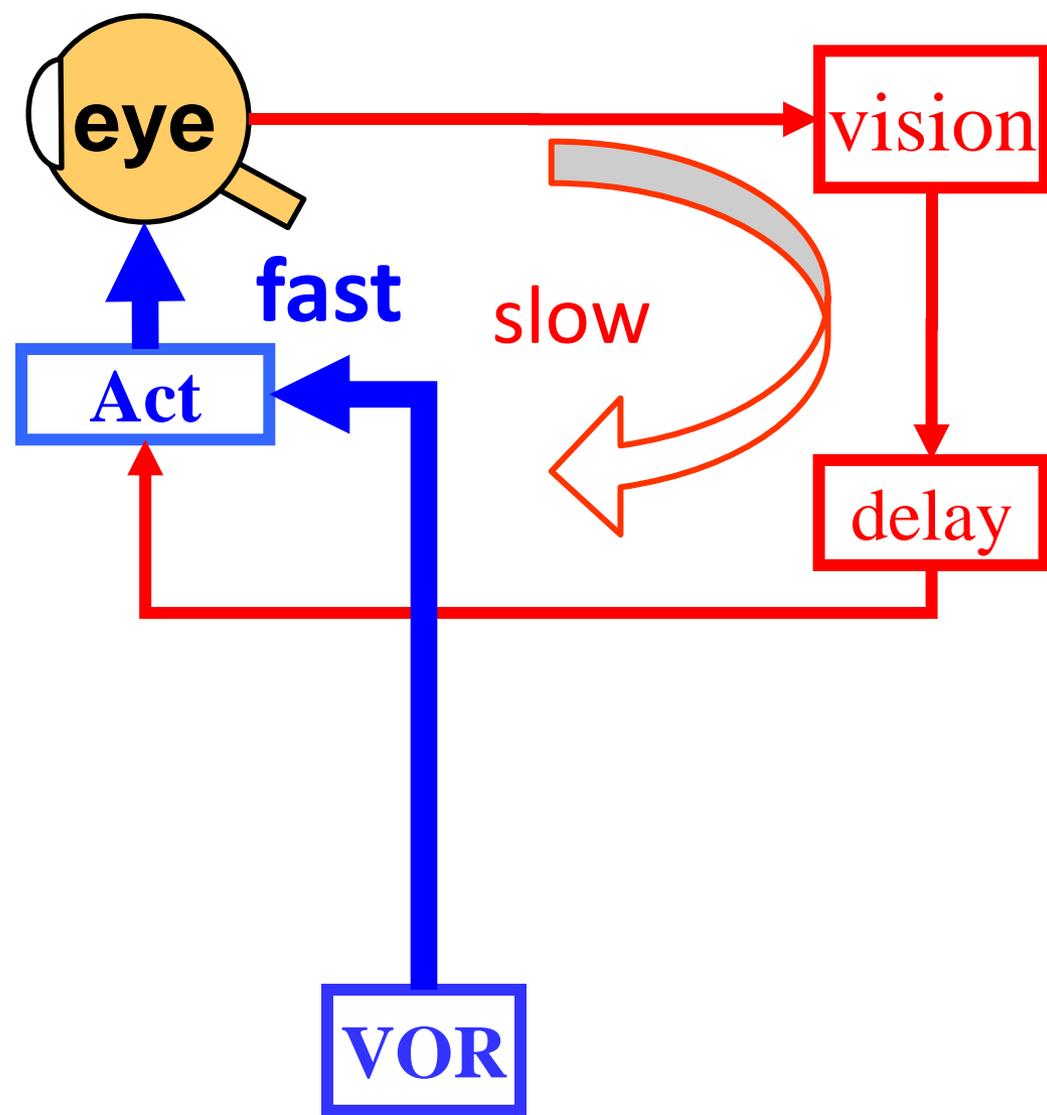
ay



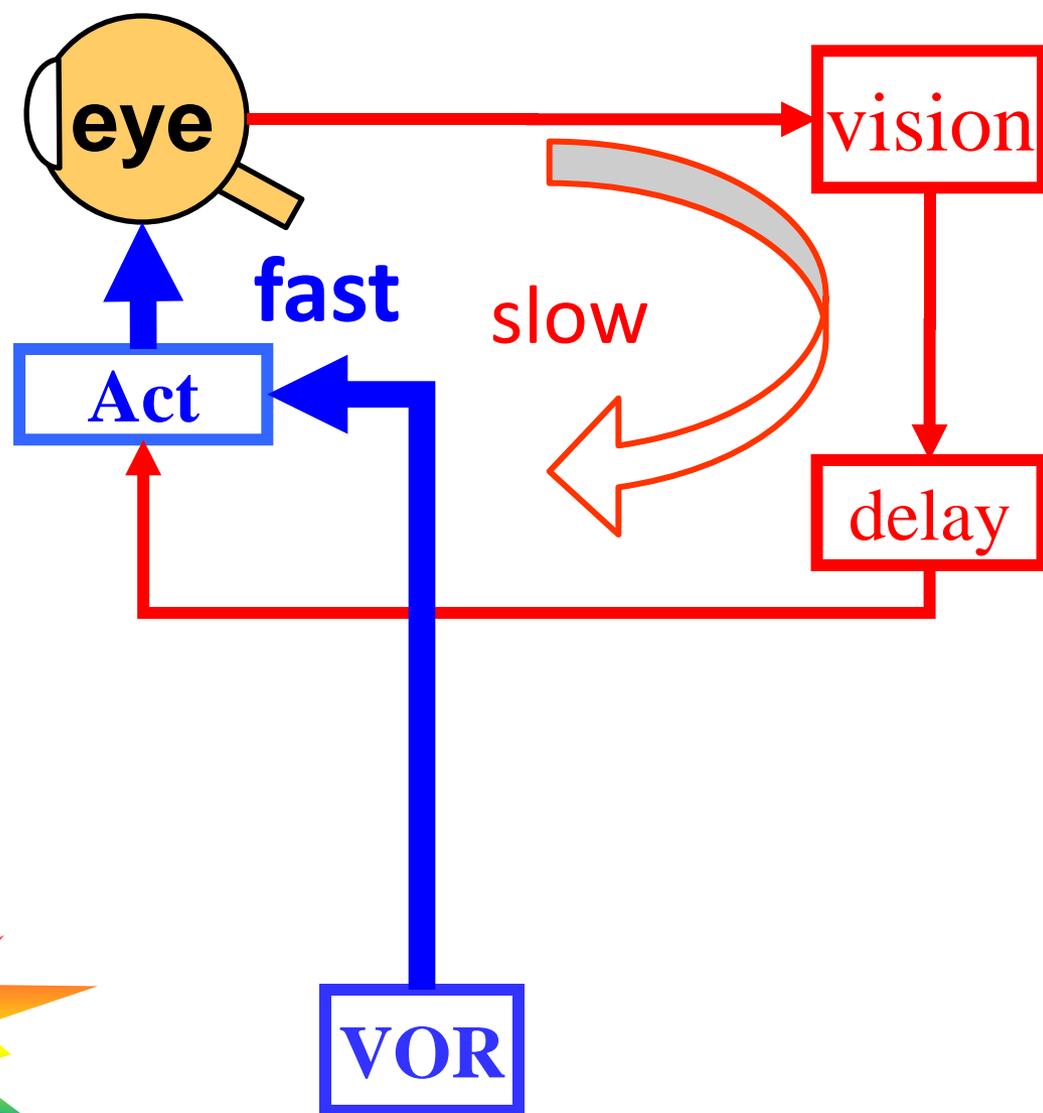
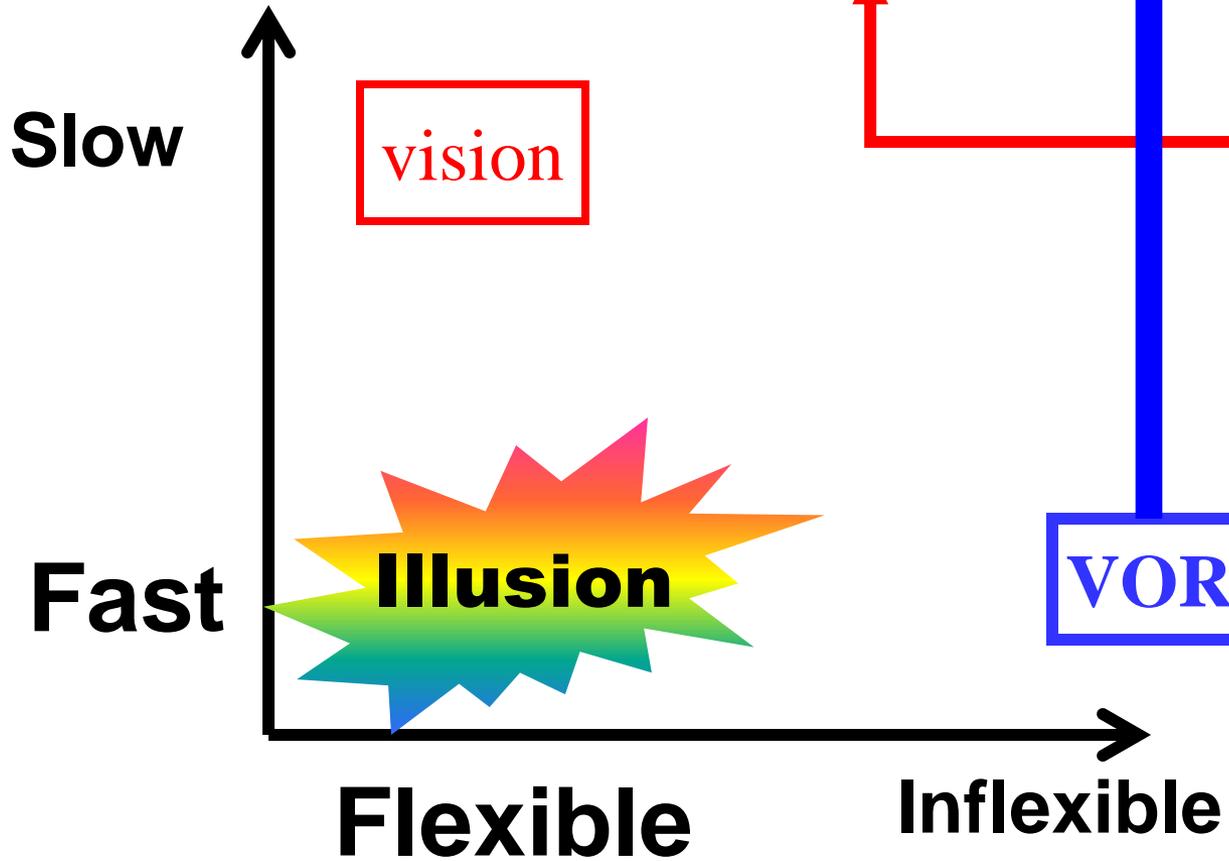
Layering



Layering Feedback

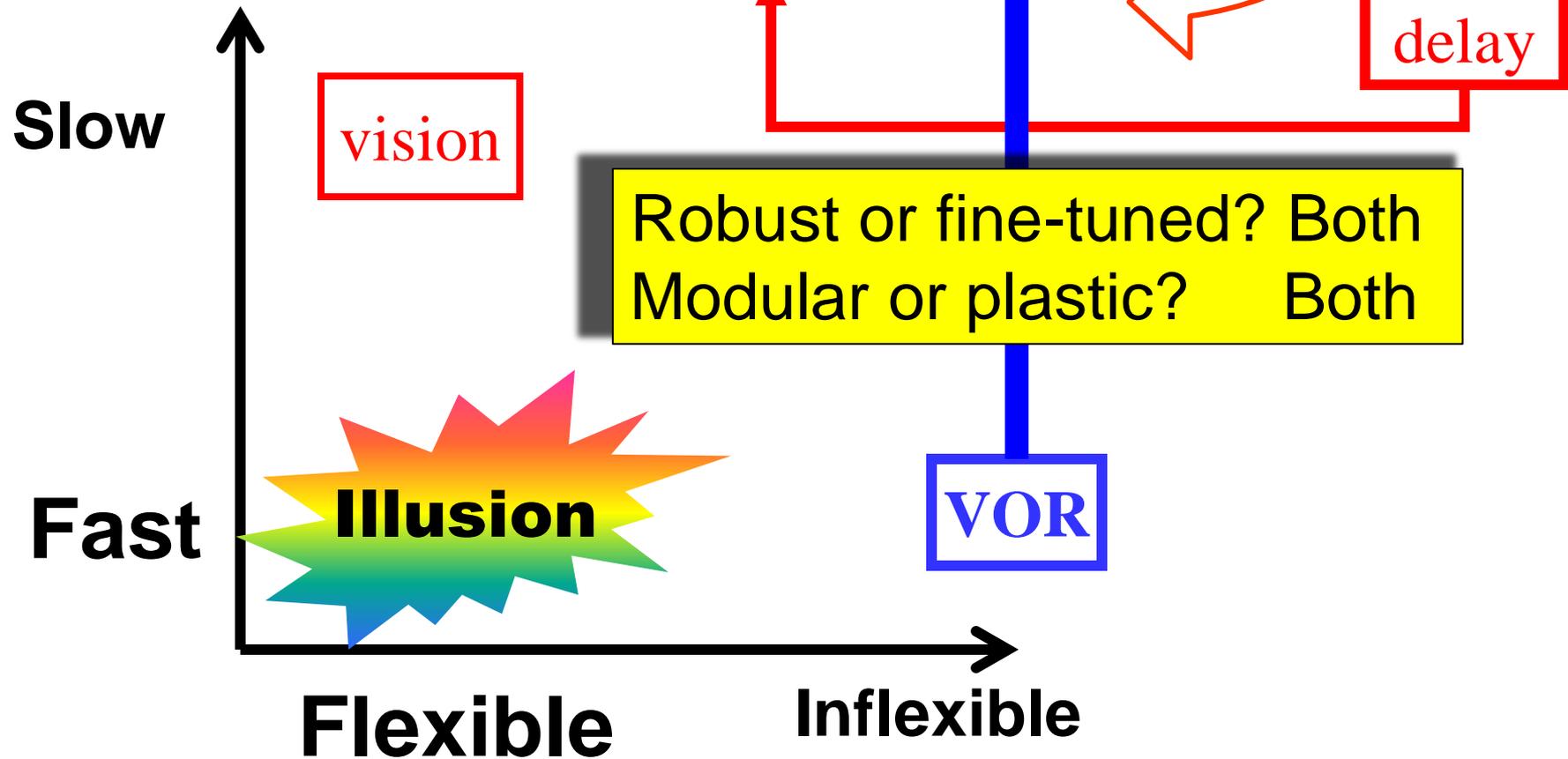


Architecture layered/ distributed

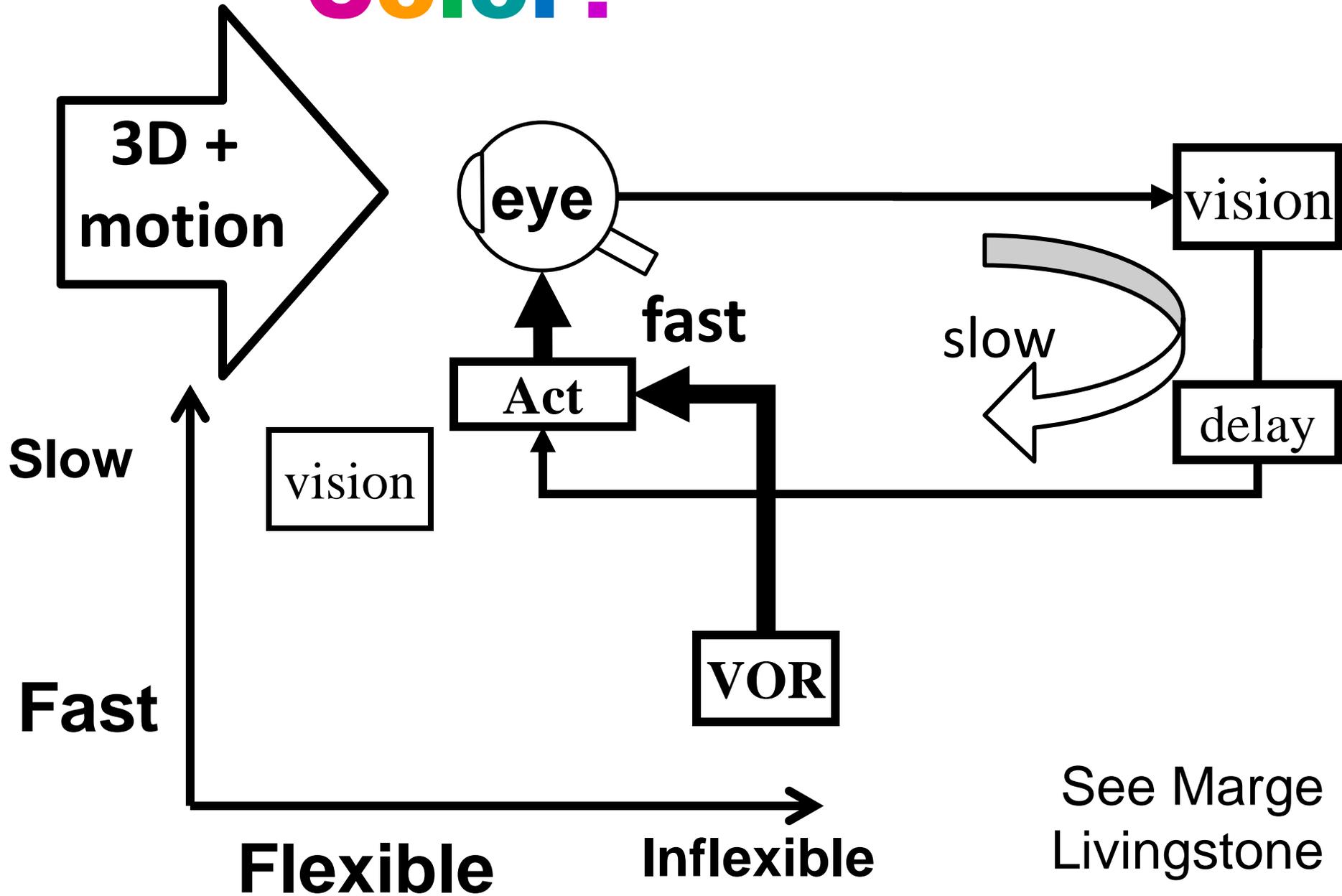


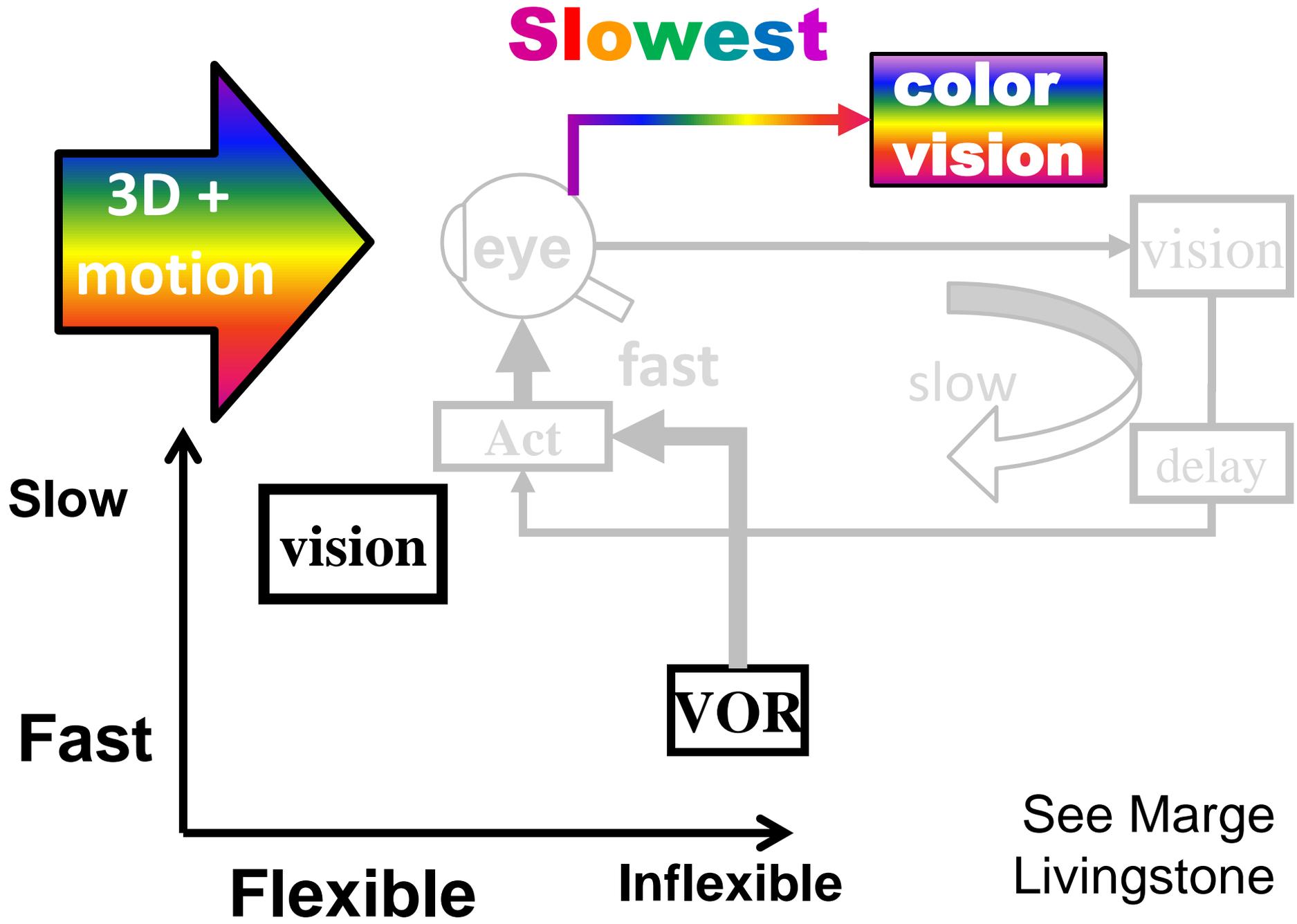
Architecture

layered/
distributed

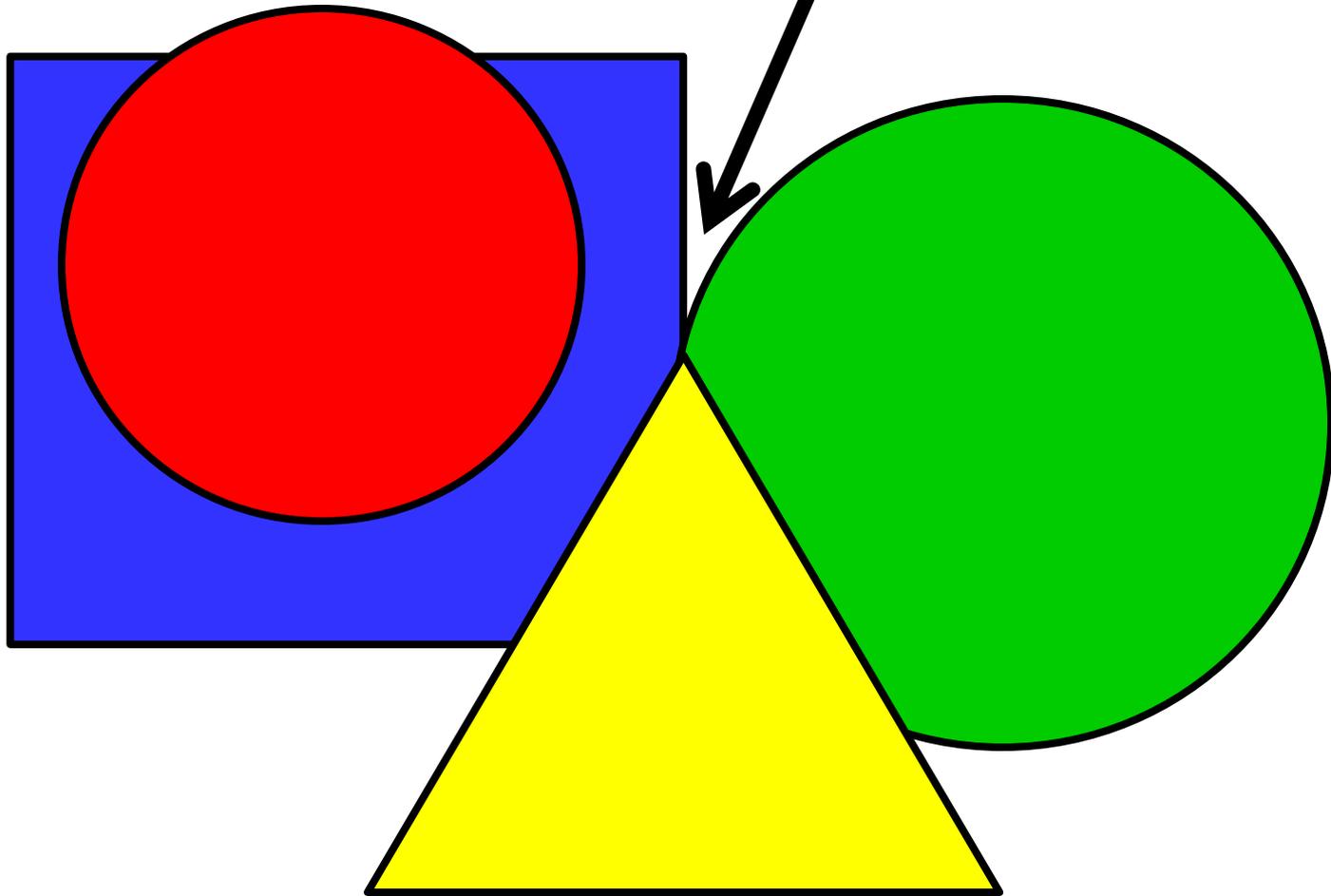


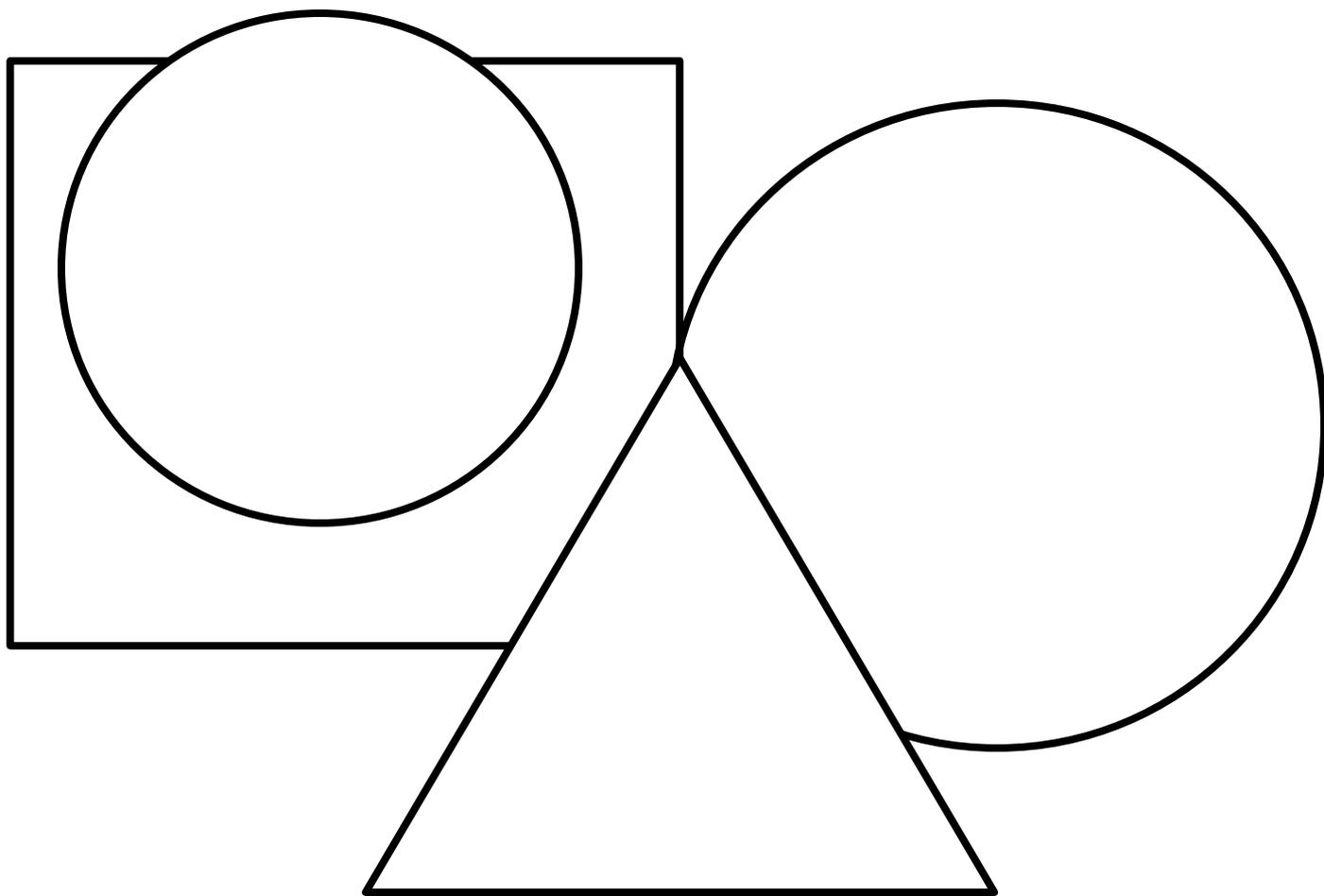
Color?



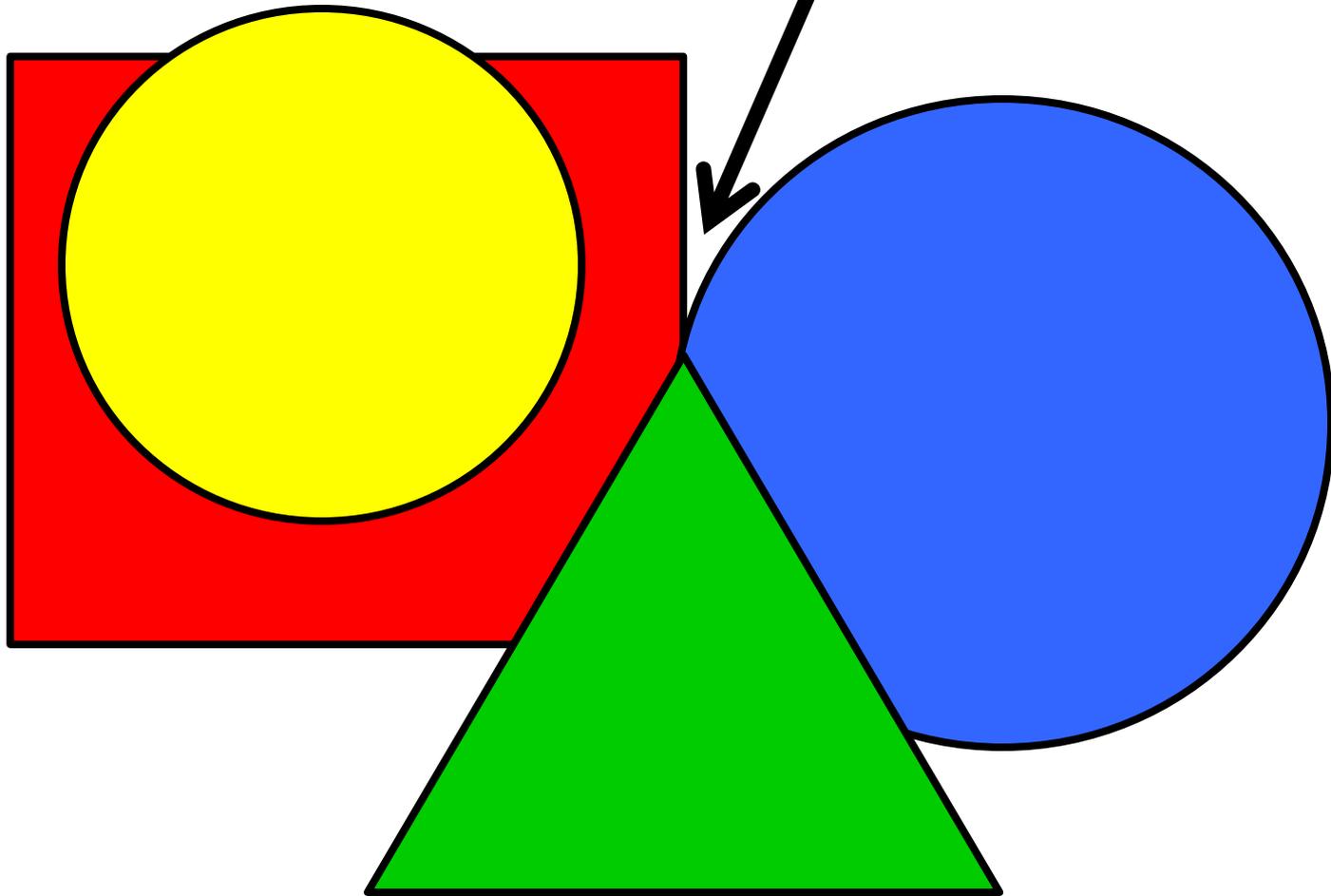


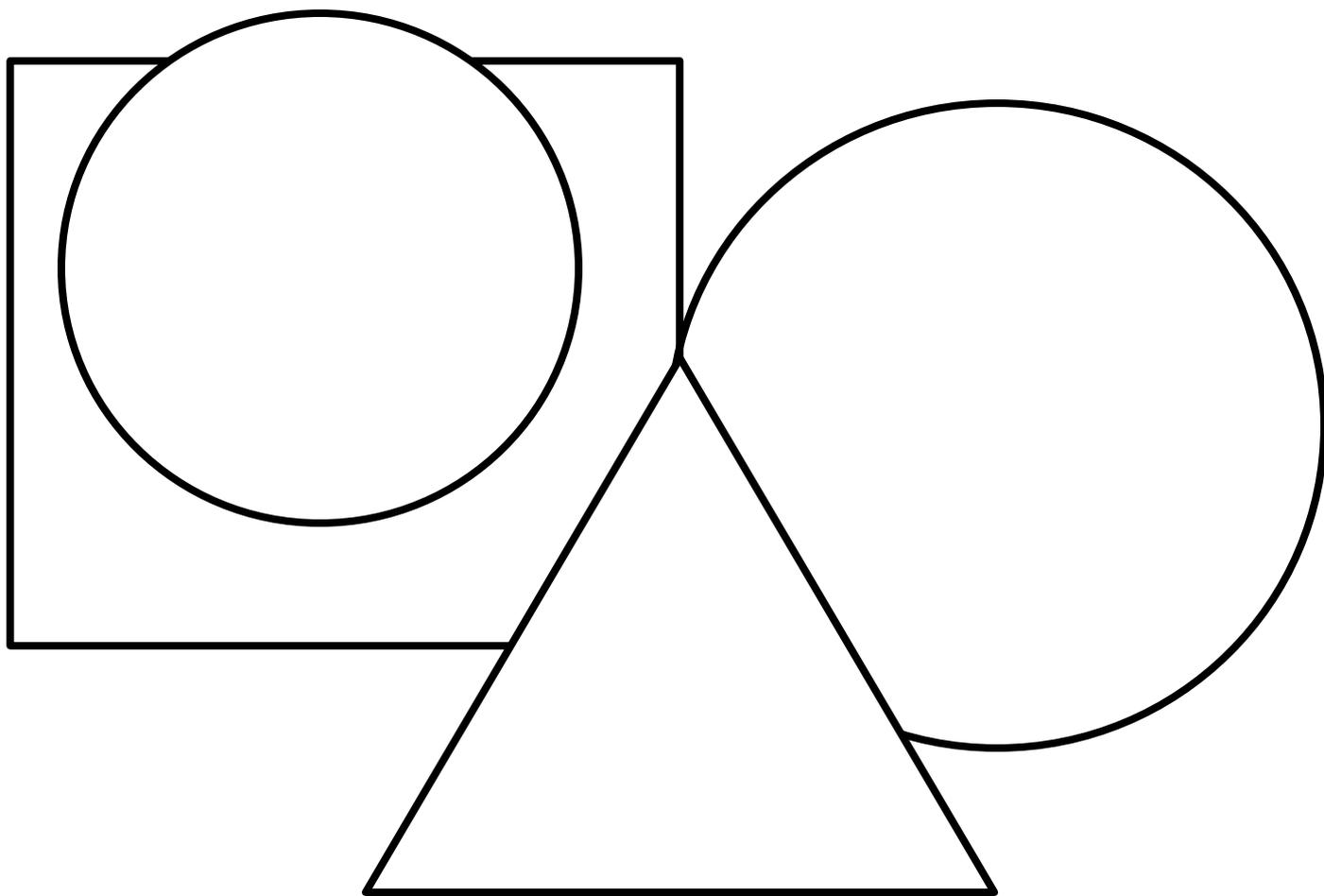
Stare at the intersection

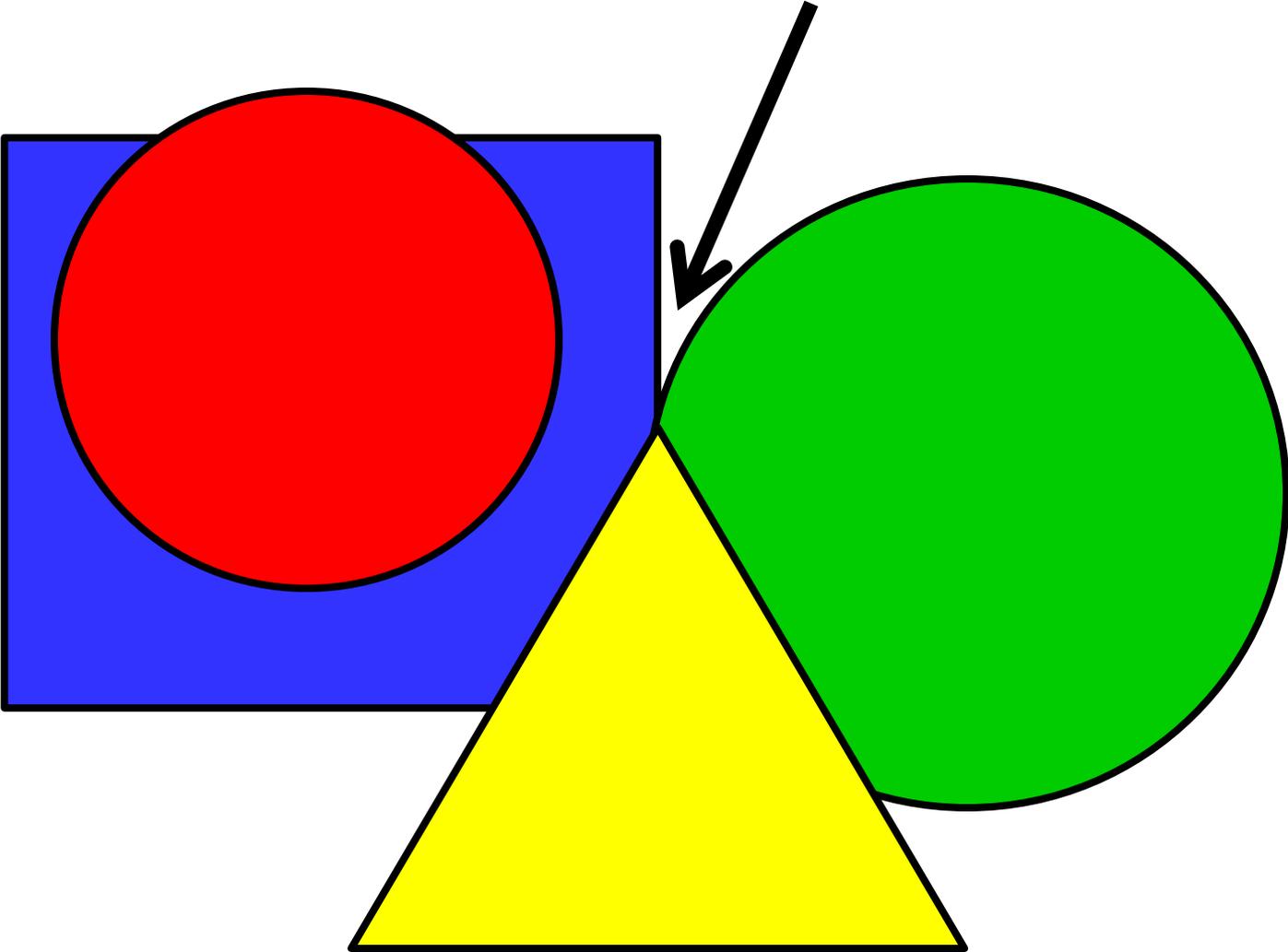


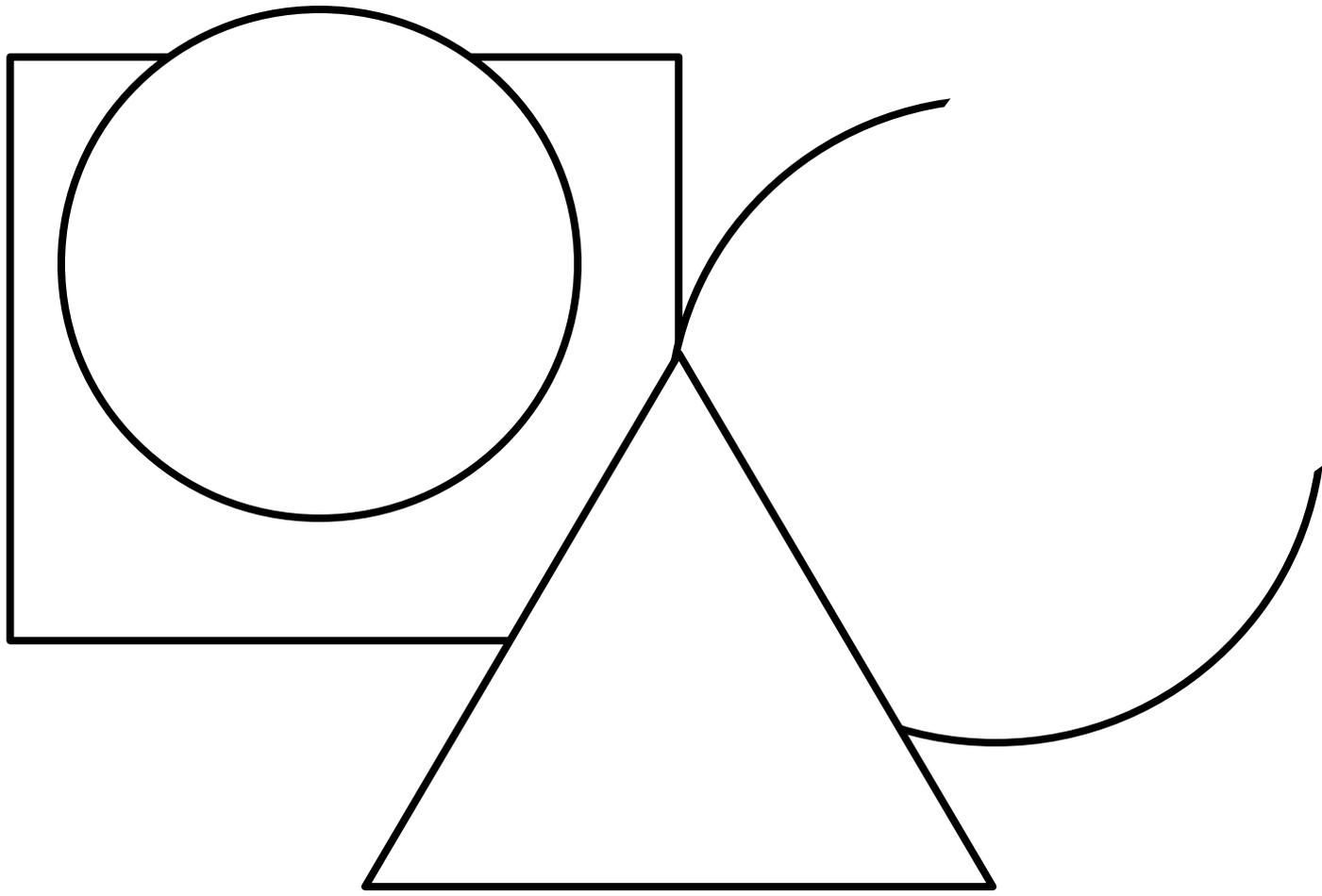


Stare at the intersection.

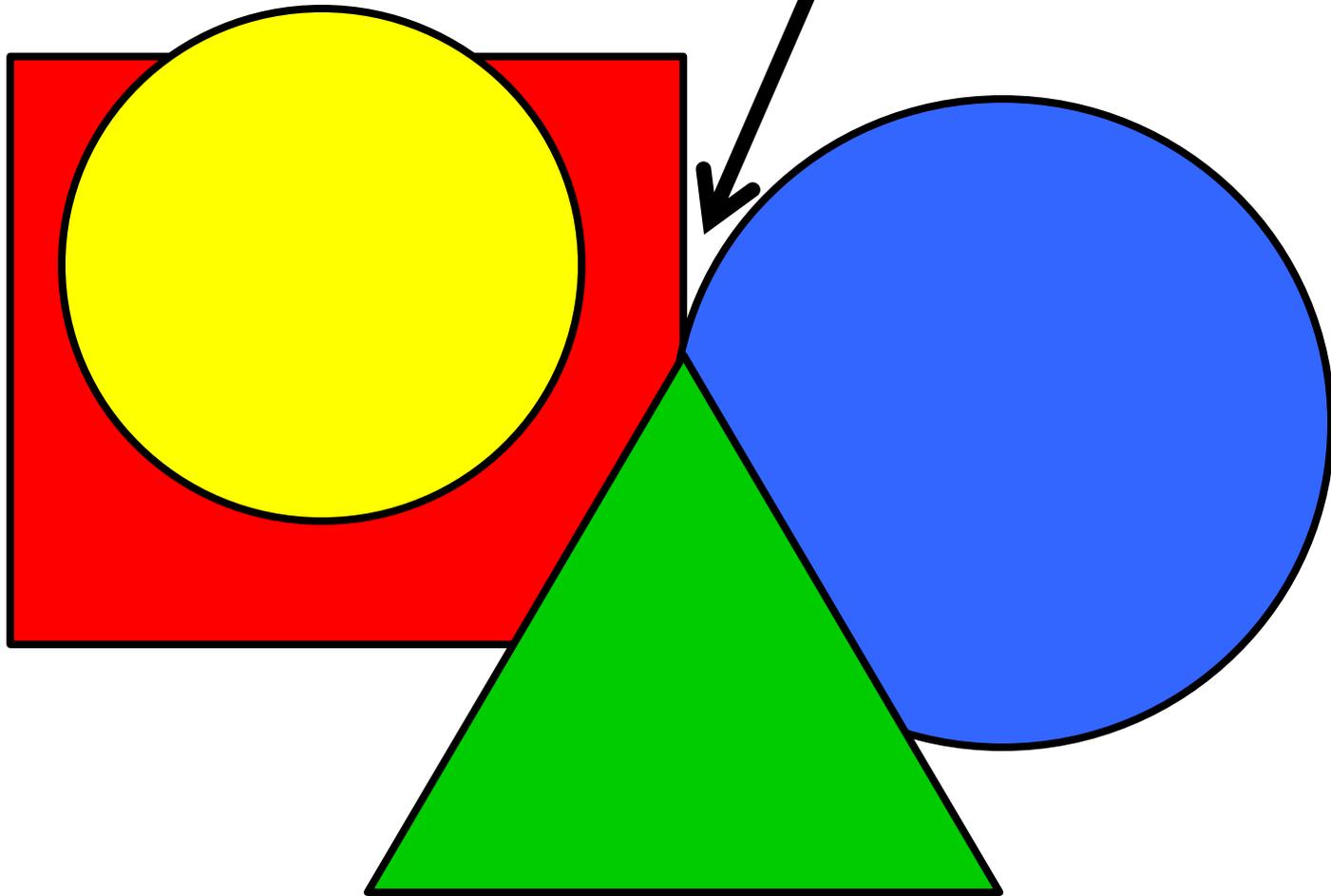


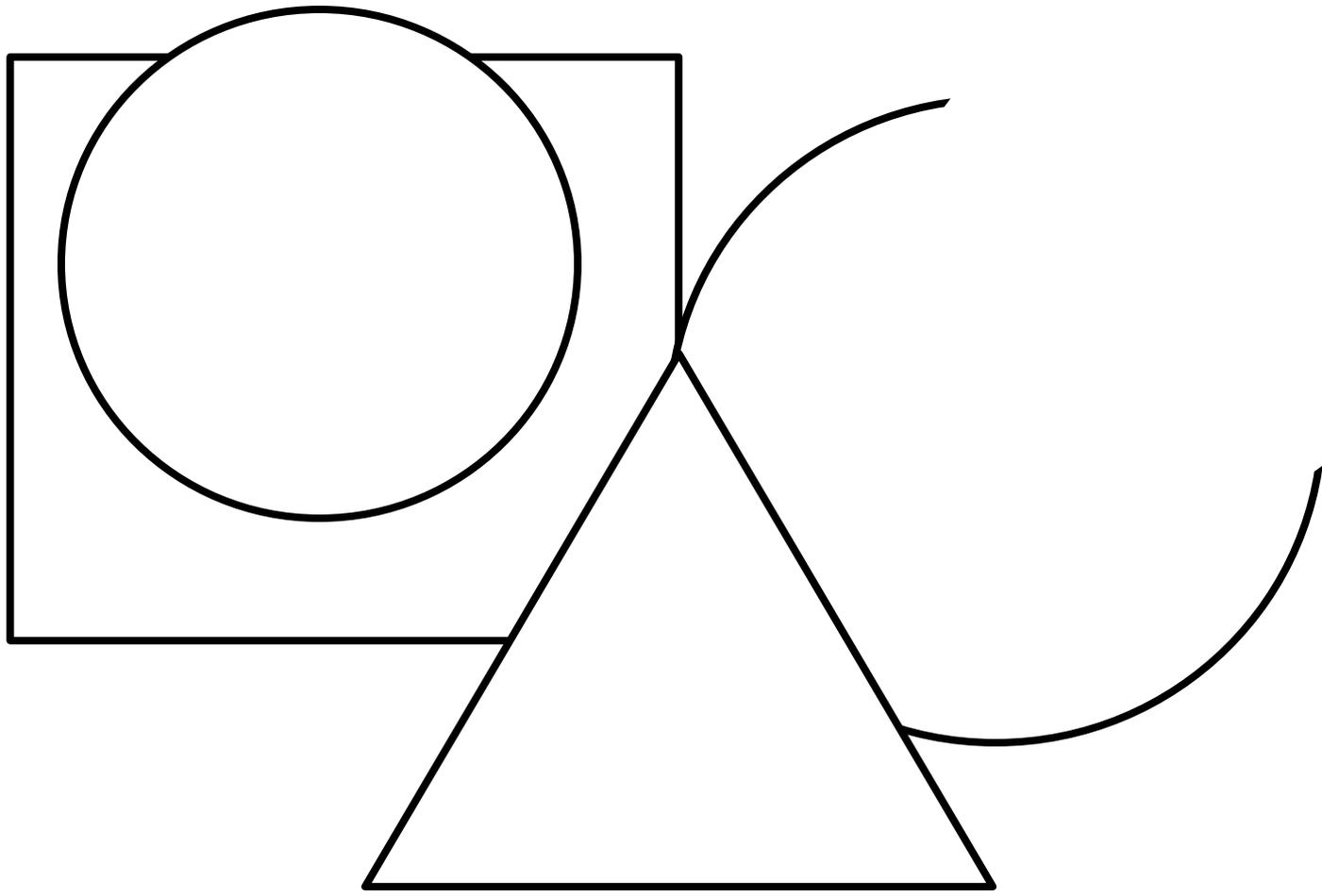


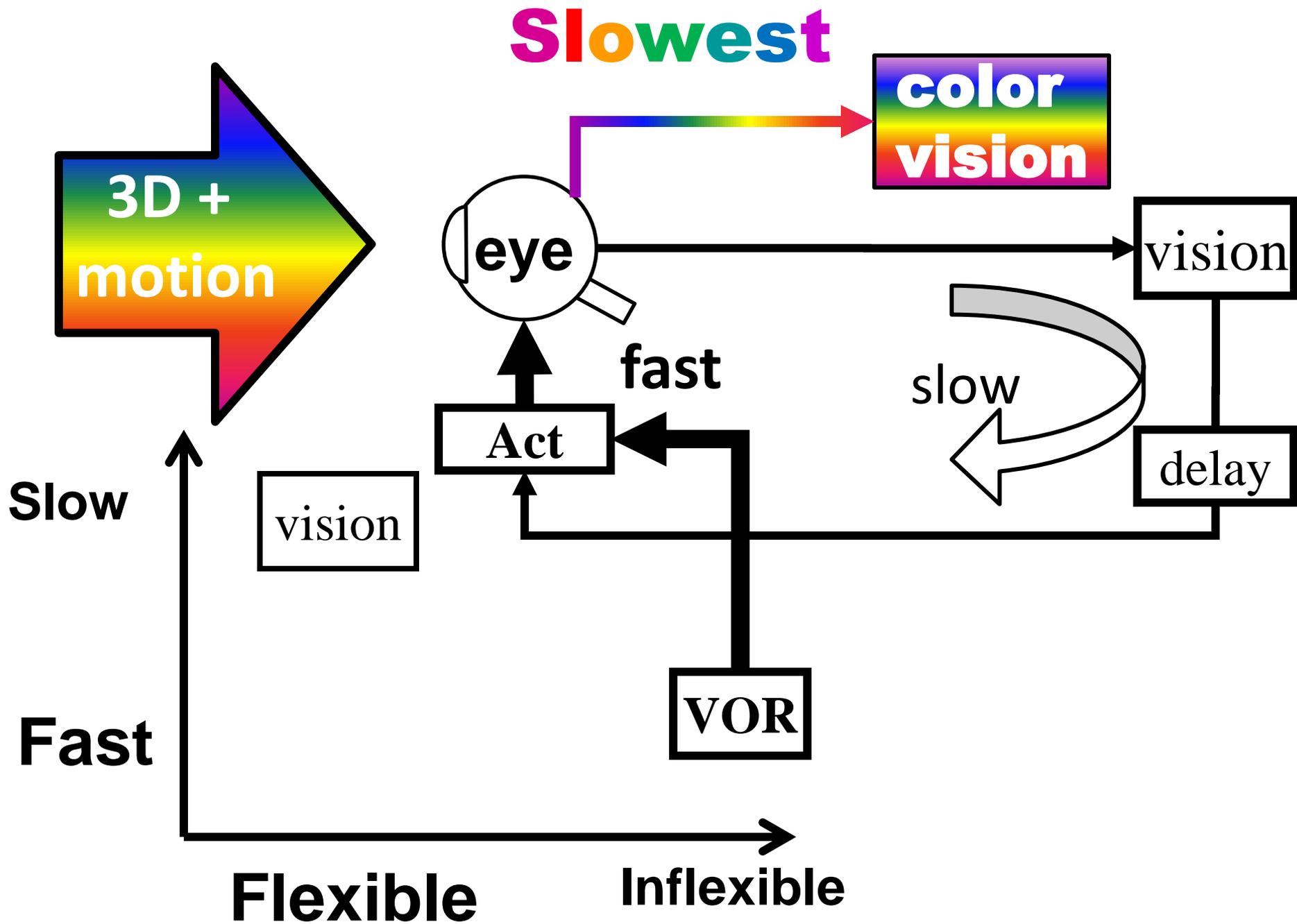




Stare at the intersection.

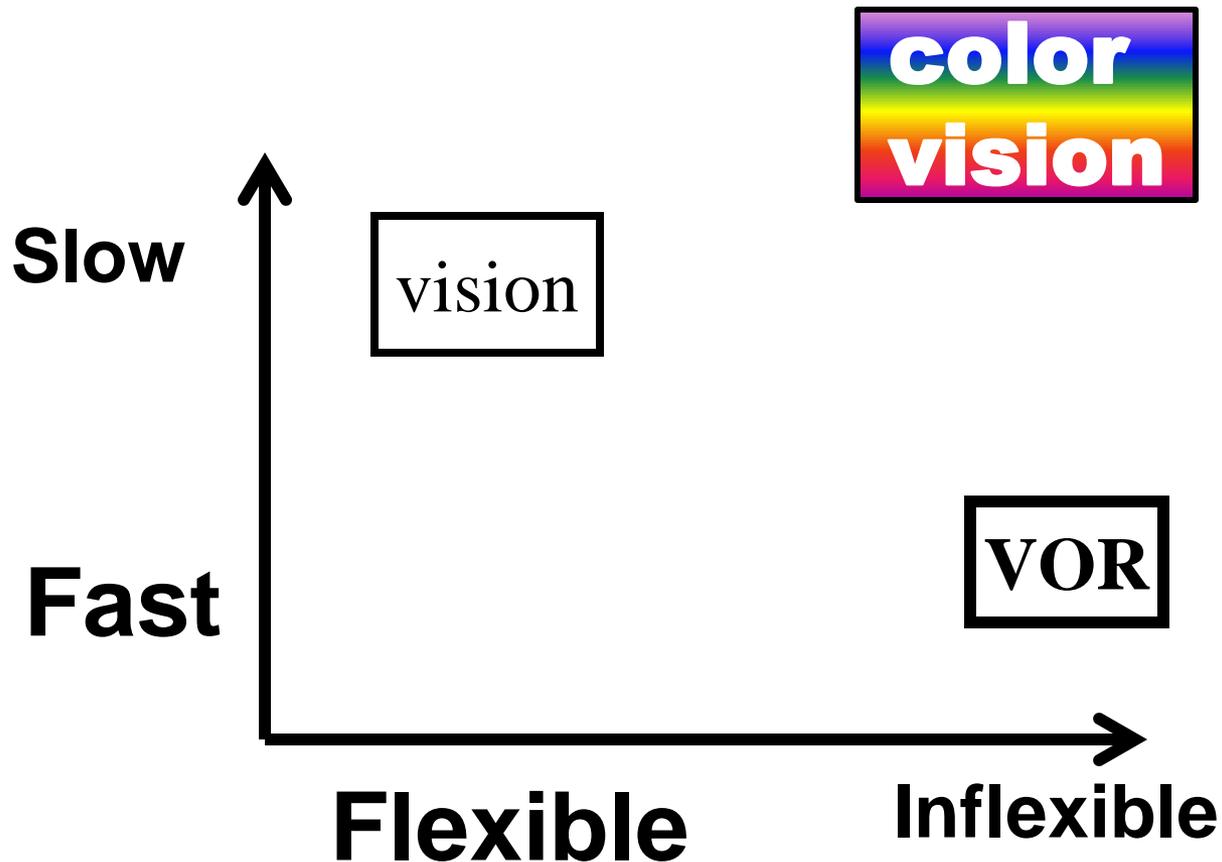


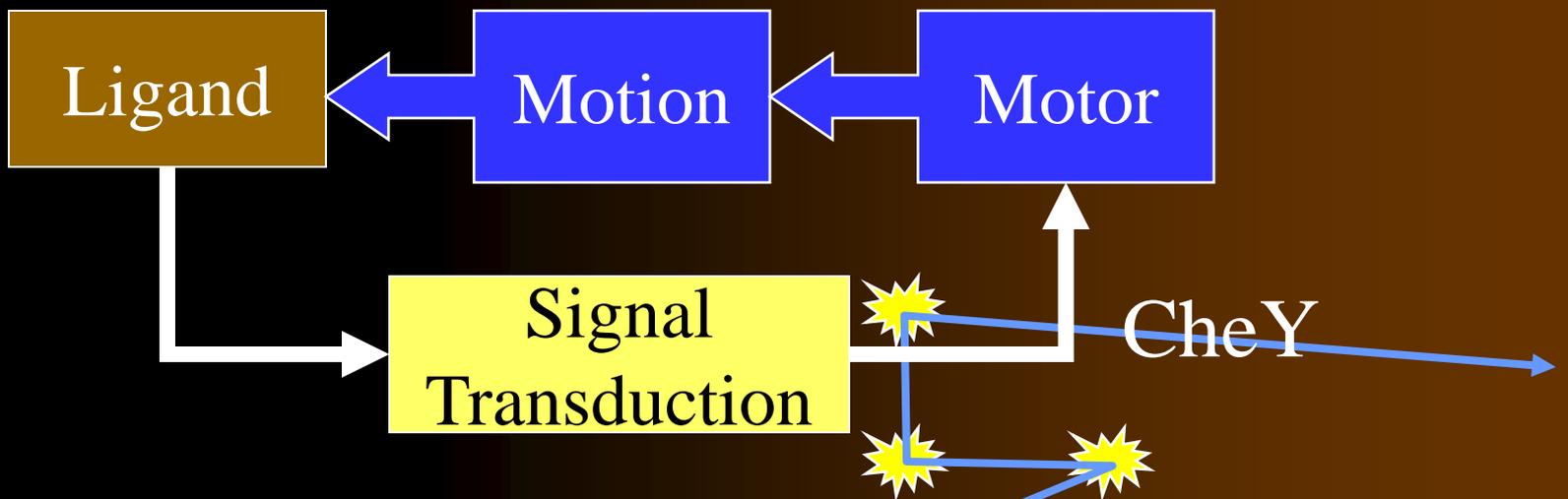




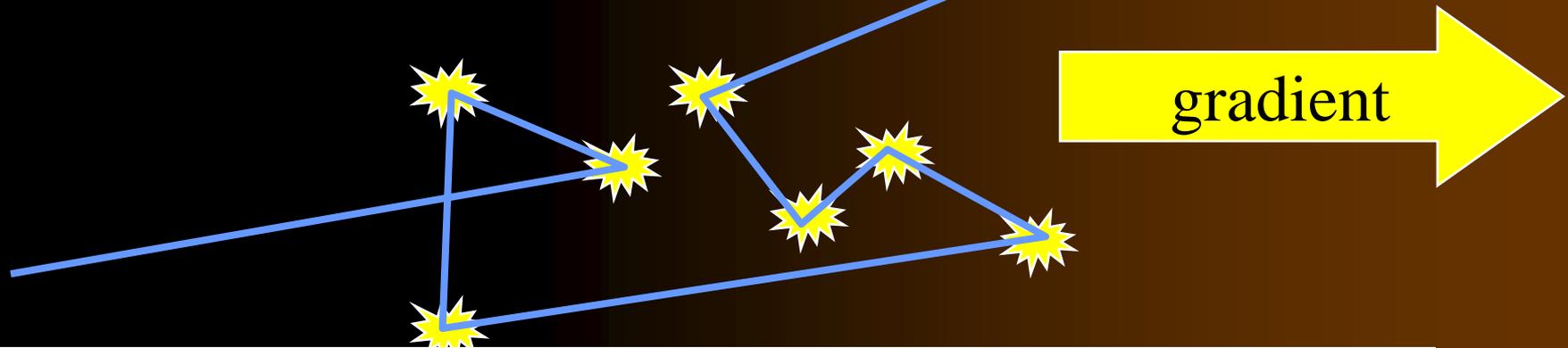
Seeing is dreaming

- is simulation
- requiring “internal model”





Biased random walk

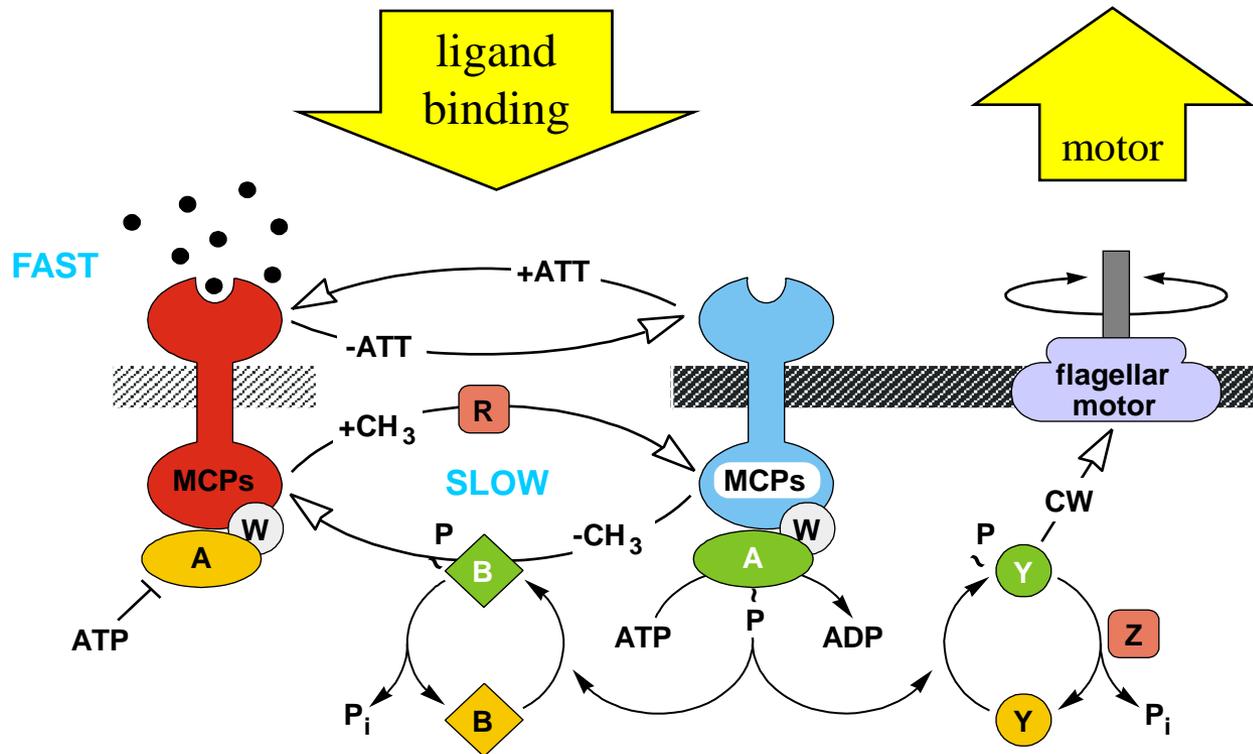


Robust perfect adaptation in bacterial chemotaxis through integral feedback control

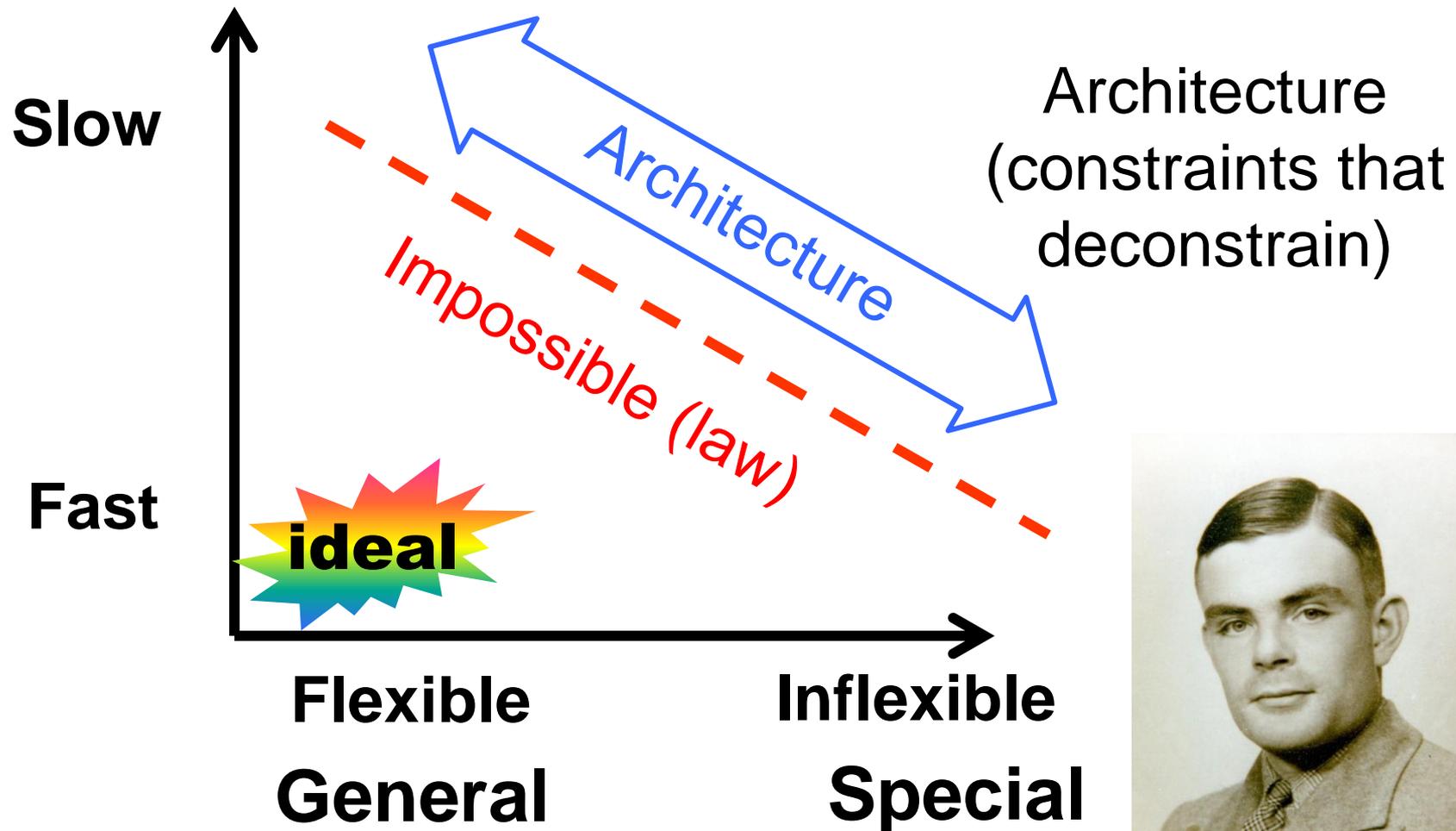
Tau-Mu Yi^{*†}, Yun Huang^{†‡}, Melvin I. Simon^{*§}, and John Doyle[‡]

Bacterial chemotaxis

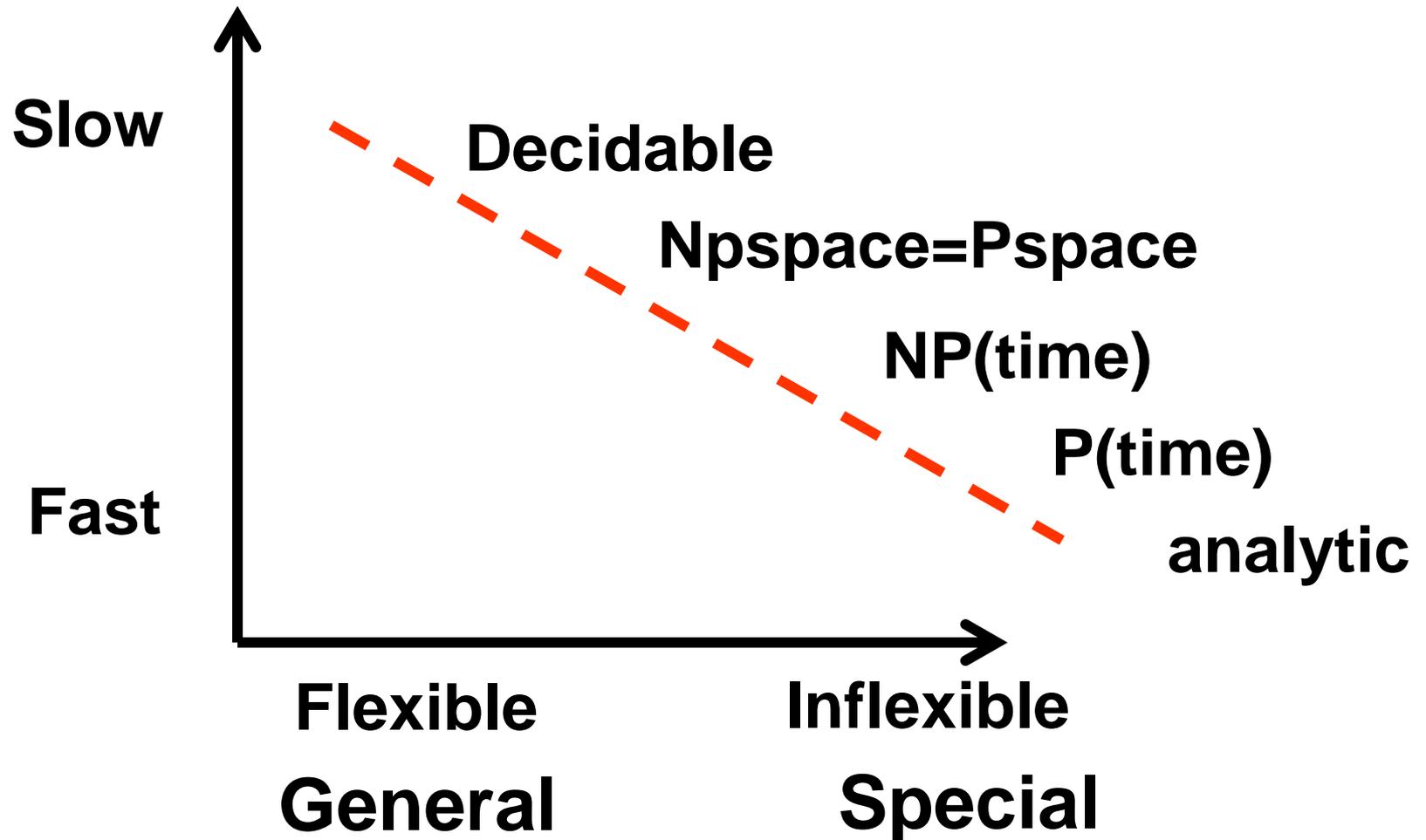
- Internal model *necessary* for robust chemotaxis
- Reality is 3d, but...
- Internal model *virtual* and 1d



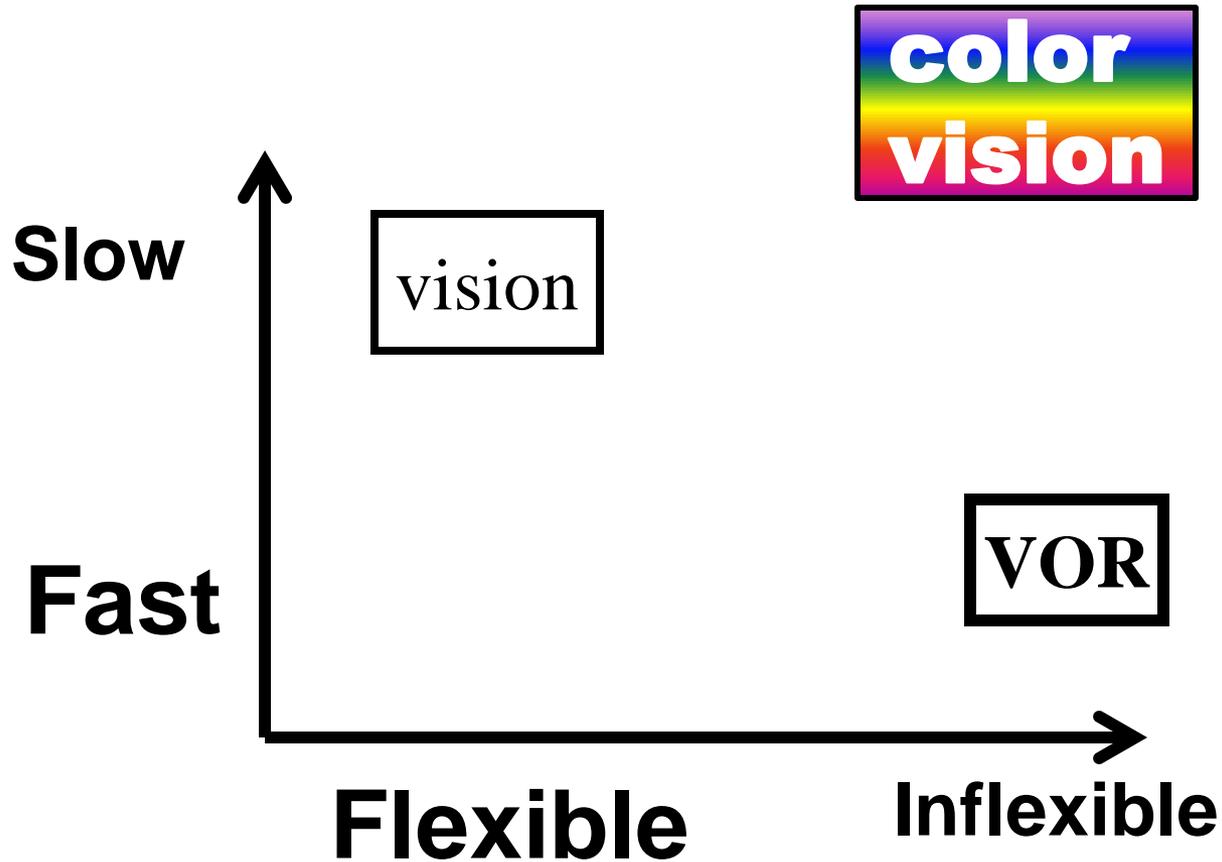
Universal laws and architectures (Turing)

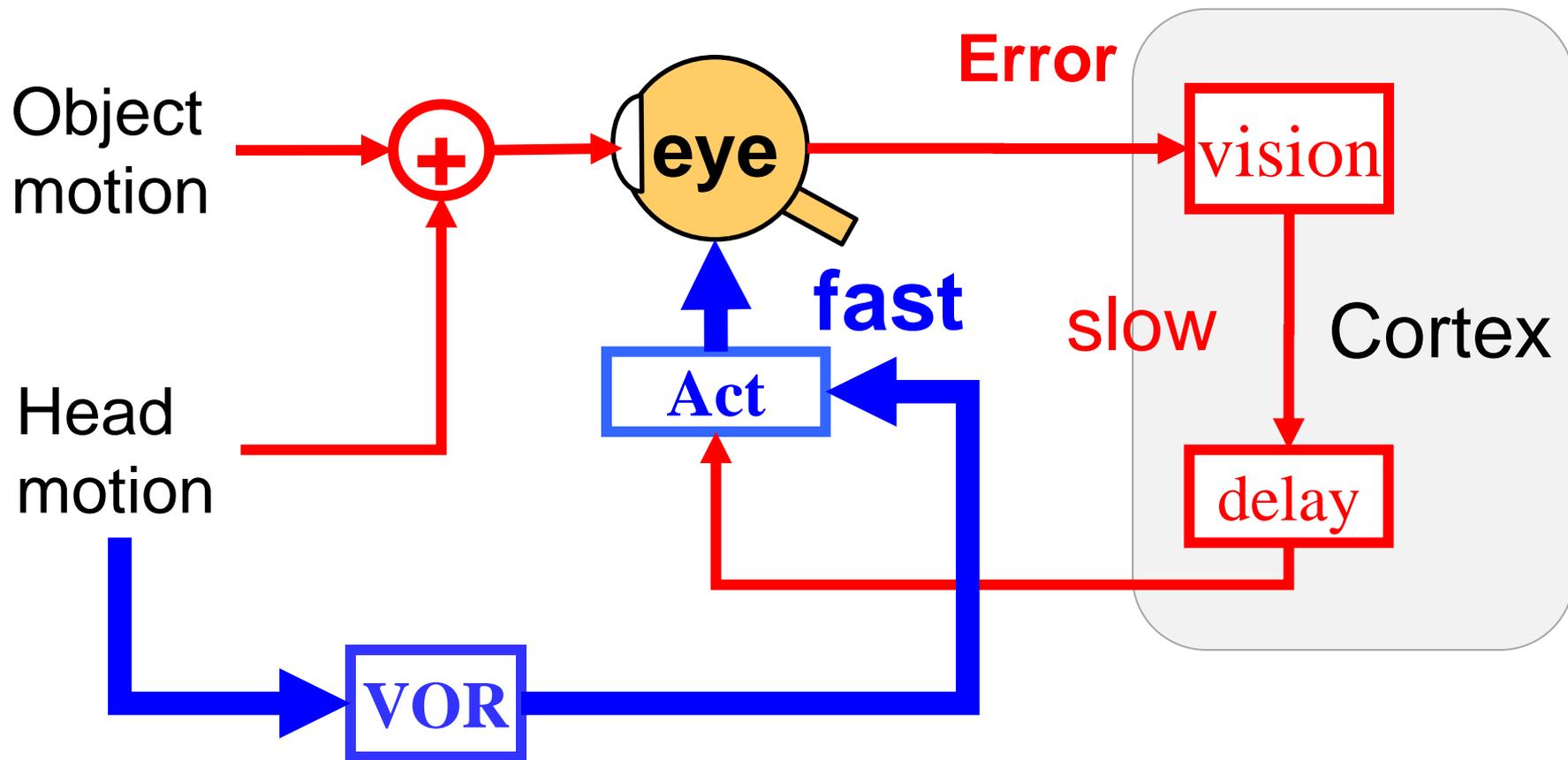


Computation (on and off-line)

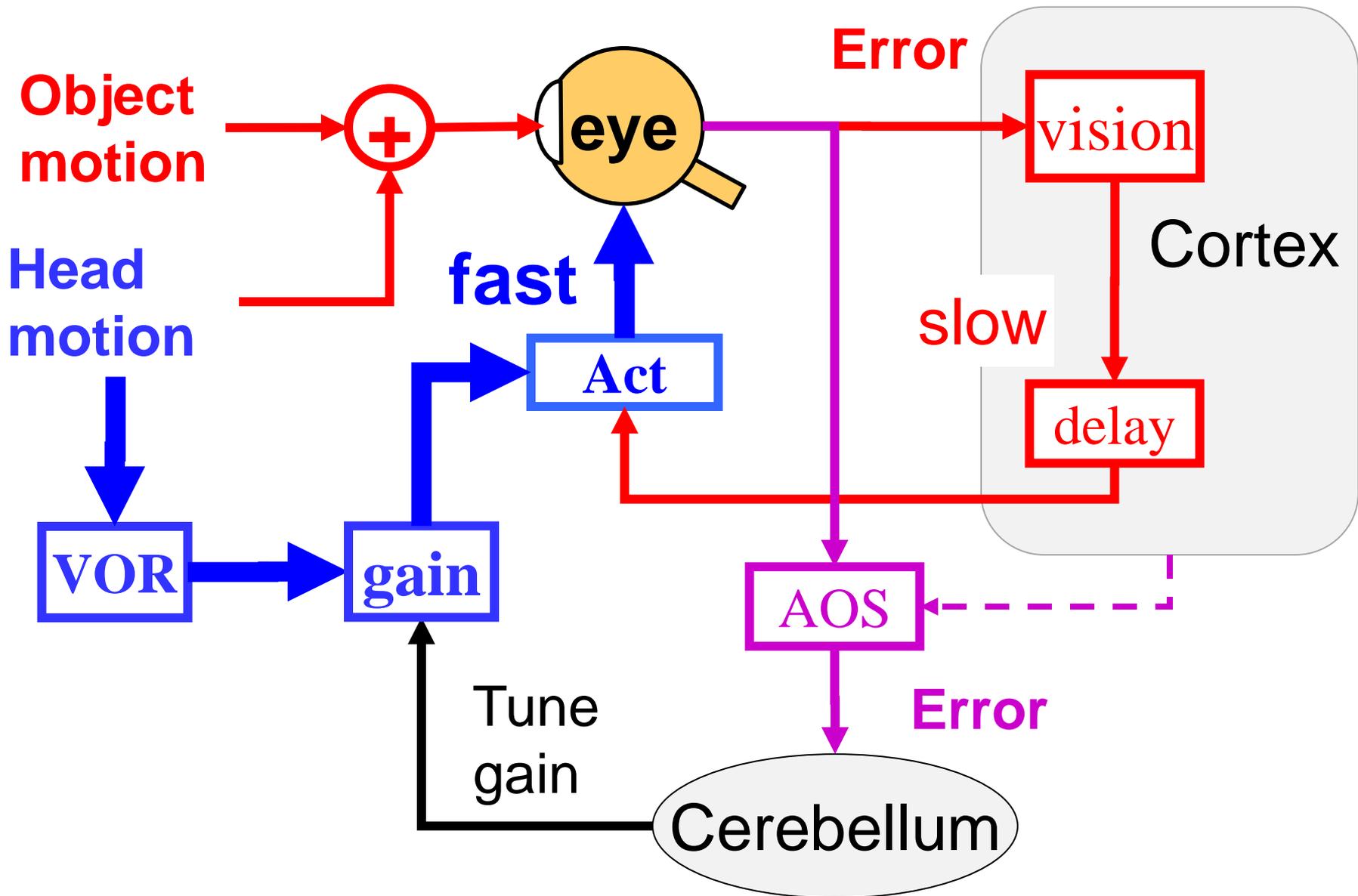


Layering Distributed Feedback



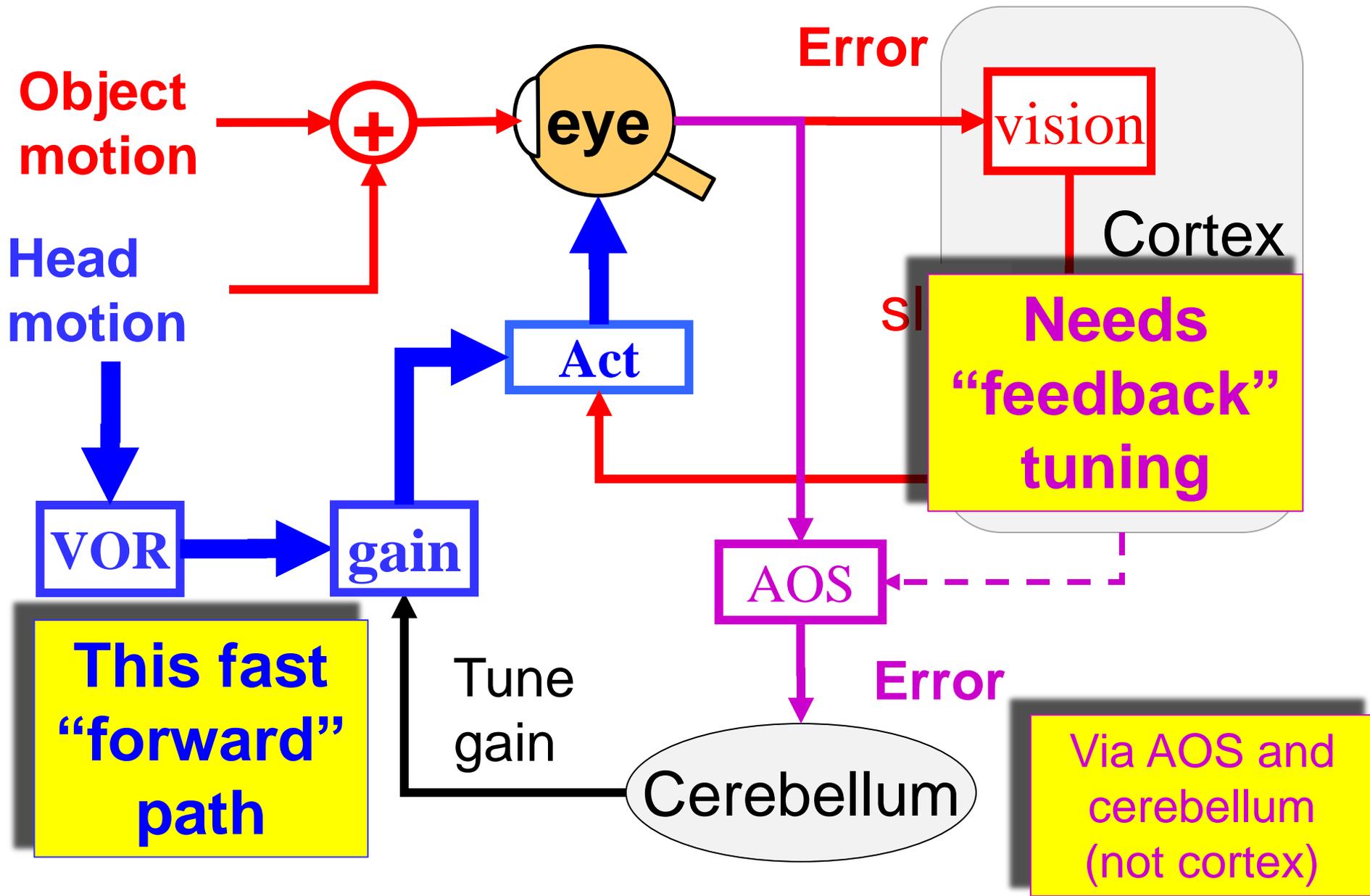


Very rough cartoon



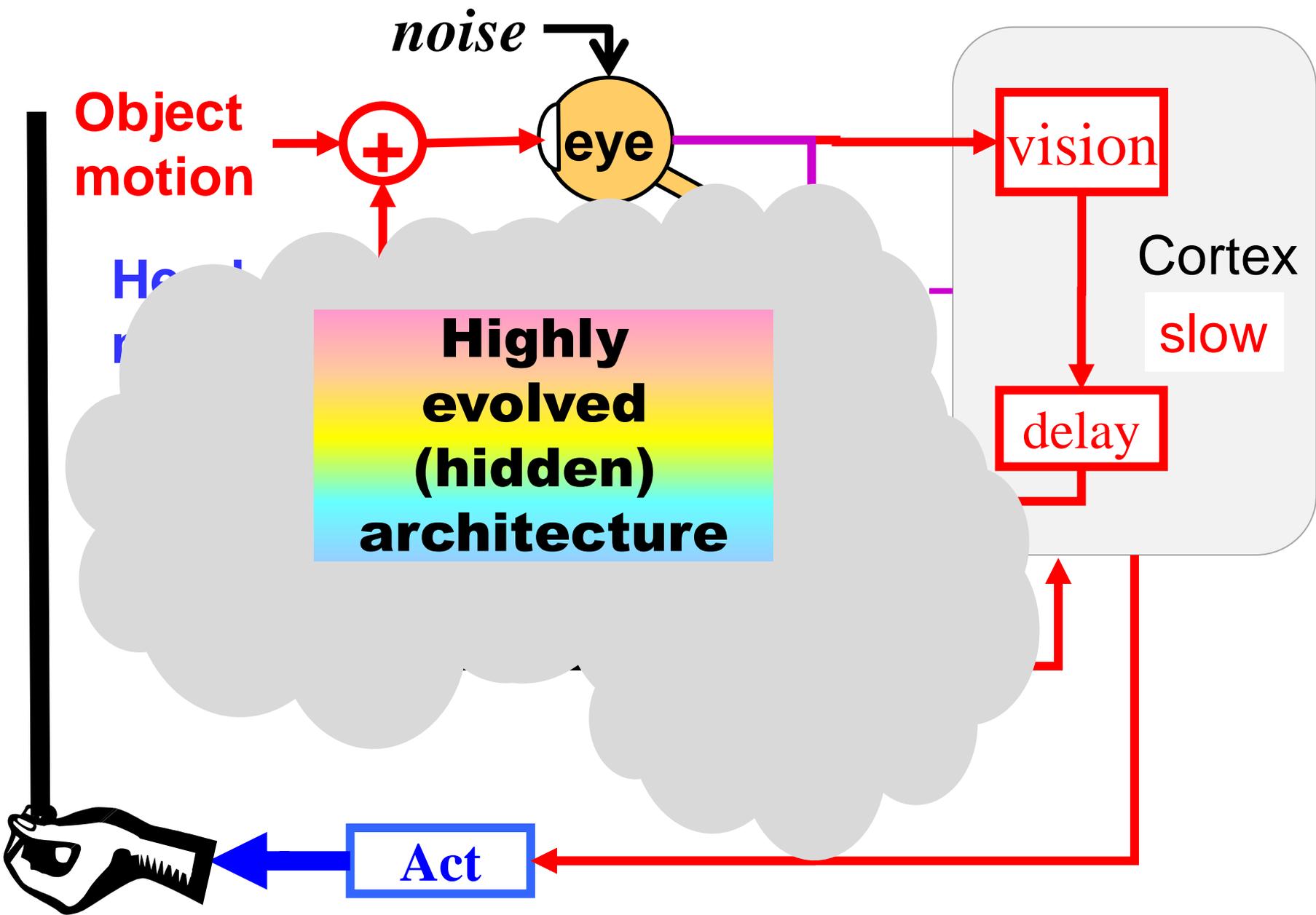
Slightly better

AOS = Accessory Optical system

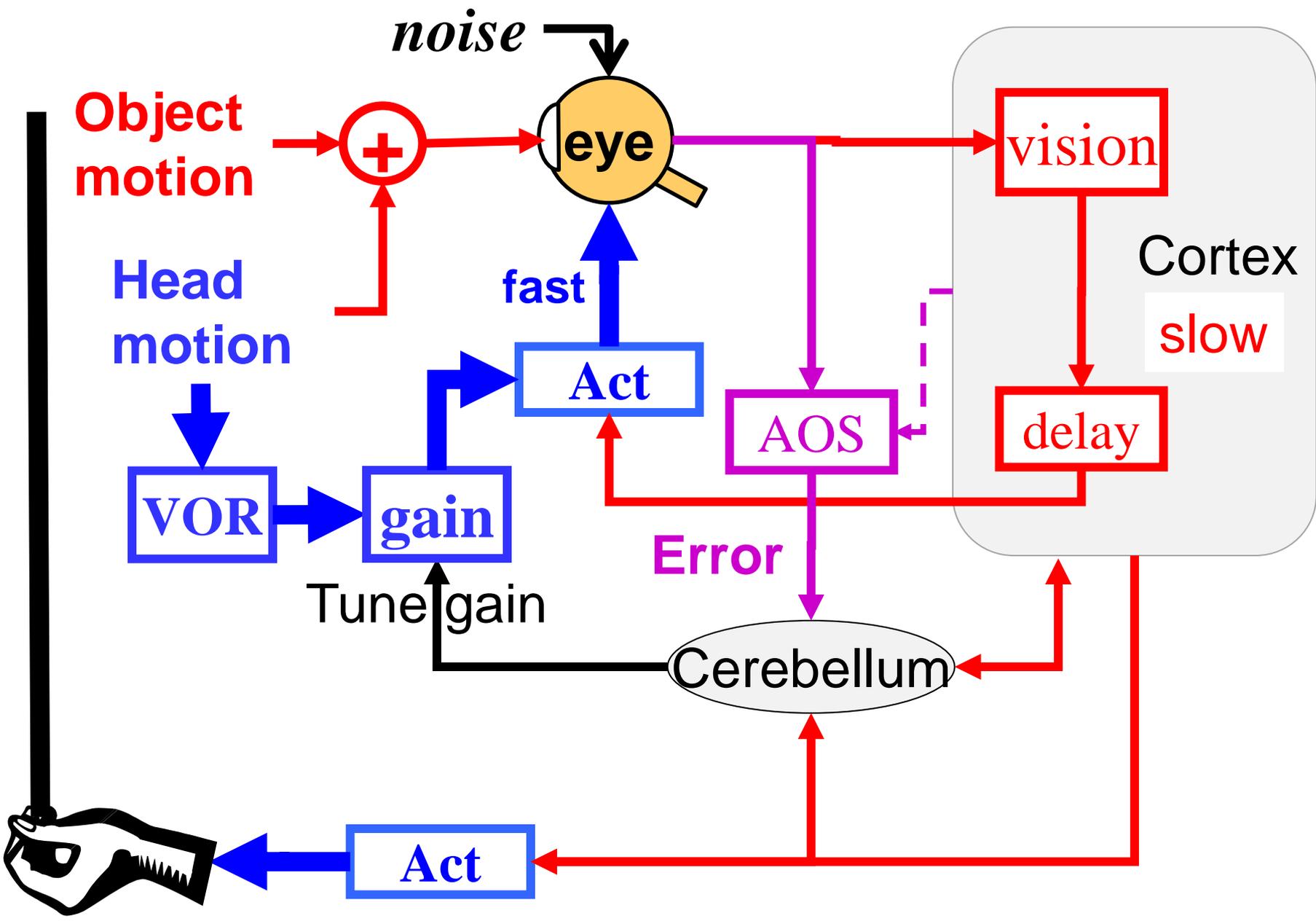


Slightly better

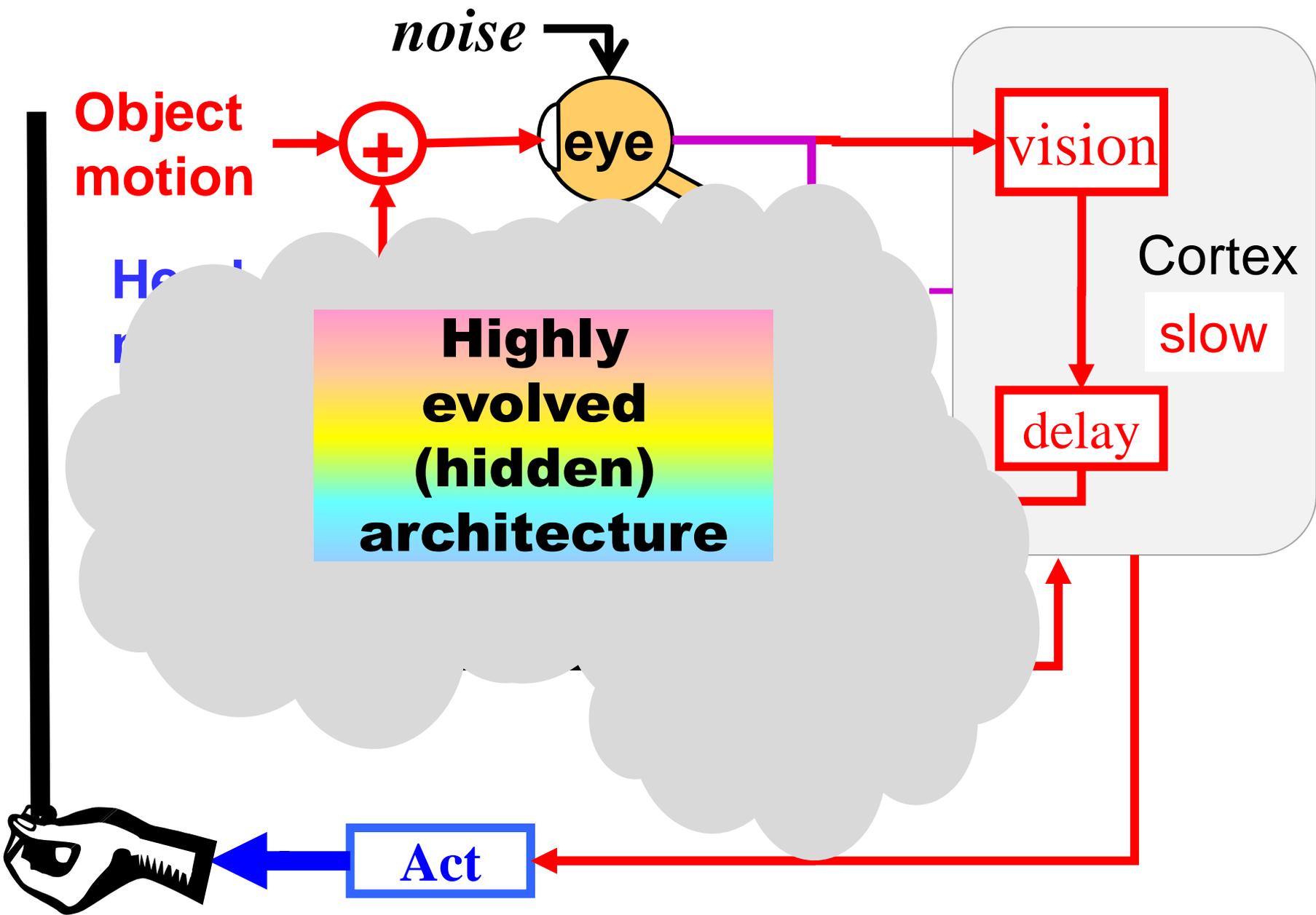
AOS = Accessory Optical system



AOS = Accessory Optical system



AOS = Accessory Optical system



AOS = Accessory Optical system

**Layering
Distributed
Feedback**

Slow

Fast

Flexible

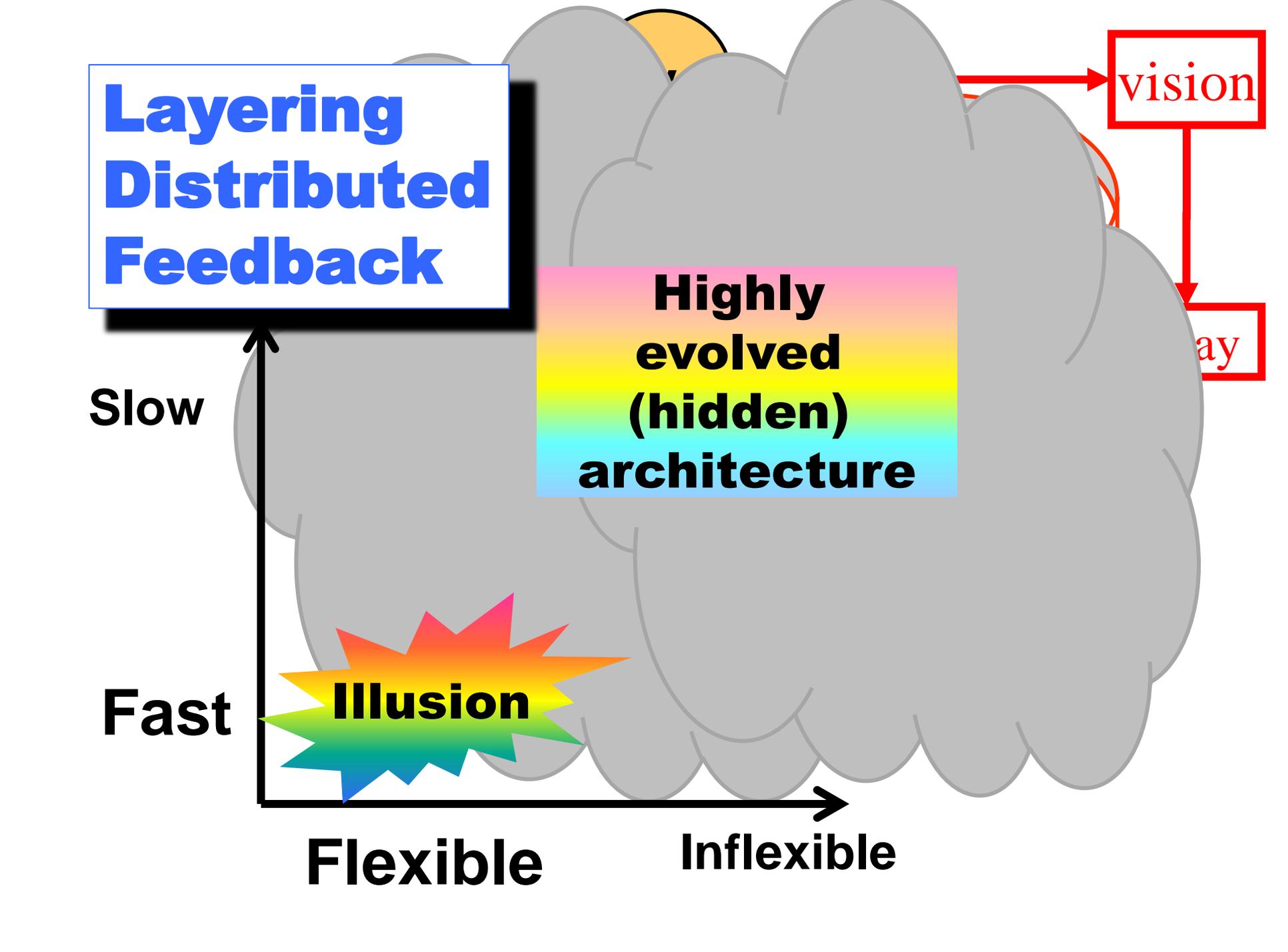
Inflexible

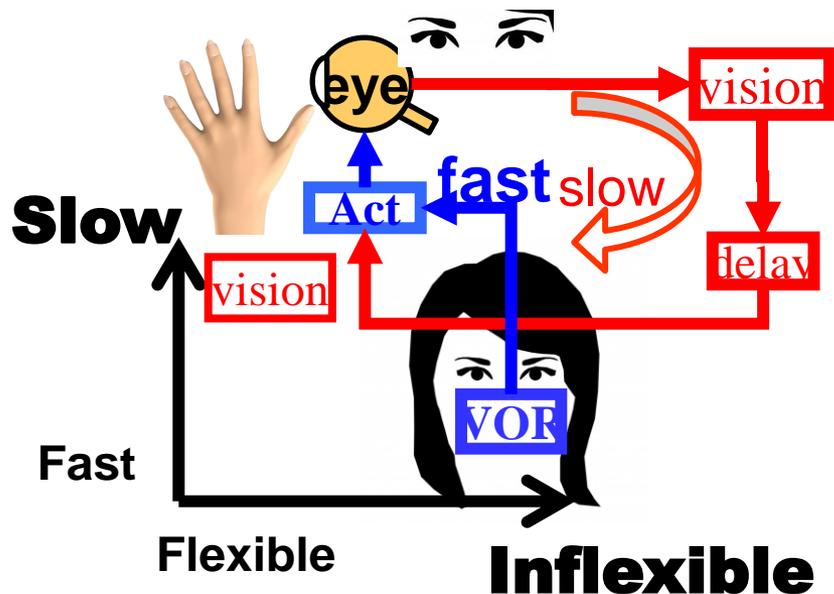
**Highly
evolved
(hidden)
architecture**

Illusion

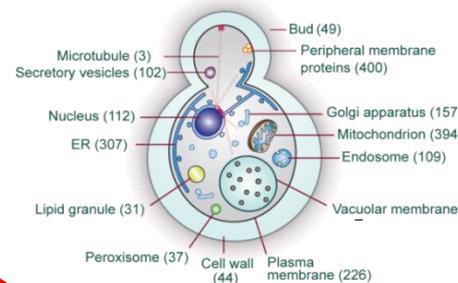
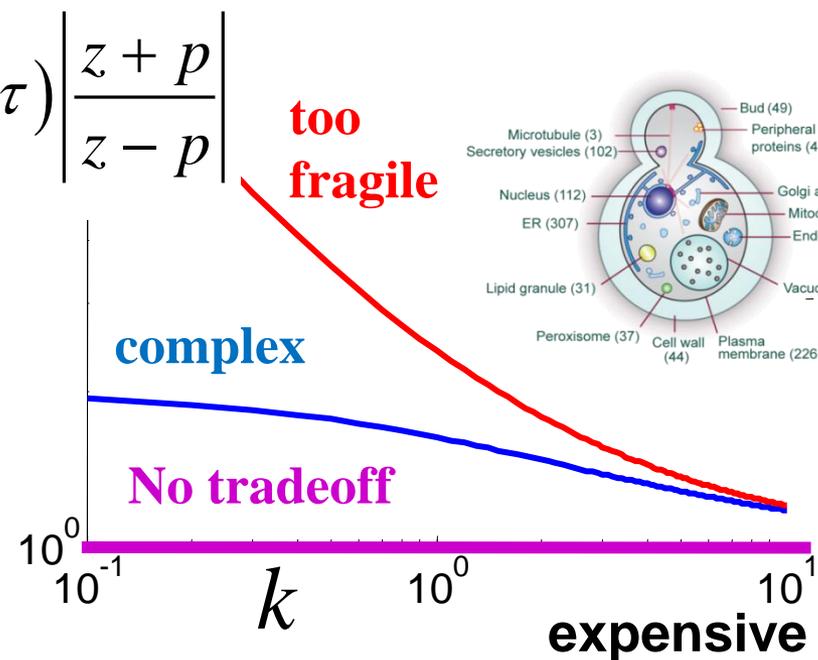
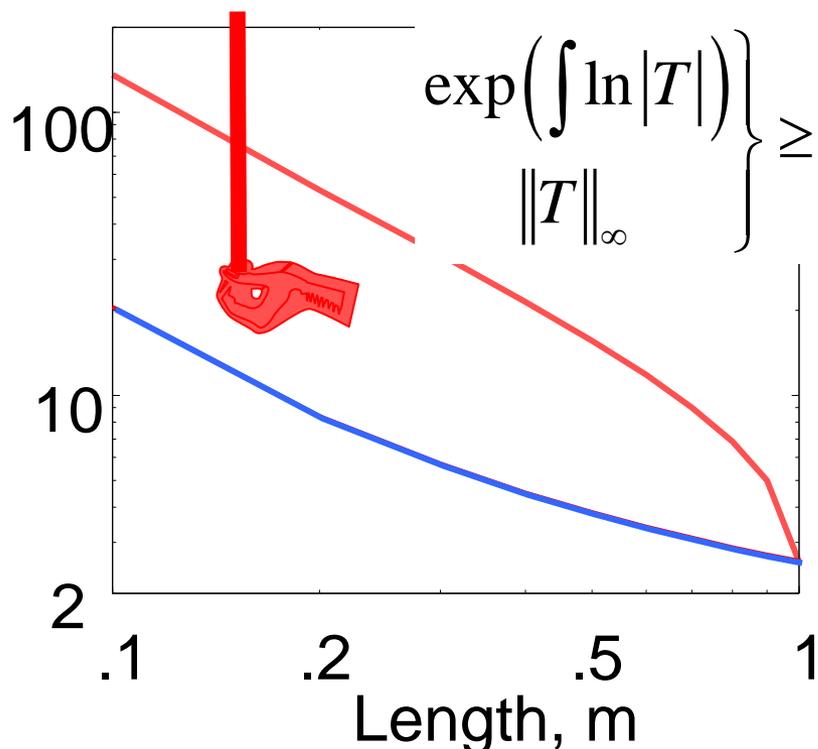
vision

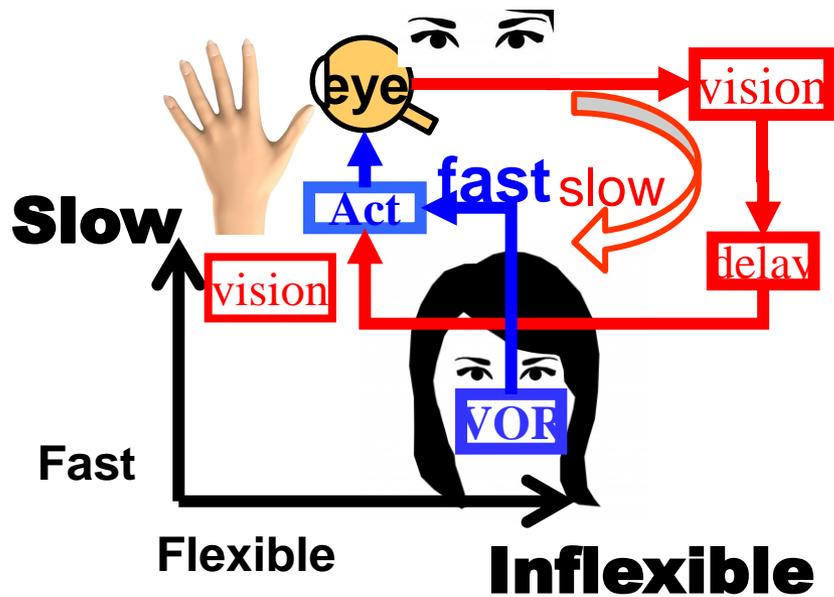
ay



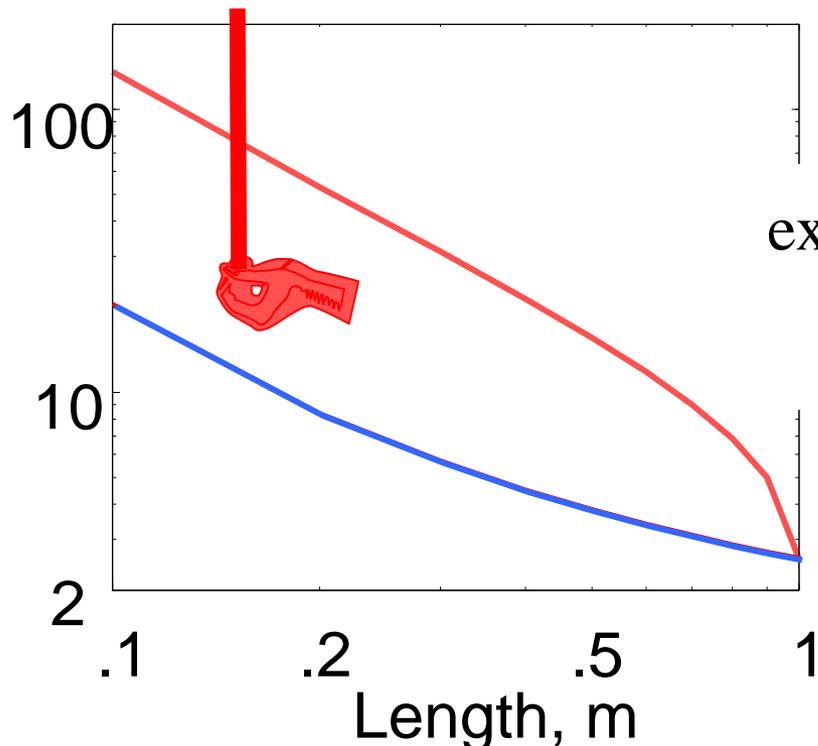


Universal laws?



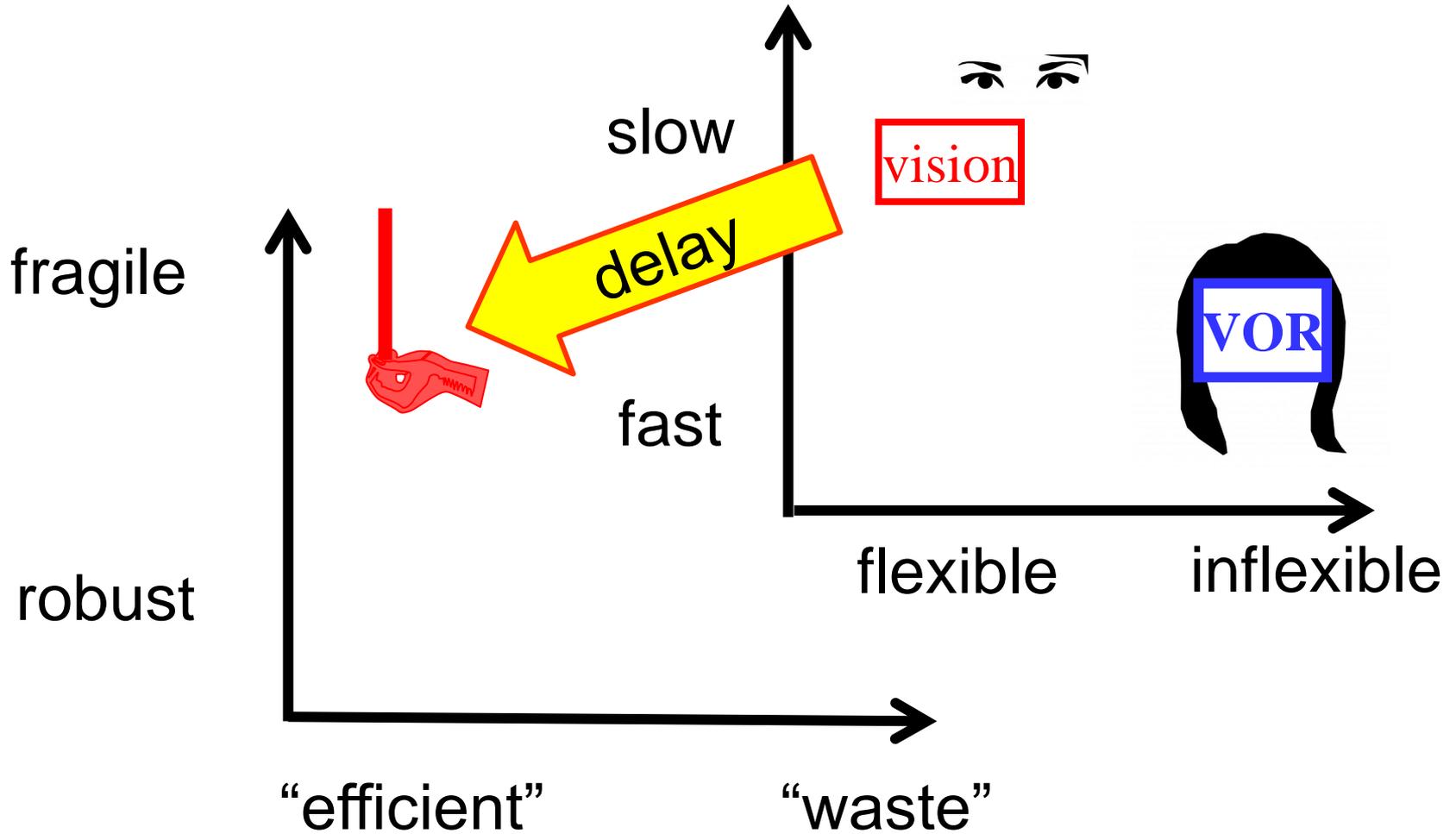


How do these fit together?

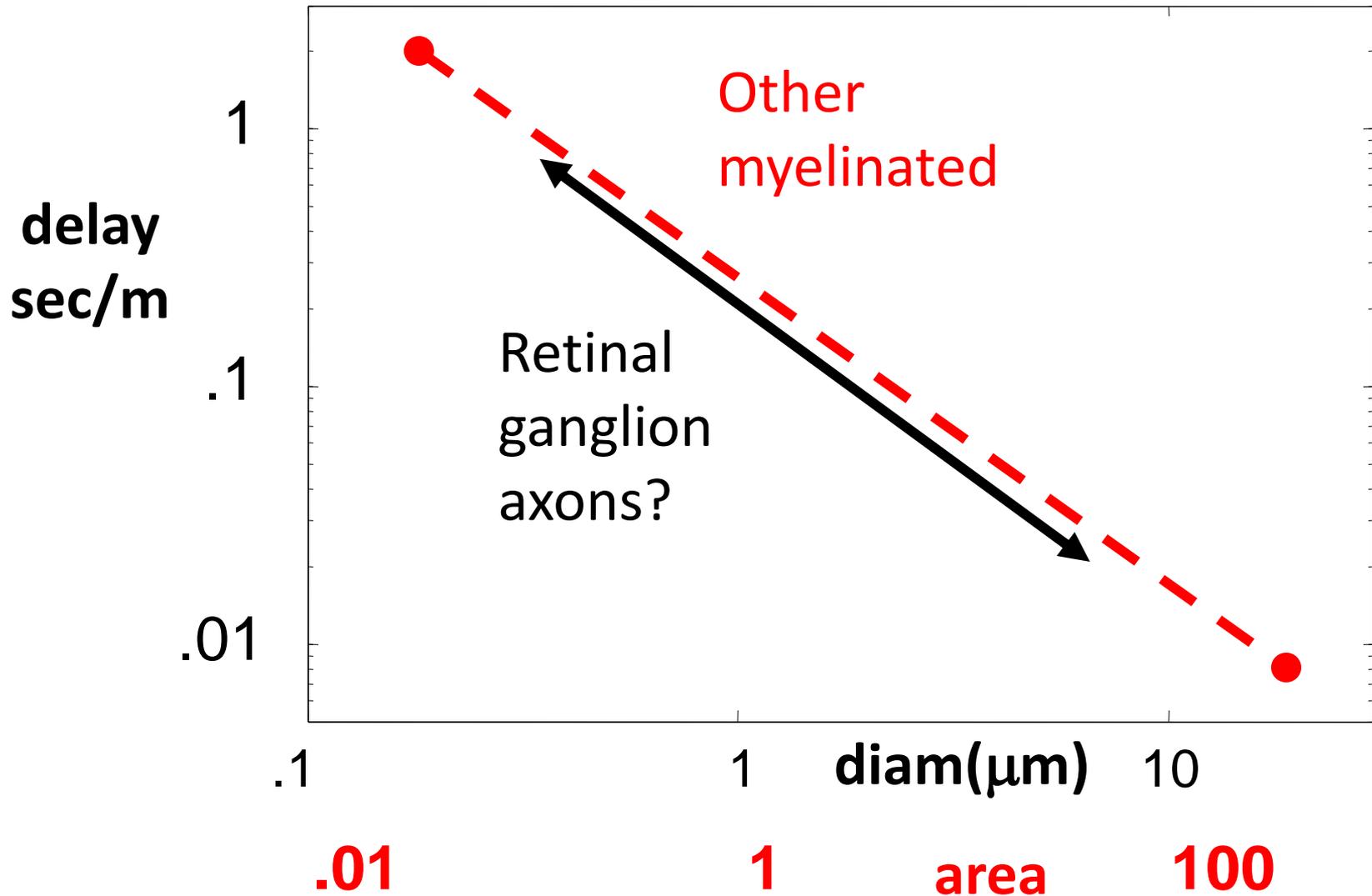


$$\left. \begin{array}{l} \exp\left(\int \ln |T|\right) \\ \|T\|_{\infty} \end{array} \right\} \geq \exp(p\tau) \left| \frac{z+p}{z-p} \right|$$

How do these fit together?



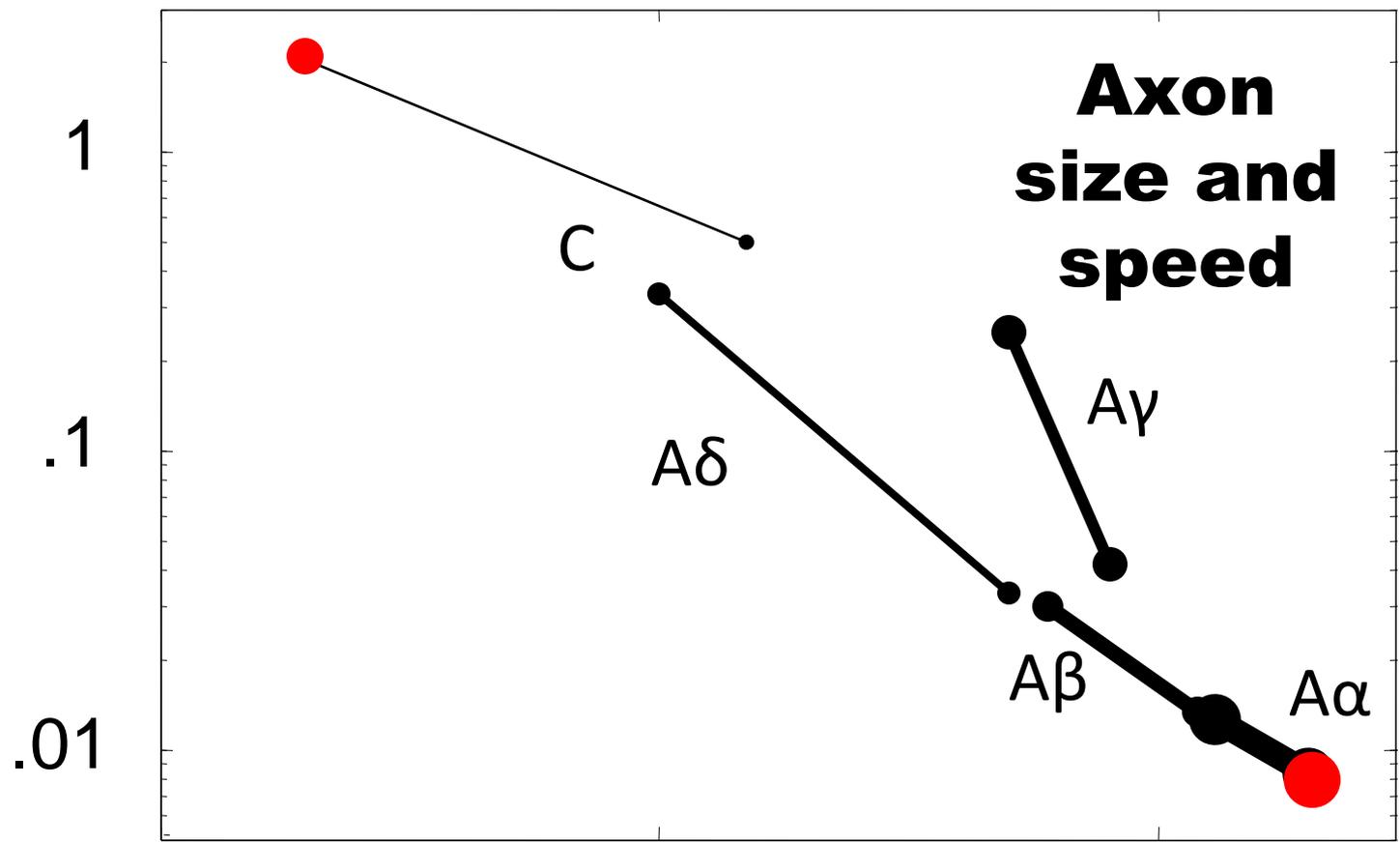
Why such extreme diversity in *axon* size and delay?



Slow

delay
sec/m

Fast



.1

1

diam(μ m)

10

Small

Large

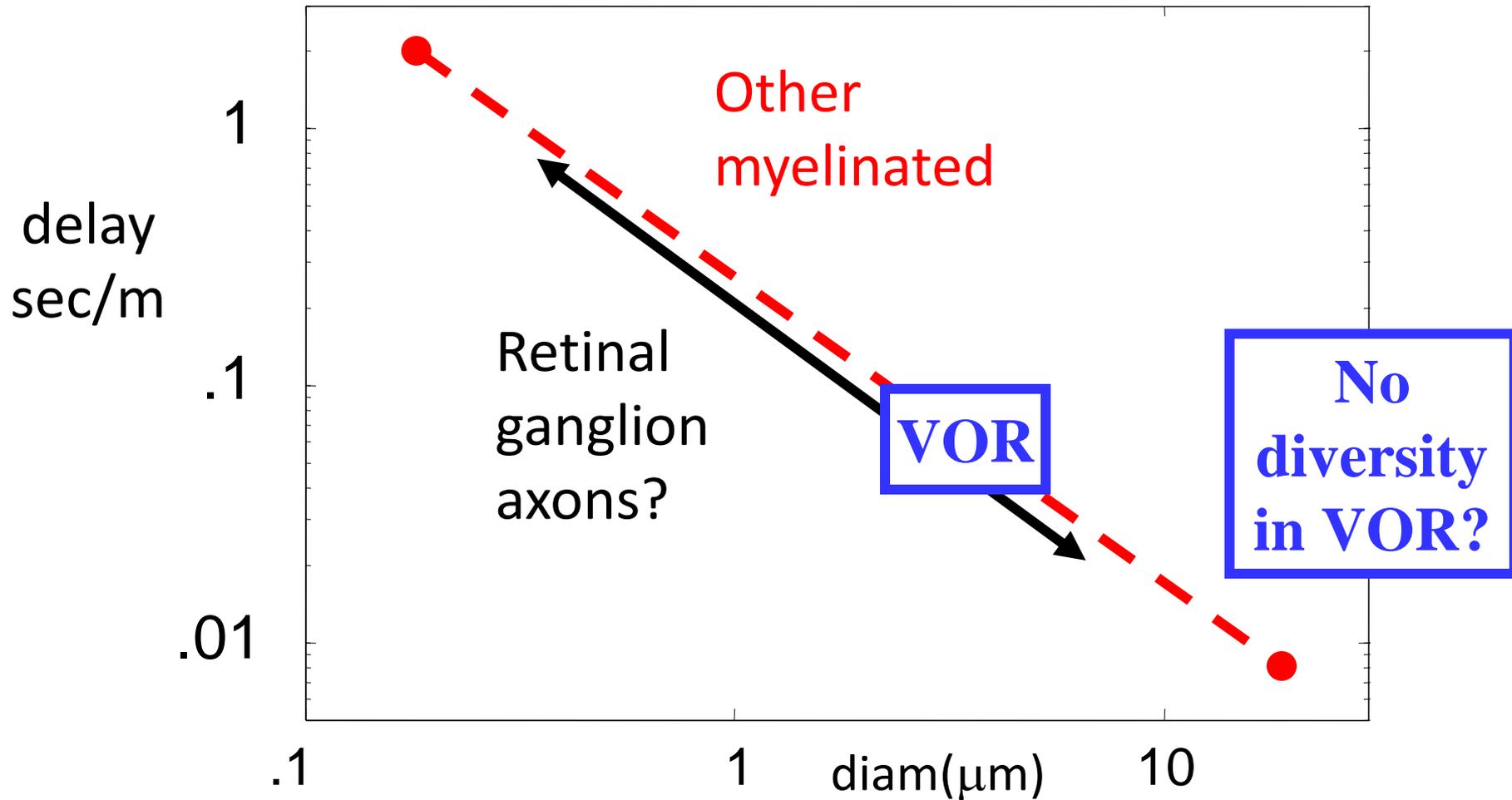
.01

1

area

100

Why such extreme diversity in *axon* size and delay?



speed costs

.01

1

area

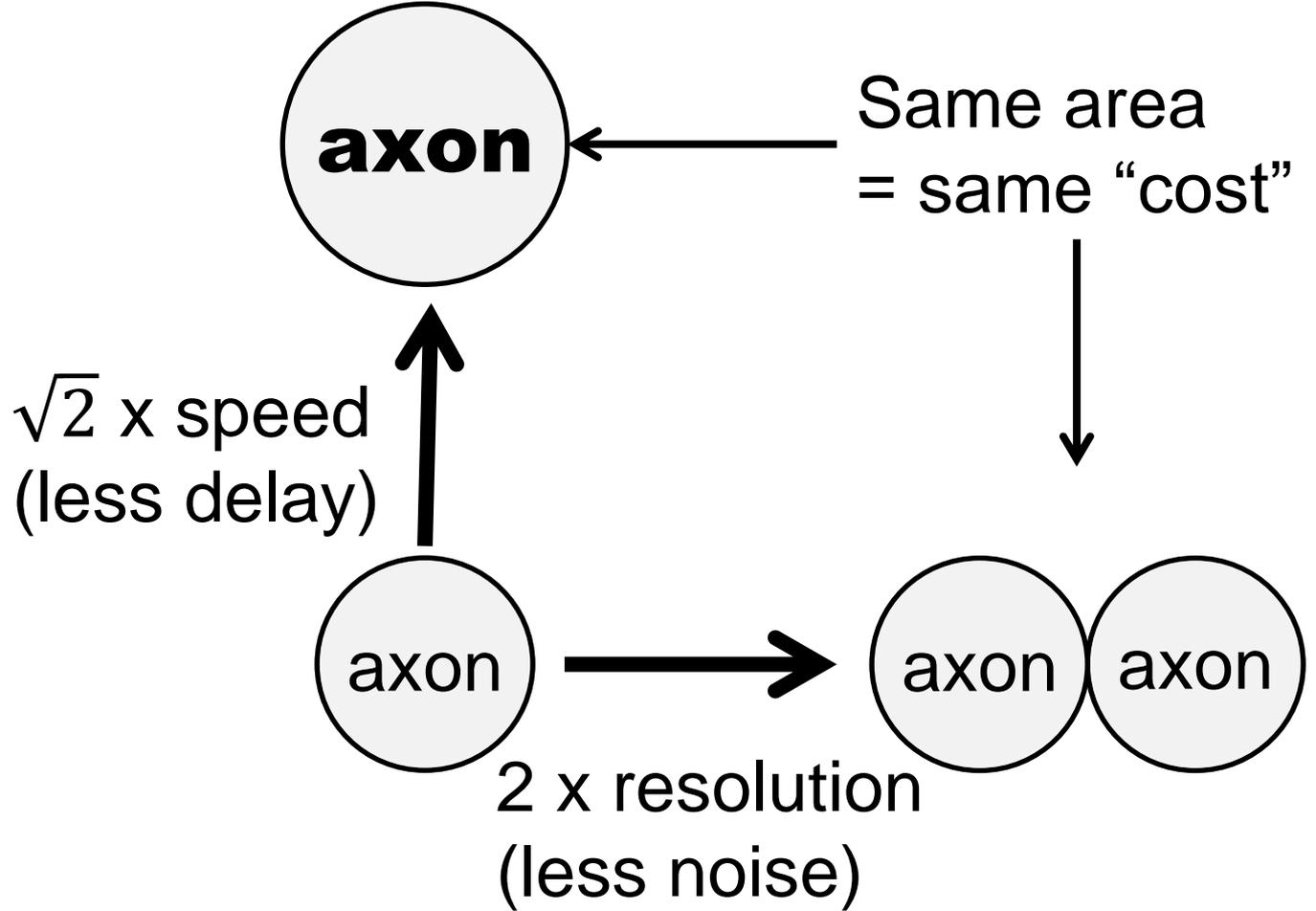
100

Resolution and bandwidth are cheap

Double the area (and cost)?

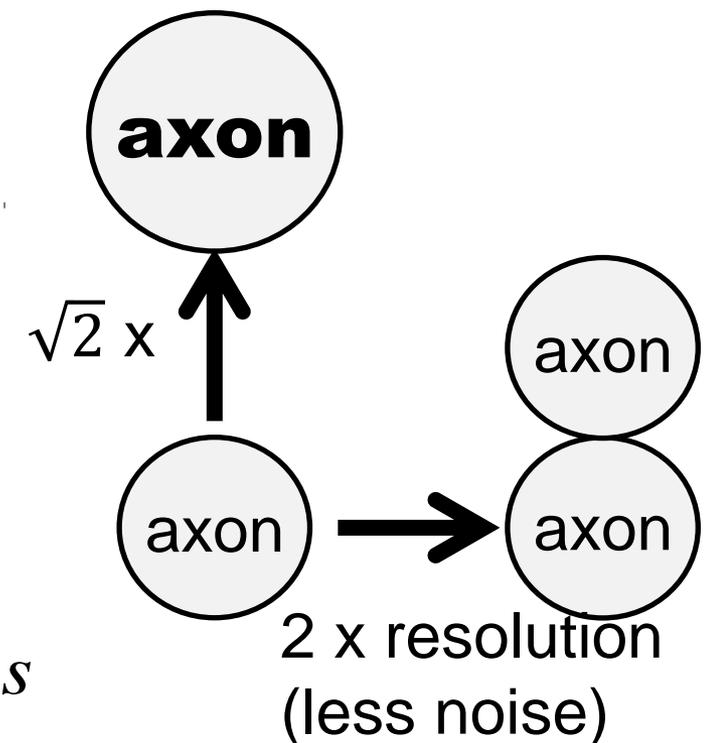
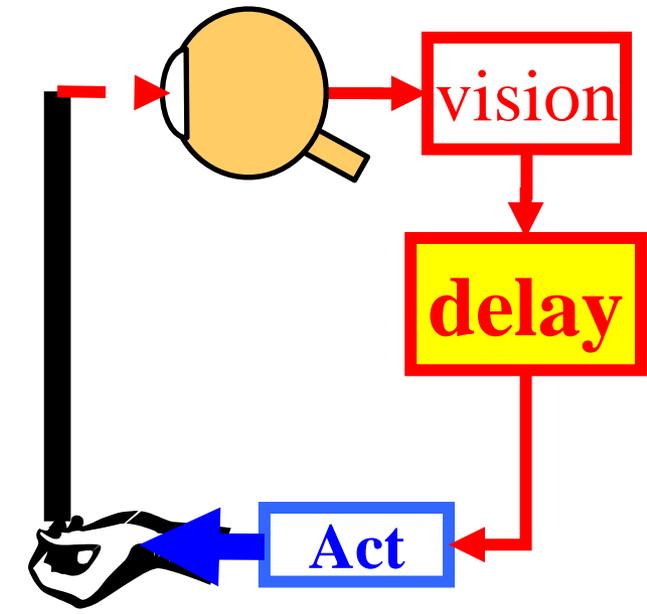
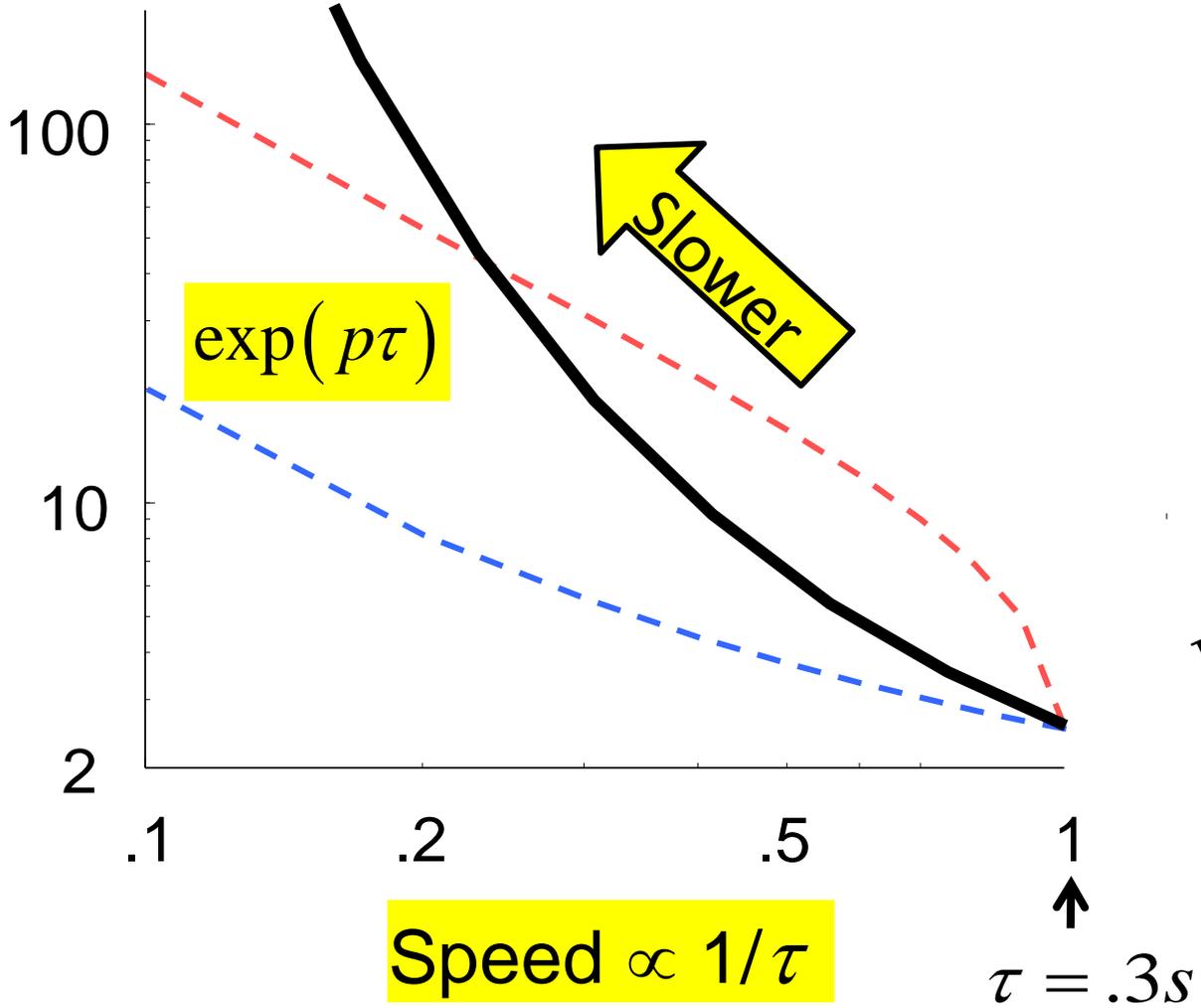
$$\text{delay} \propto \frac{1}{\text{speed}}$$

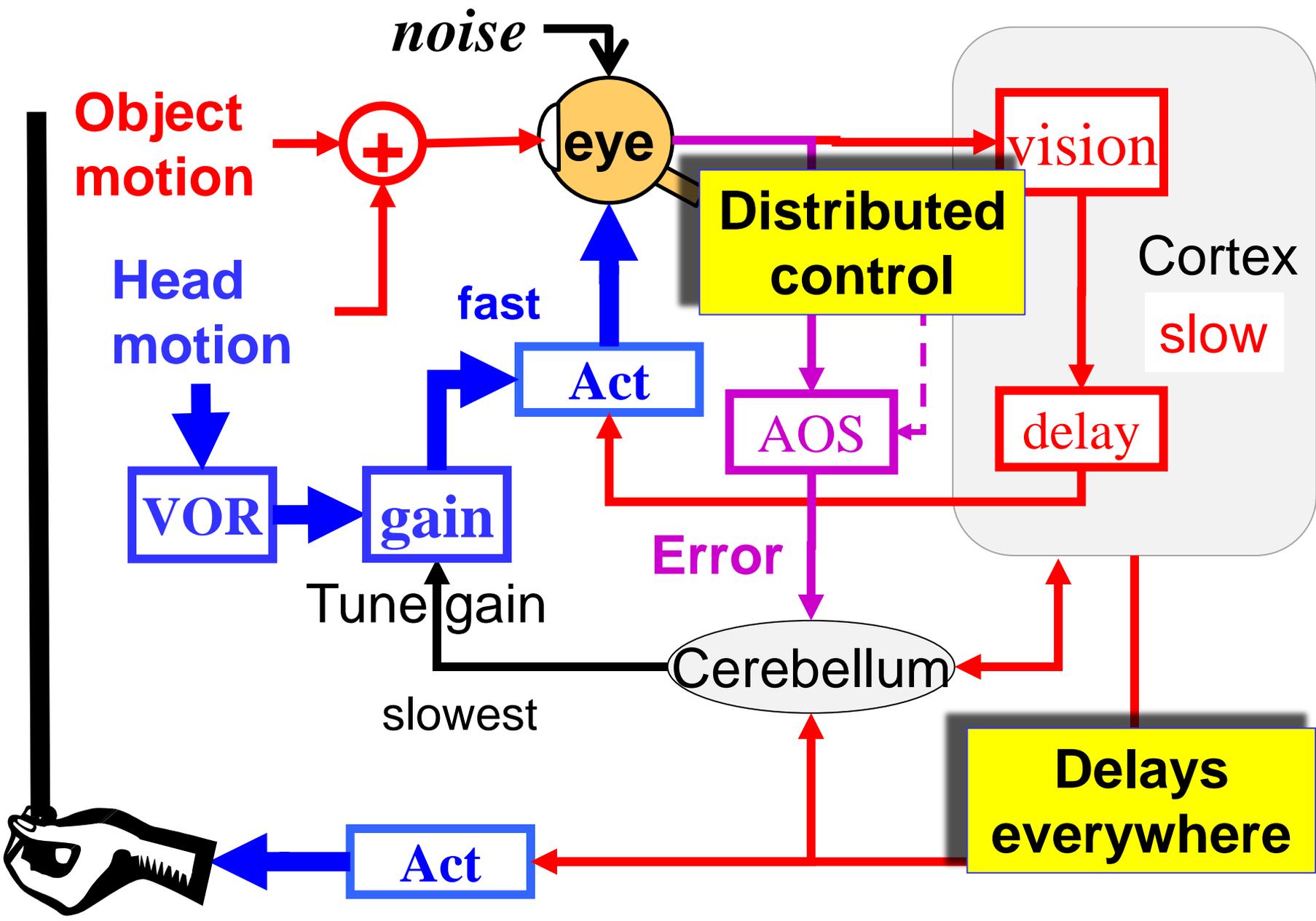
speed costs



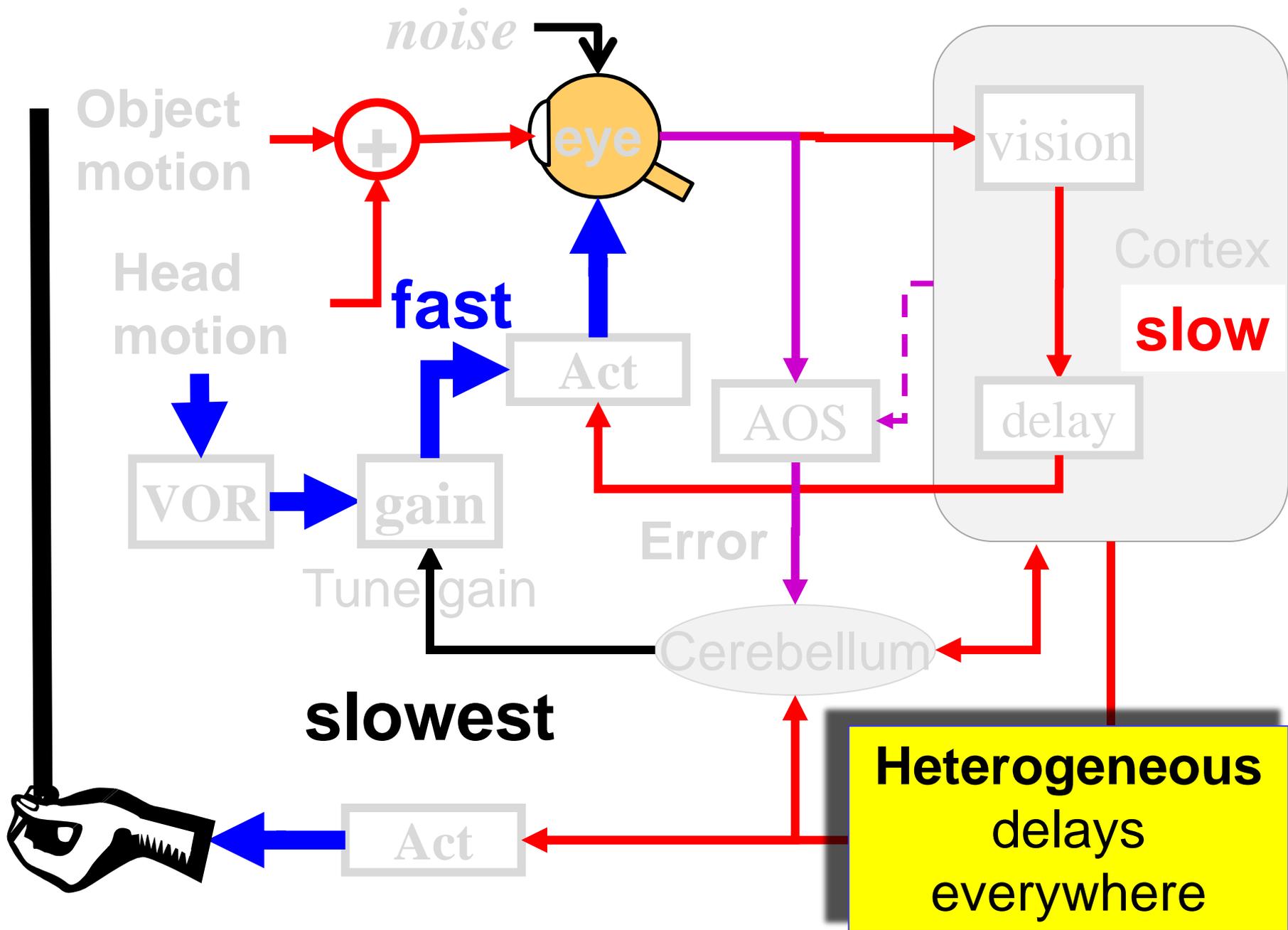
Resolution and bandwidth are cheap

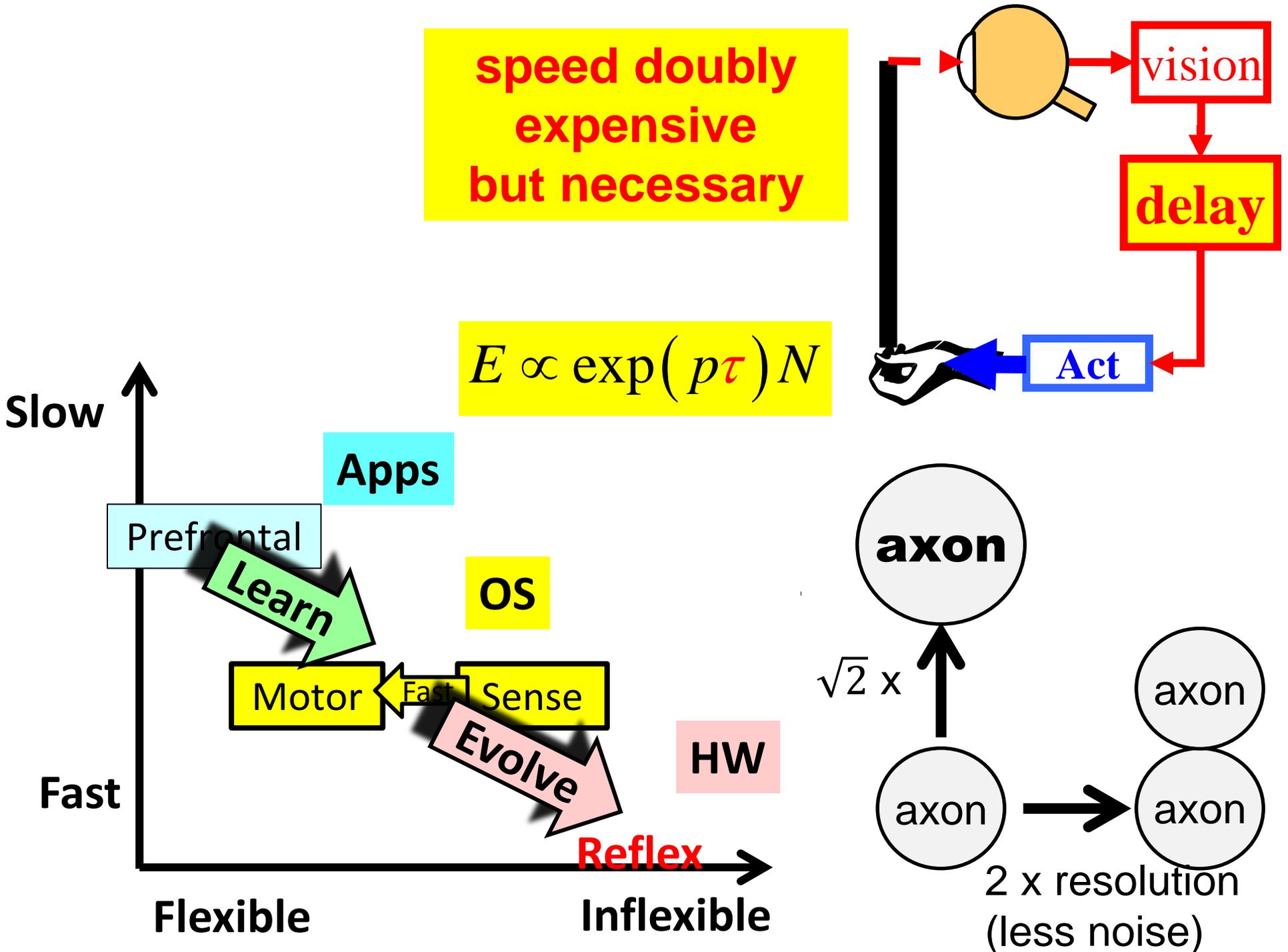
speed doubly
expensive
but necessary



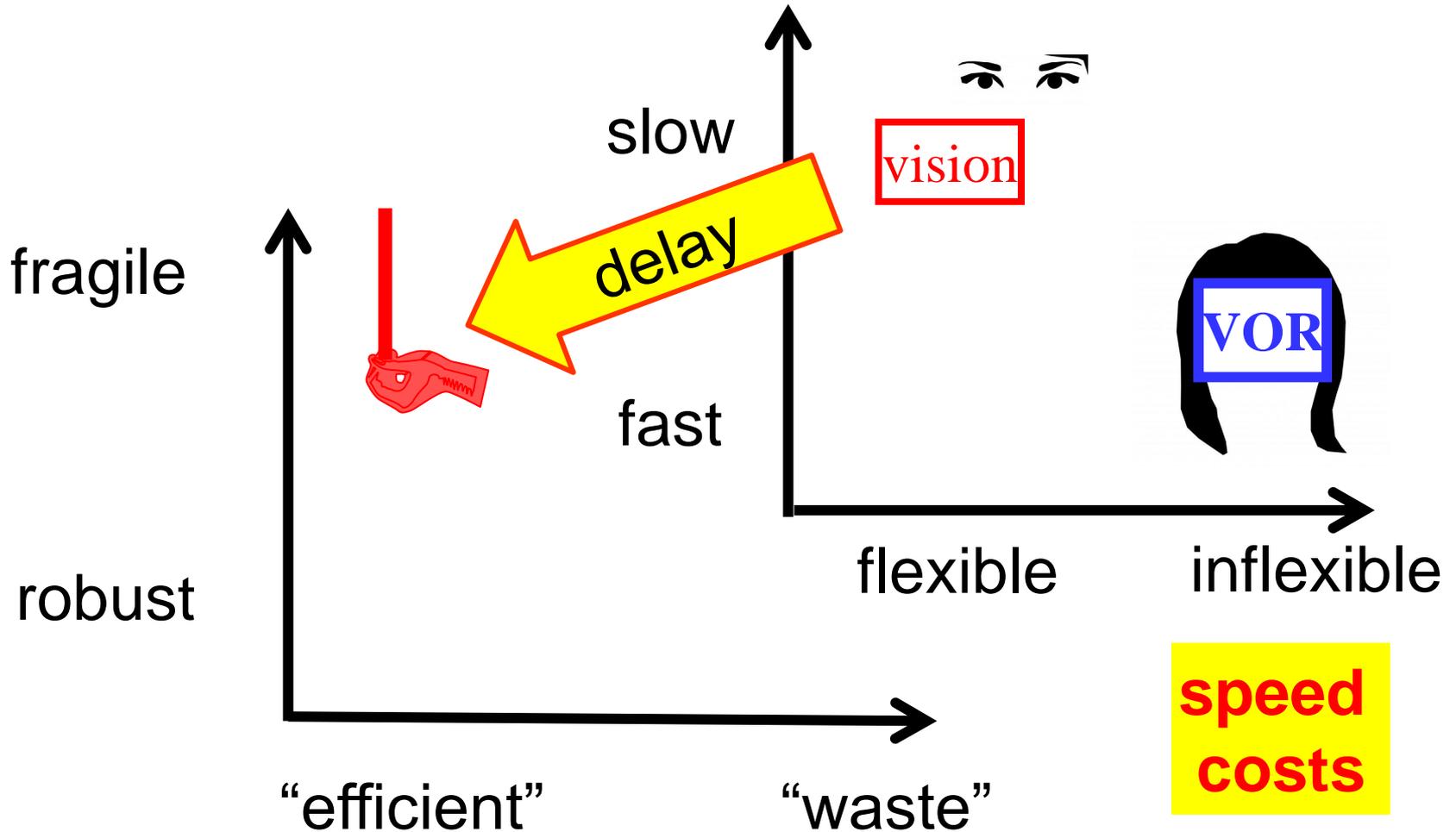


AOS = Accessory Optical system

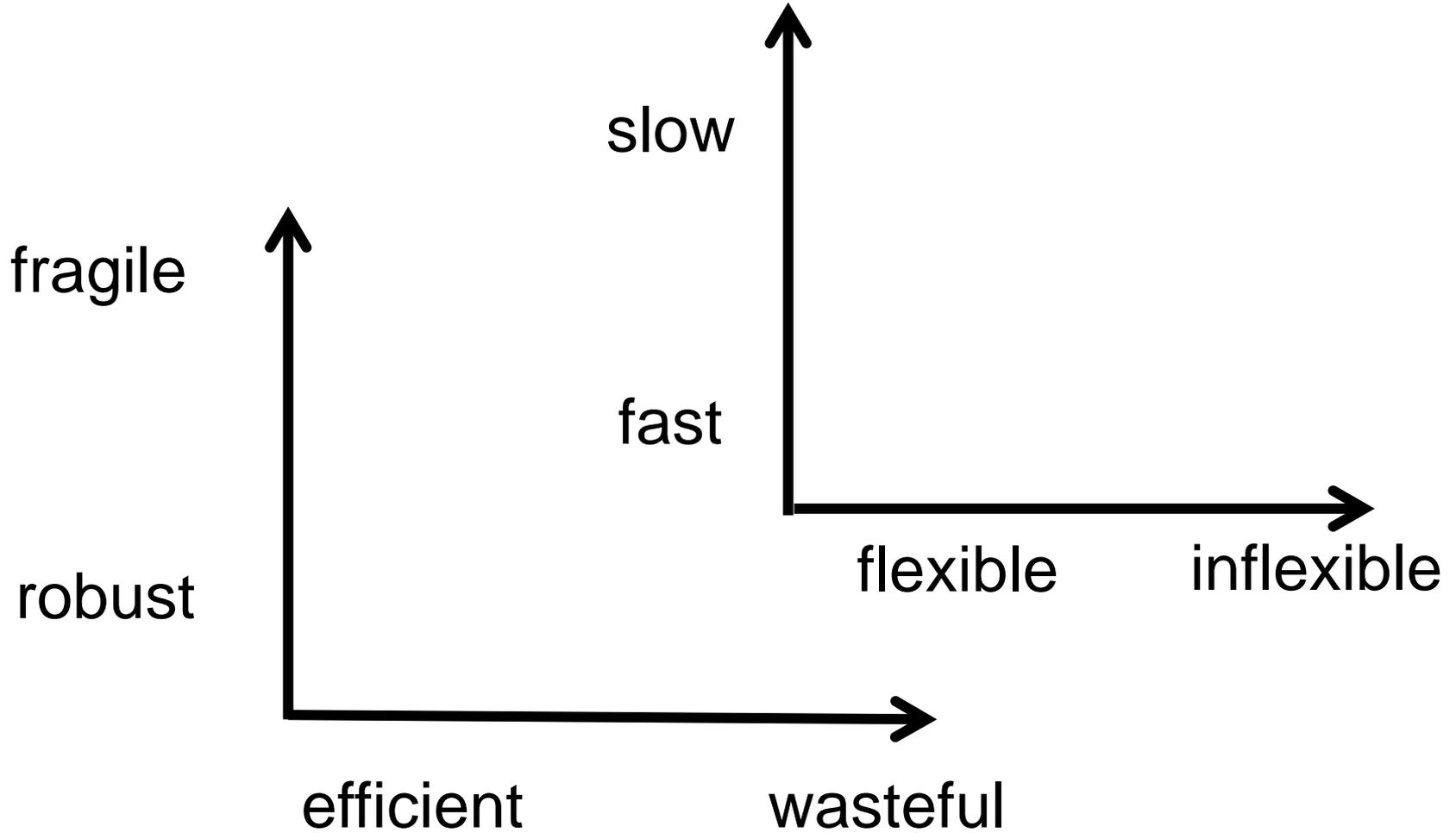


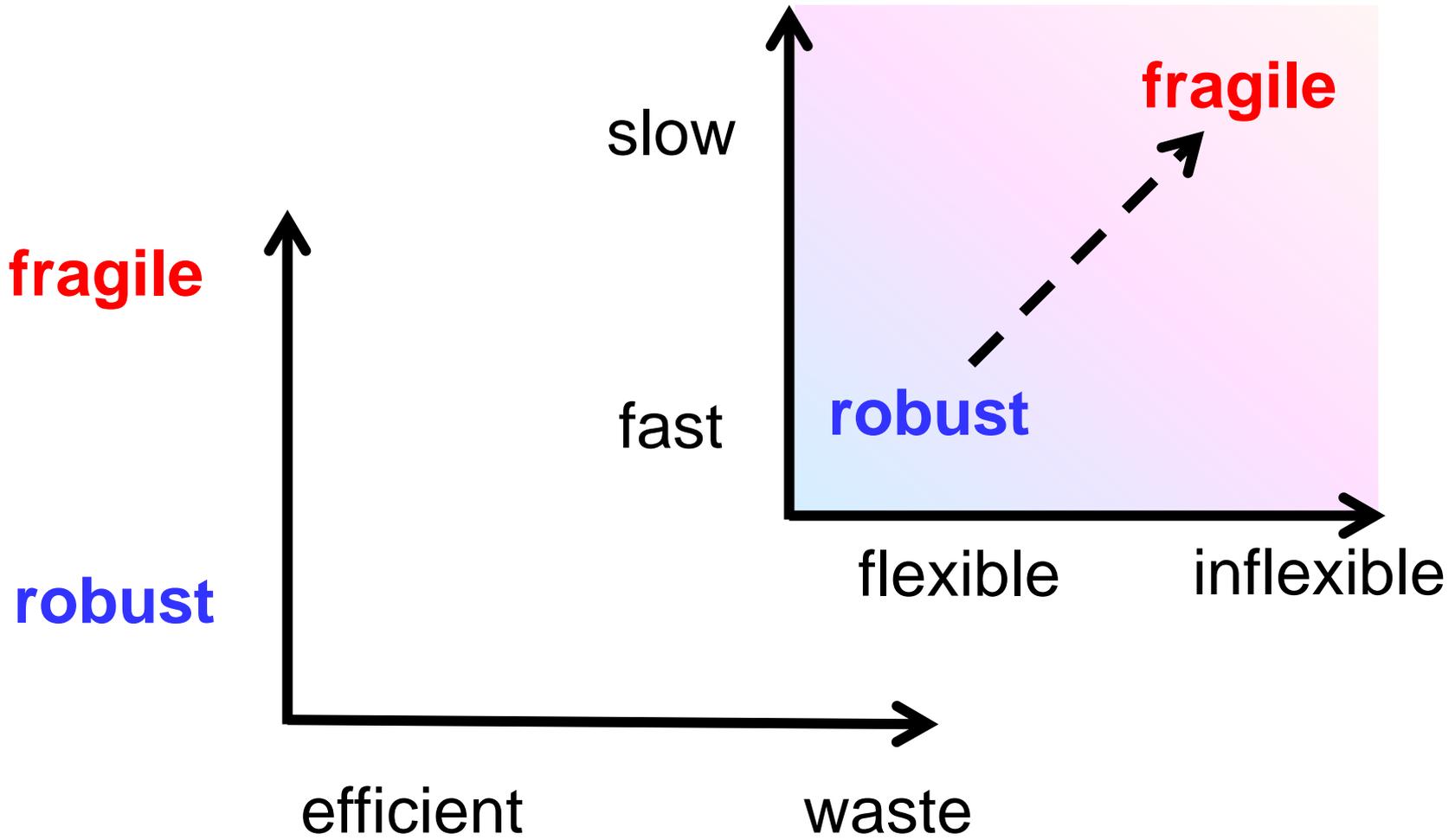


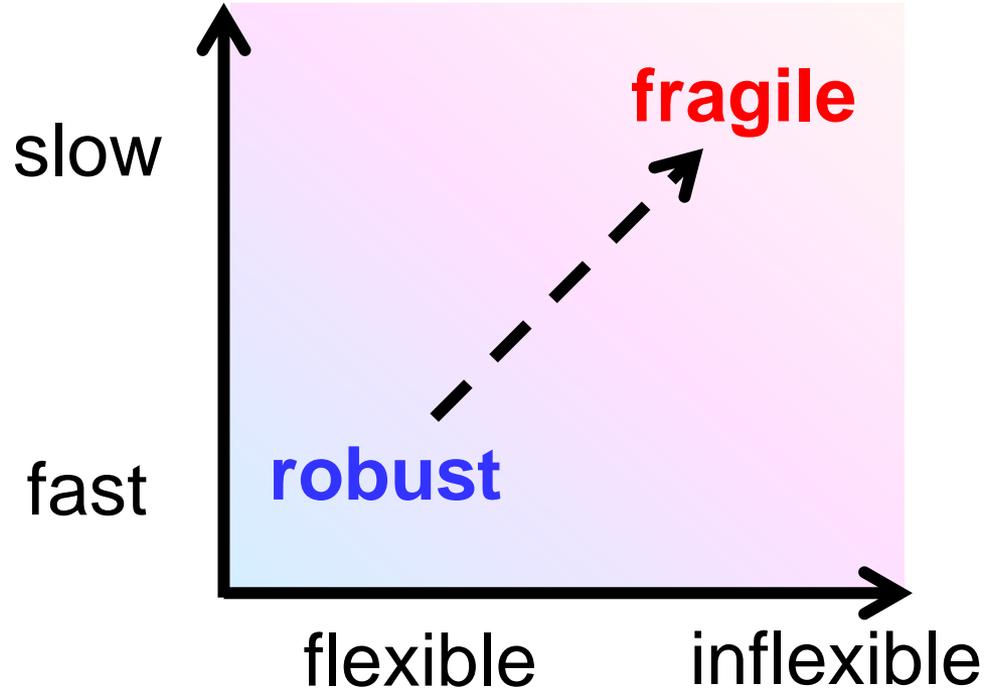
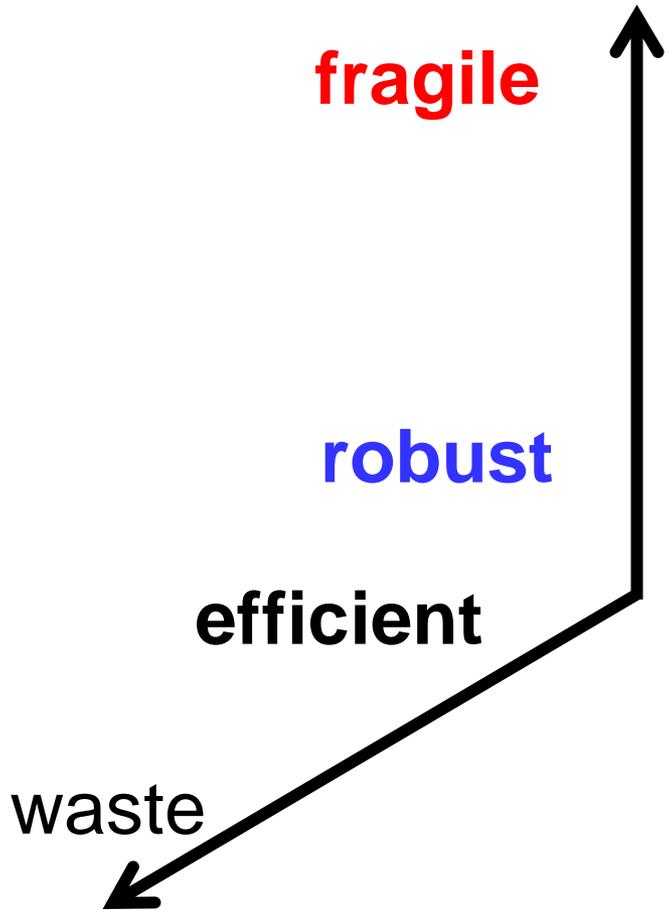
How do these fit together?

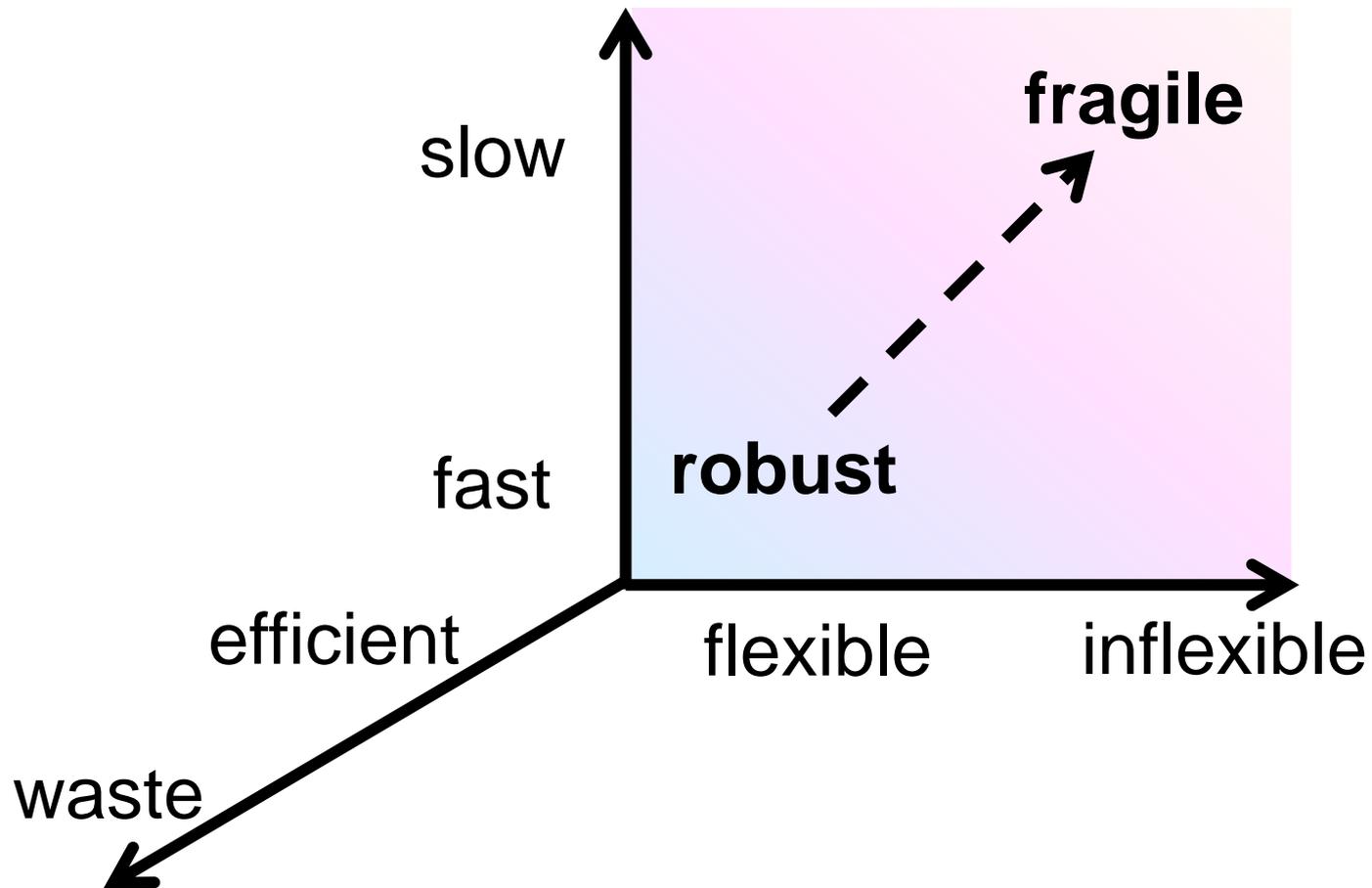


In general?









PCA ≈ Principal **Concept** Analysis ☺

accessible
accountable
accurate
adaptable
administrable
affordable
auditable
autonomy
available
compatible
composable
configurable
correctness
customizable
debugable
degradable
determinable
demonstrable

dependable
deployable
discoverable
distributable
durable

manageable
mobile
modifiable
modular
nomadic

safety
scalable
seamless
self-sustainable
serviceable

supportable
secureable
simple
able
standards
survivable

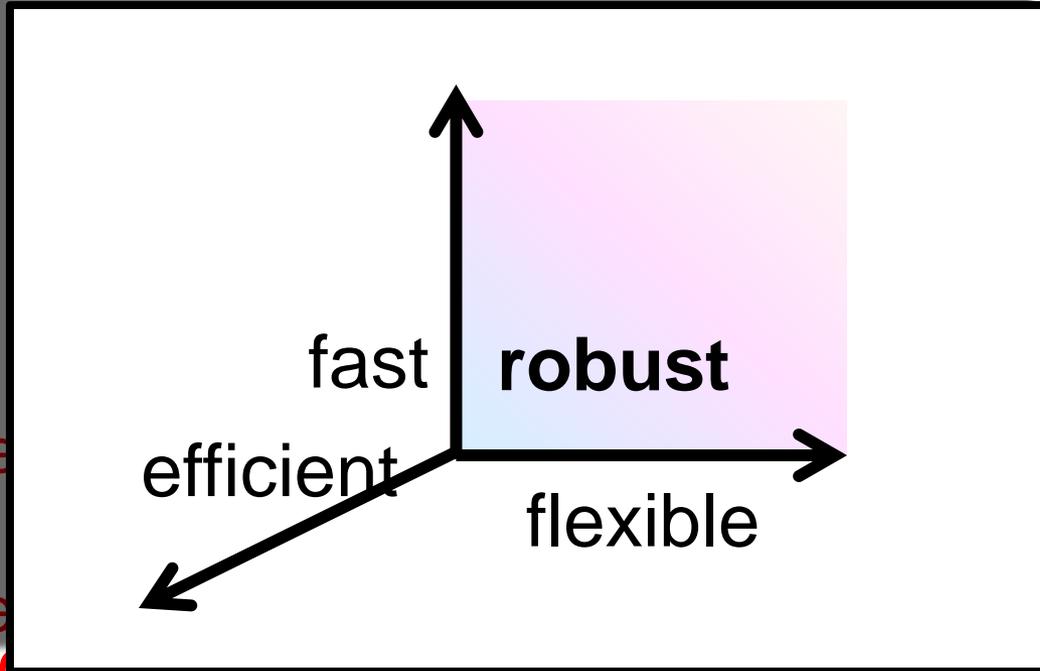
sustainable

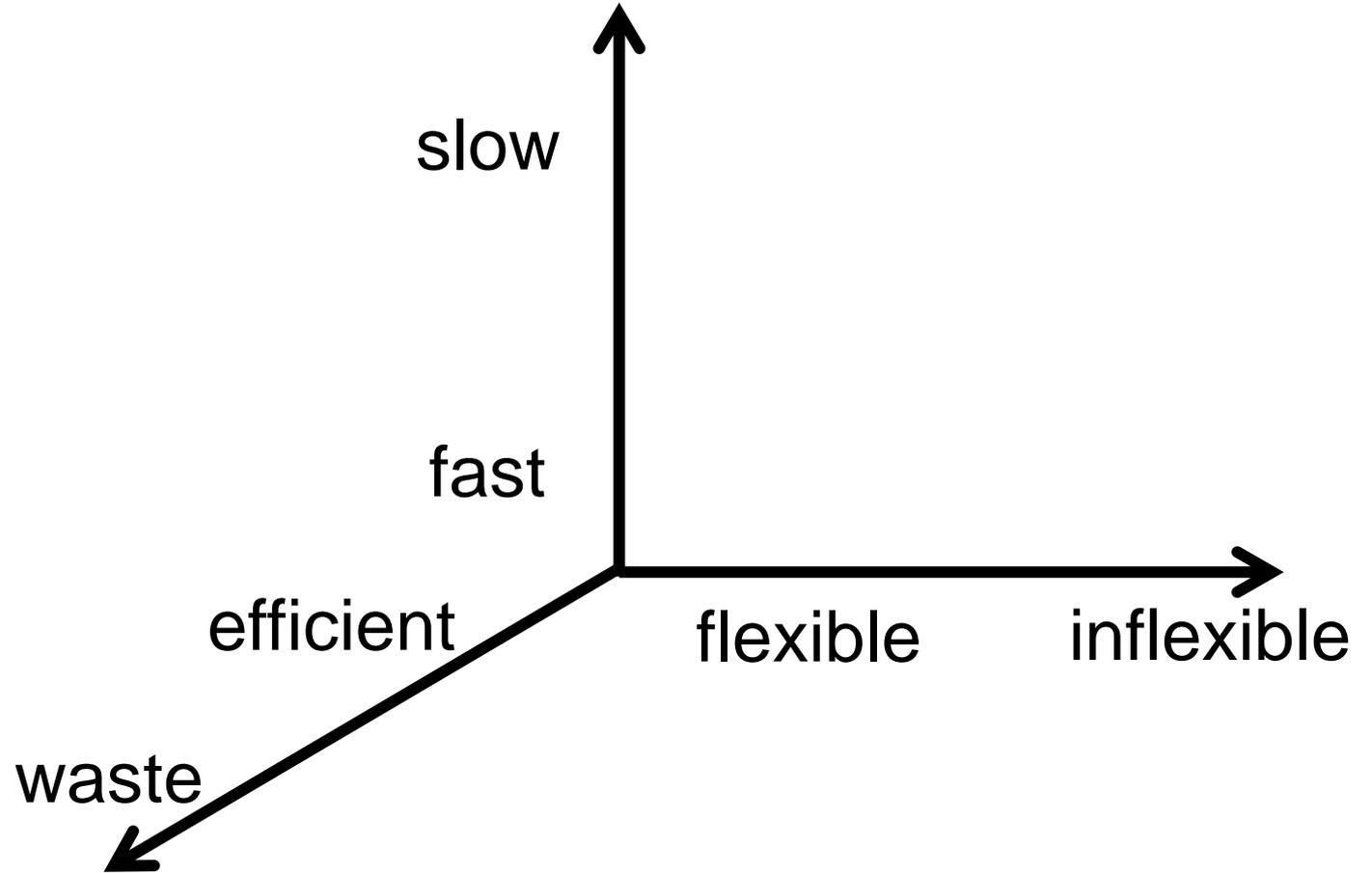
tailorable
stable
timely
traceable
 ubiquitous

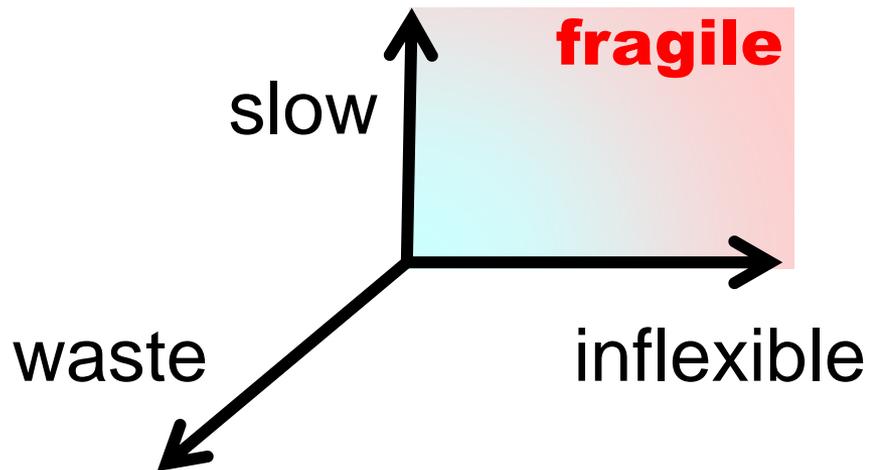
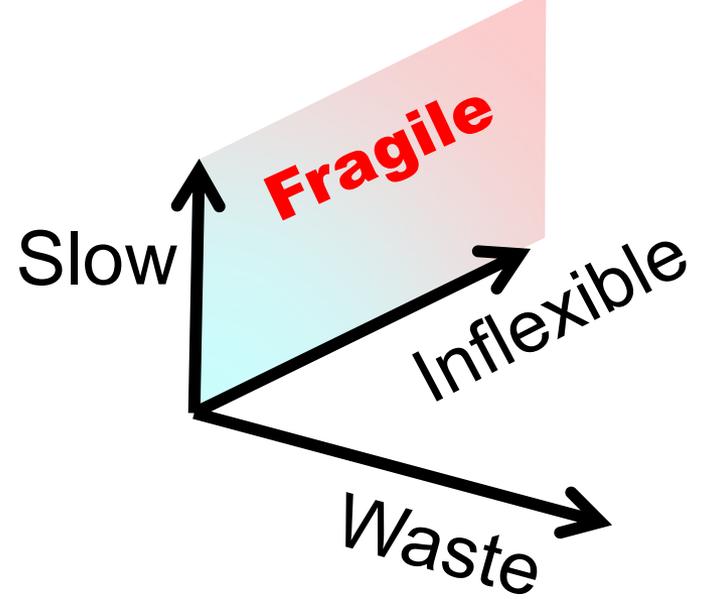
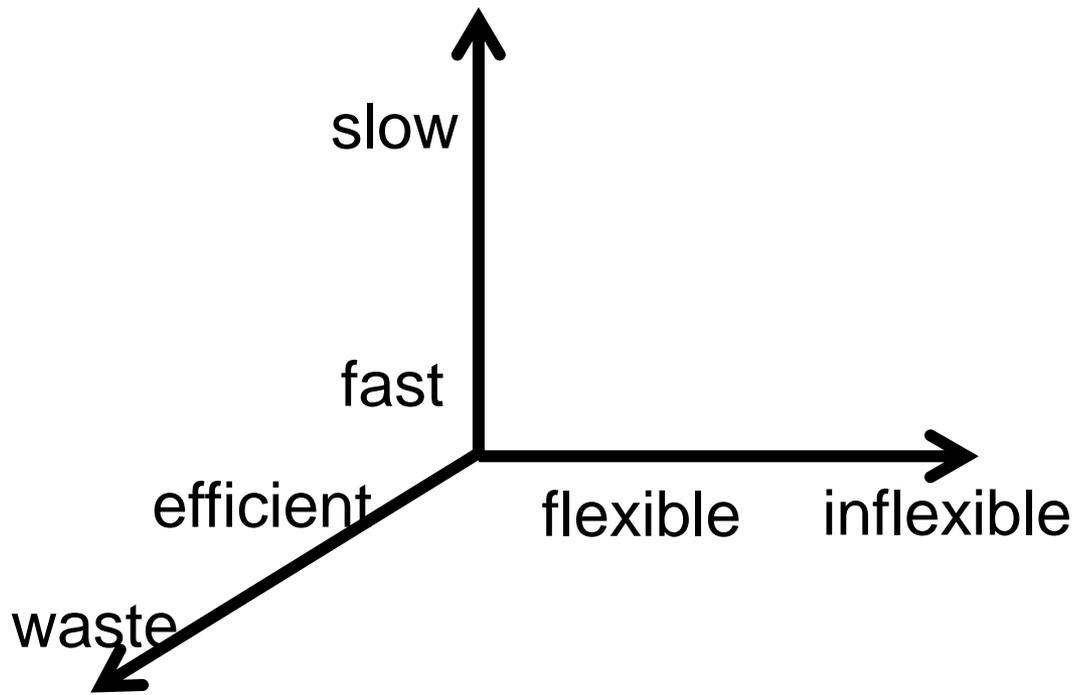
understandable
upgradable
usable

integrity
interchangeable
interoperable
learnable
maintainable

reproducible
resilient
responsive
reusable
robust







Same picture